Elementary Particle Physics: Assignment # 11 (two pages)

Due Thursday April 21

1 Assume that the vertex for the coupling of a X spin 1 boson to a pair of massless fermions f_1 and $\bar{f}_2$ is $-ig_X\gamma^\mu\frac{1}{2}(g_V - g_A\gamma^5)$. Compute the Feynman amplitude for $X \to f_1\bar{f}_2$, its square average and show that the decay width

$$\Gamma[X \to f_1\bar{f}_2] = \frac{g_X^2 M_X}{48\pi}(g_V^2 + g_A^2) \tag{1}$$

Use this to obtain the analytical expressions of $\Gamma[Z \to e^+e^-]$, $\Gamma[Z \to \nu\overline{\nu}]$, $\Gamma[Z \to hadrons]$,$\Gamma_{\rm tot}[Z]$, $\Gamma[W^+ \to e^+\nu_e]$, $\Gamma[W^+ \to \overline{h}adrons]$, $\Gamma_{\rm tot}[W]$. (neglect the mass of all fermions and remember that $m_{top} =$ 174 so it cannot be produced in the decay of the Z of the W). Evaluate them numerically using $G = 1.16 \times 10^{-5}$ GeV^{-}2 $M_W = 80.4$ GeV, $M_Z = 91.2$ GeV, $\sin^2\theta_W = 0.22$. Compare with the results in PDB and comment on the comparison.

2 Rewrite the standard model electroweak lagrangian for the 1st generation:

$$
\begin{aligned}
\mathcal{L} &= \tfrac{1}{4}W^i_{\mu\nu}W^{\mu\nu,i} - \tfrac{1}{4}B_{\mu\nu}B^{\mu\nu} \\
&+\overline{Q_L}\gamma^\mu\left(i\partial_\mu - g\tfrac{\sigma_k}{2}W^k_\mu - g'\tfrac{1}{6}B_\mu\right)Q_L \\
&+\overline{u_R}\gamma^\mu\left(i\partial_\mu - g'\tfrac{2}{3}B_\mu\right)u_R \\
&+\overline{d_R}\gamma^\mu\left(i\partial_\mu + g'\tfrac{1}{3}B_\mu\right)d_R \\
&+\overline{L_L}\gamma^\mu\left(i\partial_\mu - g\tfrac{\sigma_i}{2}W^i_\mu + g'\tfrac{1}{2}B_\mu\right)L_L + \overline{e_R}\gamma^\mu\left(i\partial_\mu + g'B_\mu\right)e_R \\
&+\left|\left(i\partial_\mu - g\tfrac{\sigma_i}{2}W^i_\mu - g'\tfrac{1}{2}B_\mu\right)\phi\right|^2 - \mu^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2 \\
&-\left(\lambda^u\overline{Q}_L(i\sigma_2)\phi^* u_R + \lambda^d\overline{Q}_L\phi d_R + \lambda^e\overline{L}_L\phi e_R + h.c.\right)
\end{aligned}
$$

after spontaneous symmetry breaking $\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}$ (in the unitary gauge) in terms of the physical states

$$
\begin{aligned}
&W^\pm_\mu = \tfrac{1}{\sqrt{2}}(W^1_\mu \mp iW^2_\mu)\,, \qquad Z^\mu = \cos\theta_w W^3_\mu - \sin\theta_w B^\mu\,, \\
&A^\mu = \sin\theta_w W^3_\mu + \cos\theta_w B^\mu\,, \qquad h \\
&e = e_L + e_R\,, \quad u = u_L + u_R\,, \quad d = d_L + d_R\,, \quad \nu
\end{aligned}
$$

In particular:

(2.1) show that if the field A^μ corresponds to the photon then $\tan\theta_w = \frac{g'}{g}$ and $\frac{1}{e^2} = \frac{1}{g^2} + \frac{1}{g'^2}$

(2.2) Obtain the expressions for the masses of all the particles and in particular show

$$M_W = \frac{1}{2} g v \qquad M_Z = \frac{1}{2\cos\theta_w} g v \qquad m_h^2 = 2\lambda v^2 \quad m_f = \frac{\lambda^f}{\sqrt{2}} v$$

(2.3) If we write the terms of the coupling of the Z^0 to any of the fermions as

$$\mathcal{L}_{Z0,f} = -\frac{g_Z}{2} \bar{f}\gamma^\mu (g_V^f + \gamma^5 g_A^f) f Z_\mu \equiv -\frac{g_Z}{4} \bar{f}\gamma^\mu (R_f(1+\gamma^5) + L_f(1-\gamma^5) f Z_\mu \qquad (2)$$

with $g_Z = g/\cos\theta_w$ obtain g_V^f, g_A^f, L_f and R_f for $f = u, d, e$ and ν.

(3.1) Show that if the SM contained several Higgs multiplets with weak isospins T_i and hypercharges Y_i whose neutral states adquire a vev v_i, then the ρ parameter

$$\rho = \frac{M_W^2}{\cos^2\theta_w M_Z^2} = \frac{\sum_i v_i^2 [T_i(T_i+1) - Y_i^2]}{2\sum_i Y_i^2 v_i^2} \qquad (3)$$

(where we have used that in our notation $Q = Y + T_3$)

(3.2) Assume that the SM contains a doublet with vev v_w and another scalar field with weak isospin T and hypercharge Y and vev v, show what bounds can be derived on the ratios of the vevs if we know that the ρ parameter has been meassured to be $0.01 \geq \rho - 1 \geq -0.03$. (For your amusement you can check Phys.Lett.B232:383,1989).