Particle Physics: Assignment # 3

Due Tuesday 2/16/16, before class

1. By direct use of the transformation properties of the fields

$$\begin{aligned}
\mathcal{P}\,\psi(x)\,\mathcal{P}^{-1} &= \alpha_P^{\psi}\,\gamma^0\,\psi(x_P)\\
\mathcal{P}\,\bar{\psi}(x)\mathcal{P}^{-1} &= \alpha_P^{\psi^*}\,\bar{\psi}(x_P)\gamma^0\\
\mathcal{C}\,\psi(x)\,\mathcal{C}^{-1} &= \alpha_C^{\psi}\,C\bar{\psi}(x)^T\\
\mathcal{C}\,\bar{\psi}(x)\mathcal{C}^{-1} &= -\alpha_C^{\psi^*}\,\psi(x)^T C^{-1}\\
\mathcal{P}\,A^\mu(x)\,\mathcal{P}^{-1} &= P^\mu_\nu\,A^\nu(x_P)\\
\mathcal{C}\,A^\mu(x)\,\mathcal{C}^{-1} &= -\,A^\mu(x)\\
\mathcal{P}\,V^\mu(x)\,\mathcal{P}^{-1} &= P^\mu_\nu\,V^\nu(x_P)\\
\mathcal{C}\,V^\mu(x)\,\mathcal{C}^{-1} &= -\,V^{\mu*}(x)
\end{aligned}$$

with $C = -C^{-1} = -i\gamma^2\gamma^0$, verify whether the following Lagrangiangs are invariant under $\mathcal{C}$, $\mathcal{P}$ and $\mathcal{CP}$

$$\mathcal{L} = -B\;\bar{\psi}_a(x)\gamma^\mu\psi_a(x)A_\mu \tag{1}$$
$$\mathcal{L} = -B\;\bar{\psi}_a(x)\gamma^\mu(1-\gamma^5)\psi_a(x)A_\mu \tag{2}$$
$$\mathcal{L} = -D\;\bar{\psi}_a(x)\gamma^\mu\psi_b(x)V_\mu + h.c. \tag{3}$$
$$\mathcal{L} = -D\;\bar{\psi}_a(x)\gamma^\mu(1-\gamma^5)\psi_b(x)V_\mu + h.c. \tag{4}$$

a and b are two types of fermions, A^μ is a real vector field, while V^μ is a complex vector field. B is real (as required by reality of the Lagrangian). D can be complex.

2. Using Wick's theorem for the interaction

$$\mathcal{L}_I(x) = -\lambda\phi(x)\bar{\psi}(x)\psi(x)$$

obtain the lowest order Feynmann amplitude for

$$f\bar{f} \to H$$

where $\bar{f}$ has momentum p_1 and helcity s_1, f has momentum p_2 and helicity s_2 and H has momentum k. Deduce the corresponding Feymman rules for incoming fermions and antifermions.