Chapter 1

"Some basics"

1) Natural units

2) Relativistic kinematics
   - Lorentz transformations : 4-vector notation
   - Energy momentum 4-vector

3) the elementary particle content of the SM

1) Natural units

In physics 3 independent dimensionfull quantities
i.e

Quantity	Dimension	uniti in IS
Mass	$[M]$	kg
Length	$[L]$	m
Time	$[T]$	s

the dimension of any other quantity is expresed as powers of these 3.

			unit in IS
i.e	velocity	$[v] \equiv \left[\frac{L}{T}\right]$	m/s
	Energy	$[E] \equiv \left[M \frac{L^2}{T^2}\right]$	$kg \frac{m^2}{s^2} \equiv J$ (Joules)
	Action	$[S] \equiv [ET] \equiv \left[M \frac{L^2}{T}\right]$	$kg \frac{m}{s} \equiv J \cdot s$

In particle physis we use as unit of energ the

$$eV \equiv 1.602 \times 10^{-19} \ J$$

Two universal constants relevant in particle phys

- speed of light $C \overset{IS}{=} 2.999 \times 10^{8} \ m/s$
- reduced Plank constant $\hbar = 1.055 \times 10^{-34} \ J\,s$ (action)

$$= 6.582 \times 10^{-25} \underbrace{GeV}_{10^9 eV} s$$

Most particles move with velocities close to c and for quantum systems actions are $O(\hbar)$
So it is convenient to define a system of units
≡ natural units in which velocities are given in units of c and action in units of $\hbar$

⇒ In NU   Speed of light = 1
          plank reduced const = 1

In NU one can express any quantity in units of a unique dimension full quantity. We use energy and express any quantity in power of eV

Quantitatively

$$\hbar \overset{IS}{\overset{\downarrow}{=}} 6.58 \times 10^{-25} \text{GeV s} \overset{\text{in NU}}{\equiv} 1 \Rightarrow 1s = 1.52 \times 10^{15} eV^{-1} \text{ (in NU)}$$

$$\hbar c = 1.97 \times 10^{-16} \text{GeV m} = 1 \Rightarrow 1m = 5.08 \times 10^{6} eV^{-1}$$

⇒ we express time and length in eV^{-1}

$$\frac{\hbar}{c^2} \overset{IS}{\overset{\downarrow}{=}} \frac{1}{8.47 \times 10^{50}} \frac{kg}{s} \overset{\text{in NU}}{\overset{\downarrow}{\equiv}} 1 \Rightarrow 1 kg \overset{\text{in NU}}{=} 8.47 \times 10^{50} s^{-1}$$
$$= 5.57 \times 10^{35} eV$$

express mass in multiples of eV
We will use NU and do not write "c_s" nor "$\hbar_s$" in our eqs.

2) Relativistic Kinematics (Griffiths chaper 3 for details)

2.1) Lorentz transformation

Relativity principle ≡ same laws of physics apply in any inertial system (inertial ≡ moving at constant velocity) ⇒ light (em wave) must travel at same speed in any inertial system

