1 The lagrangian for electromagnetic interactions of an electron $\psi$ (charge $-1$ and mass $m$) and a scalar $\phi$ of charge $e_i$ and mass $m_s$ with an electric field (photon) $A$ is

$$L = \bar{\psi} (i\partial_\mu \gamma^\mu - e\gamma^\mu A_\mu - m) \psi + \left( (\partial^\mu + i e_i A^\mu) \phi \right) \left( (\partial^\mu + i e_i A^\mu) \phi^\dagger \right) - m_s^2 |\phi|^2$$

- With this Lagrangian the amplitude for $e^-(k, r) + s(p) \rightarrow e^-(k', r') + s(p')$ is

$$M = e^2 e_i \frac{q^2}{q^2} u^\dagger(r') (\not{p} + \not{p}') u(r)$$

- Obtain the unpolarized squared amplitude and the corresponding differential cross section $\frac{d\sigma}{dE'd\Omega}$ in the LAB system (where $p = (m_s, 0)$). Neglect the electron mass. As usual $E'$ and $\Omega$ are the corresponding energy and solid angle of the outgoing electron.

- With the results above obtain the differential cross section $\frac{d\sigma}{dE'd\Omega}$ for the DIS $e^- p \rightarrow e^- X$ in a parton model with partons being scalars.

- Predict the expected scaling and relations between the form factors $F_1^{ep}$ and $F_2^{ep}$ in this scalar-parton model.

2 If we define the variables $x = \frac{Q^2}{2M\nu}$ and $y = \frac{\nu}{E}$ show that in the LAB frame

$$E' = E(1 - y) , \quad \sin^2 \frac{\theta}{2} = \frac{M x y}{2E(1-y)} , \quad dE'd\Omega = 2M\pi \frac{y}{(1 - y)} dxdy$$

Write the prediction of the parton model for

$$\frac{d\sigma}{dx dy} (ep \rightarrow eX)$$

in the lab frame in terms of the “$x$” and “$y$” variables.

3 Suppose that you are looking for a heavy 4th-generation fermion $F$ with electric charge $-1$ and mass $M$ which can be pair produced in
quark-antiquark collisions $q_i\bar{q}_i \to \bar{F}F$ via electromagnetic interactions. QED predicts the fundamental cross section to be

$$
\sigma(\hat{s}) = \frac{4\pi\alpha^2}{3\hat{s}} \sqrt{1 - 4m_F^2/\hat{s}}
$$

(1)

Compute the cross section $pp \to F\bar{F}X$ in nb (nanobarns) and for $p\bar{p} \to F\bar{F}X$ for $\sqrt{s} = 7$ TeV (center of mass energy of the hadron-hadron collision) for masses $M=100, 1000$ GeV. Suppose that the up and down valence quark distribution in the proton are given by $u_v(x) = 2d_v(x) = 6(1 - x)^2$ and that all the sea are $u_s(x) = \bar{u}_s(x) = d_s(x) = \bar{d}_s(x) = (1 - x)^3/(4x)$. Neglect the contribution of the strange quark. Discuss the difference between the result in $pp$ and $p\bar{p}$ (see Hint behind)

Hint: You are going to need to evaluate first some integrals which can be done analytically. And then a second integral has to be done numerically (for example with Mathematica)

Here are the answers of the first integrals (you may also check them):

$$
I_1(\tau) = \int_\tau^1 dx \frac{1}{x^3}(1 - x)^2(x - \tau)^2 = 3(\tau^2 - 1) - (\tau^2 + 4\tau + 1)\ln(\tau)
$$

$$
I_2(\tau) = \int_\tau^1 dx \frac{1}{x^4}(1 - x)^3(x - \tau)^2 = \frac{1}{\tau} \int_\tau^1 dx \frac{1}{x^3}(1 - x)^2(x - \tau)^3 = \frac{1}{3}(10\tau^2 - 9\tau + \frac{1}{\tau} + 18) + \left(\tau^2 + 6\tau + 3\right)\ln(\tau)
$$

$$
I_3(\tau) = \int_\tau^1 dx \frac{1}{x^4}(1 - x)^3(x - \tau)^3 = \frac{11}{3}(\tau^3 - 1) + 9\tau^2 - \left(\tau^3 + 9\tau^2 + 9\tau + 1\right)\ln(\tau)
$$

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