Take 2 systems of coordinates

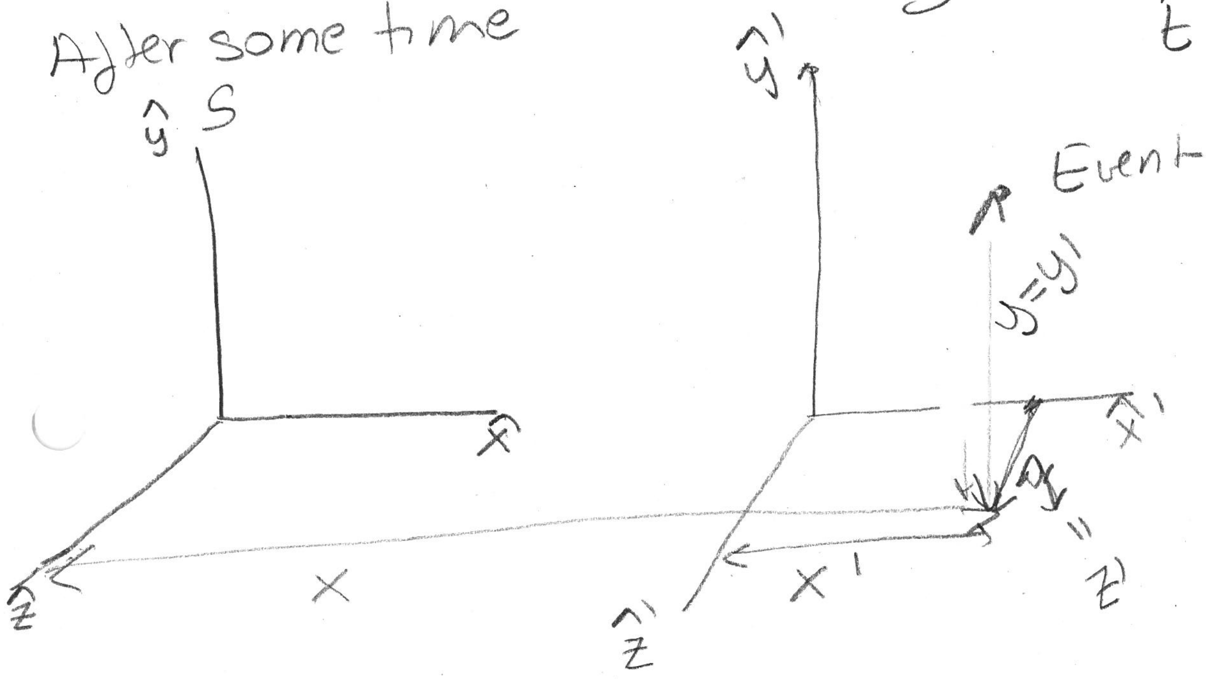
- S with spatial coordinates (x, y, z) and time coordinate t
- S' " " " (x', y', z') " " " t'

S' is moving wrt S at velocity $\vec{v} = v\,\hat{x}$

At $t = t' = 0$ $x = x' = y = y' = z = z' = 0$

After some time

an event occurs in (x, y, z) at time t at S



In S' it is seen in (x', y', z') at t'

At S' the event is obseued at x', y', z', at time t'

with

$$x' = \gamma(x - vt) \overset{NU}{\equiv} \gamma(x - \beta t)$$

$$y' = y$$

$$z' = z$$

$$t' = \gamma(t - \frac{v}{c^2} x) \equiv \gamma(t - \beta x)$$

with

$$\beta = \frac{v}{c} < 1 \quad \gamma = \frac{1}{\sqrt{1-\beta^2}} \geq 1$$

when $\beta \to 1$ $\gamma \to \infty$

$\equiv$ Lorentz transf.

or equivalently

$$x = \gamma(x' + \beta t')$$

$$y = y'$$

$$z = z'$$

$$t = \gamma(t' + \beta x')$$

Most ubiqutuous consequences in particle physics

- time dilatation: if at rest (S') a particle lives has a lifetime $\tau \equiv t'_2 - t'_1$ in a system in which it is moving with velocity β (S) it lives a time

$$t_2 - t_1 \equiv \Delta t = \gamma[\tau + \beta \underbrace{(x'_2 - x'_1)}_{0 \text{ (at rest in } S')}] = \gamma\tau > \tau$$

For ultra relativistic particles $\beta \simeq 1$ $\Delta t \gg \tau$

So in S' the particle travels a distance

$$L = \beta \Delta t = \beta \gamma \tau \gg \beta \tau$$ ← naive non relativistic estimate

- Length contraction: an object that at rest (S) measures $L = x_2 - x_1$ in a sistem in which is (S') moving at velocity β is seen (both extremes are seen at same time so $t'_1 = t'_2$) with length

$$L' = x'_2 - x'_1 \quad \text{with} \quad \begin{array}{l} x_1 = \gamma(x'_1 + \beta t'_1) \\ x_2 = \gamma(x'_2 + \beta t'_2) \end{array} \Rightarrow L = \gamma L'$$

$$\Rightarrow L' = \frac{L}{\gamma} \ll L \quad \text{contracted.}$$

So that is why the distance traveled in S

$$L = \beta \Delta t = \beta \gamma \tau$$

is seen much shorter at the rest frame of the particle (S') $L' = \dfrac{\beta \tau \gamma}{\gamma} = \beta \tau$

No time inclass

To simplify notation we introduce 4-vectors

the contravariant position-time 4 vector with

coordinates x^μ ← upper index

$\mu = 0, 1, 2, 3$

$x^0 = ct \equiv t$, $x^1 = x$, $x^2 = y$, $x^3 = z$

which we write as a column 4-vector

~~my~~ notation for 3-vector

$$\cancel{x} = \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix} \equiv \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix} \equiv \begin{pmatrix} x^0 \\ \vec{x} \end{pmatrix} \quad \text{with } \vec{x} \equiv \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

In this notation a Lorentz transformation can

be written in matrix form

$$\begin{pmatrix} x'^0 \\ x'^1 \\ x'^2 \\ x'^3 \end{pmatrix} = \underbrace{\begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{4\times 4 \text{ matrix } \Lambda} \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix}$$

which can be written as

Einstein's convention

$$x'^\mu = \sum_{\nu=0}^{3} \Lambda^\mu{}_\nu x^\nu \equiv \Lambda^\mu{}_\nu x^\nu$$

repeated upper lower index ($\equiv$ contracted) are assumed to be summed

We define ~~the~~ covariant position-time 4-vector

$$x_\mu = \sum_{\nu=0}^{3} g_{\mu\nu} x^\nu \equiv g_{\mu\nu} x^\nu$$

lower index

where $g_{\mu\nu} \equiv$ metric tenson which in matrix for

$$g \equiv \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \Rightarrow x_0 = x^0,\ x_1 = -x^1,\ x_2 = -x^2,\ x^3 = x^3$$

and we write the covariant 4-vector as line 4-vect

$$(x_0, x_1, x_2, x_3) \equiv (x_0, -x^1, -x^2, -x^3)$$

the 4-norm of the space-time 4 vector

$$(x)^2 \equiv x^{0\,2} - x^{1\,2} - x^{2\,2} - x^{3\,2} \equiv x^0 x_0 + x^1 x_1 + x^2 x_2 + x^3 x_3 = x^\mu x_\mu$$

$$= x_\mu x^\mu \equiv x'^\mu x'_\mu \quad \text{(check)}$$

So the 4 norm of the 4-vector is invariant under a Lorentz transformation

Lorent group is the group of transformations which leave the 4-norm invariant

It includes

- boost of velocity $\vec{\beta}$
- rotations in 3 dimension $\vec{X}' = R_{3\times3} \vec{X}$ (3x3 rotation matrix) $\rightarrow \Lambda_{ROT} \begin{pmatrix} 1 & \\ & R \end{pmatrix}$
- Parity $\vec{X}' = -\vec{X}$, $X^{0\prime} = X^0$ $\rightarrow \Lambda_{Par} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$
- Time revers $\vec{X}' = \vec{X}$, $X^{0\prime} = -X^0$ $\rightarrow \Lambda_{TR} = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$

We define a contravariant 4-vector "a" a 4 dimens column object

$$\begin{pmatrix} a^0 \\ a^1 \\ a^2 \\ a^3 \end{pmatrix}$$

which under Lorent transf transforms as

$$a'^{\mu} = \Lambda^{\mu}{}_{\nu} a^{\nu}$$

and the corresponding covariant 4-vecter

$$a_{\nu} = g_{\mu\nu} a^{\nu}$$

2

An the scalar product of 4-vecs "a, b" is invariant under Lorent transf.

$$a \cdot b \equiv a^0 b^0 - a^1 b^1 - a^2 b^2 - a^3 b^3$$
$$= a^0 b_0 + a^1 b_1 + a^2 b_2 + a^3 b_3 \equiv a^{\mu} b_{\mu} = a_{\mu} b^{\mu}$$

3.2 Energy - momentum 4-vector

In spatial relativity motion of a particle of mass m and velocity $\vec{\beta} = \frac{d\vec{x}}{dx^0}$ is characterized by

noto forget

- its 3 momentum $\vec{p} = m\gamma\vec{\beta} = \vec{\beta} E$
- " Energ $E = m\gamma$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \quad \beta = |\vec{\beta}| \Rightarrow \beta^2 = 1 - \frac{1}{\gamma^2}$$

$$\Rightarrow |\vec{p}|^2 = \beta^2\gamma^2 m^2 = m^2\gamma^2 - m^2 = E^2 - m^2 \Rightarrow E^2 = |\vec{p}|^2 + m^2$$

Since $\begin{pmatrix} x^0 \\ \vec{x} \end{pmatrix}$ is a 4vector $\Rightarrow \begin{pmatrix} m\gamma \\ m\vec{\beta} = m\frac{d\vec{x}}{dx^0} \end{pmatrix}$ is a 4-vector

we call it the Energy - momentum 4vect p

$$\begin{pmatrix} p^0 \equiv E = \sqrt{|\vec{p}|^2 + m^2} \\ \vec{p} \end{pmatrix}$$

Its 4-norm

$$p^2 = p^{0\,2} - |\vec{p}|^2 = E^2 - |\vec{p}|^2 = m^2$$

$\Rightarrow$ mass is invariant under Lorentz transf $\equiv$ it is the same in any inertial system.

A particle with $m=0 \Rightarrow E=|\vec{p}| \equiv \beta E \rightarrow \beta = 1$
$\Rightarrow$ moves at speed of light

Invariance under translations $\Rightarrow$ in any physical process the total Energy-momentum 4-vector is conserved.

So for any process:

N initial particles $\longrightarrow$ M final particles

with 4-momenta

$$P_i^{ini} \equiv \begin{pmatrix} E_i^{ini} = \sqrt{|\vec{p}_i^{\,ini}|^2 + m_i^2} \\ \vec{p}_i^{\,ini} \end{pmatrix} \qquad i=1 \dots N$$

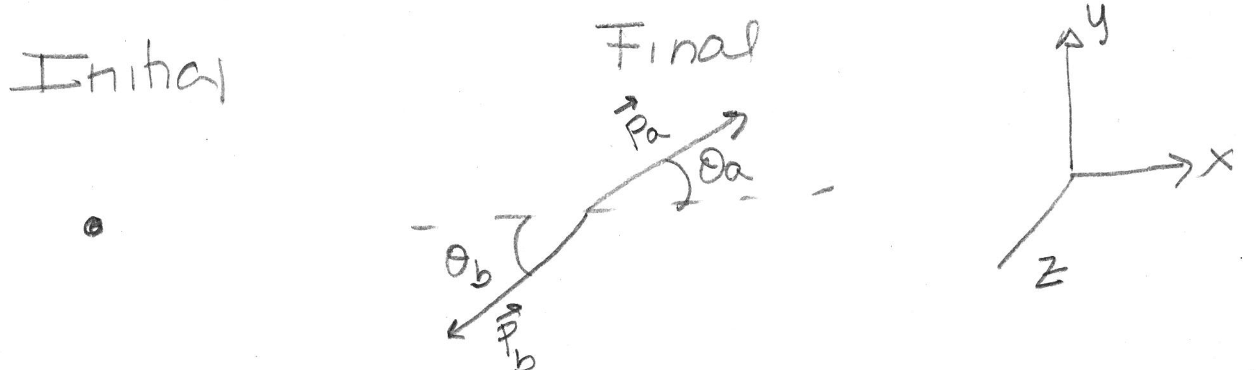
$$P_j^{fin} = \begin{pmatrix} E_j^{fin} = \sqrt{|\vec{p}_j^{\,fin}|^2 + m_j^2} \\ \vec{p}_j^{\,fin} \end{pmatrix} \qquad j=1 \dots M$$

$$P_{TOT}^{ini} = \begin{pmatrix} \sum_{i=1}^{N} E_i^{ini} \\ \sum_{i=1}^{N} \vec{p}_i^{\,ini} \end{pmatrix} = P_{TOT}^{fin} = \begin{pmatrix} \sum_{j=1}^{M} E_j^{fin} \\ \sum_{j=1}^{M} \vec{p}_j^{\,fin} \end{pmatrix} \leftarrow \text{4 equations}$$

$\Rightarrow$ If we know the energy and 3-momentum of all initial particles $\Rightarrow$ relations among energy and 3-momenta (ie their directions) of final particles $\equiv$ Kinematic constraints

Examples

1. Particle A of mass M decays at rest into two particles with mass m_a and m_b



$$P_{TOT}^{int} = \begin{pmatrix} E_A^{int} = M \\ \vec{p}_A^{\,int} = \vec{0} \end{pmatrix} \qquad P_a^{fin} = \begin{pmatrix} E_a = \sqrt{|\vec{p}_a|^2 + m_a^2} \\ \vec{p}_a \end{pmatrix} \qquad P_b^{fin} = \begin{pmatrix} E_b = \sqrt{|\vec{p}_b|^2 + m_b^2} \\ \vec{p}_b \end{pmatrix}$$

at rest

$$(1) \quad E_A = M = E_a + E_b = \sqrt{|\vec{p}_a|^2 + m_a^2} + \sqrt{|\vec{p}_b|^2 + m_b^2}$$

$$(2) \quad 0 = \vec{p}_a + \vec{p}_b \Rightarrow \vec{p}_a = -\vec{p}_b \Rightarrow a, b \text{ produced back to back}$$

$$\Rightarrow \theta_b = \pi + \theta_a \equiv \theta_{AR}$$

$$\Rightarrow |\vec{p}_a| = |\vec{p}_b| \equiv p_f$$

$$(1) \Rightarrow M = \sqrt{p_f^2 + m_a^2} + \sqrt{p_f^2 + m_b^2} \Rightarrow p_f = \frac{\sqrt{(M-(m_a+m_b)^2)(M-(m_a-m_b)^2)}}{2M}$$

Notice that if $M < (m_a + m_b) \Rightarrow$ p_f imaginary $\equiv$ decay not possible

$$\Rightarrow E_a = \sqrt{p_f^2 + m_a^2} = \frac{M^2 + m_a^2 - m_b^2}{2M}$$

$$E_b = \sqrt{p_f^2 + m_b^2} = \frac{M^2 + m_b^2 - m_a^2}{2M}$$

Lets look at same process for another frame in which A is moving at $\vec{\beta} = +\beta\hat{x}$. Lets call this frame AF (flight) (and previous AR (rest))

We can obtain $P_{a,b}^{AF}$ from $P_{a,b}^{AR}$ using that these are 4-vectors so under the Lorentz transf from AR to AF they are

$$\begin{pmatrix} E_a^{AF} \\ P_{ax}^{AF} \end{pmatrix} = \begin{pmatrix} \gamma & \gamma\beta \\ \gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} E_a^{AR} \\ \underbrace{P_{ax}^{AR}}_{P_f \cos\theta_{AR}} \end{pmatrix}$$

$$P_{ay}^{AF} = P_{ax}^{AR} = P_f \sin\theta_{AR}$$
$$P_{az}^{AF} = P_{az}^{AR} = 0$$

So $E_a^{AF} = \gamma(E_a^{AR} + \beta P_f \cos\theta_{AR})$ for b $\cos\theta_{AR} \to -\cos\theta_{AR}$, $\sin\theta_{AR} \to -\sin\theta_{AR}$

$$P_{ax}^{AF} = \gamma(P_{ax}^{AR} + \beta E_a^{AR})$$

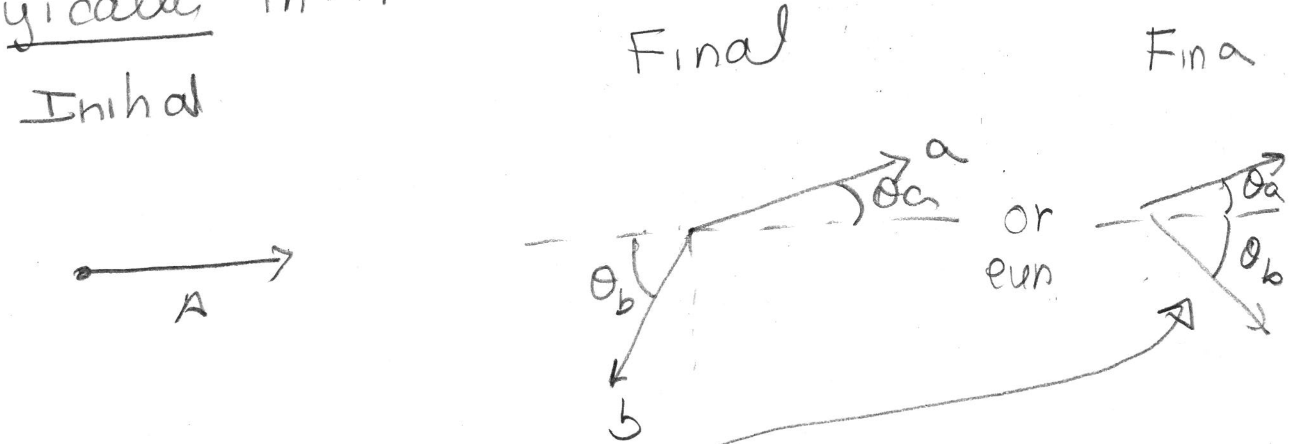
So $$\tan\theta_a^{AF} = \frac{P_{ay}^{AF}}{P_{ax}^{AF}} = \frac{P_f \sin\theta}{\gamma(P_f \cos\theta_{AR} + \beta E_a^{AR})}$$

for $m_a, m_b \ll M$

$$\tan\theta_A^{AF} \triangleq \frac{\sin\theta}{\gamma(\cos\theta + \beta)} \ll \tan\theta$$

$\Rightarrow$ emission closer to A moving direction

Graphically in AF

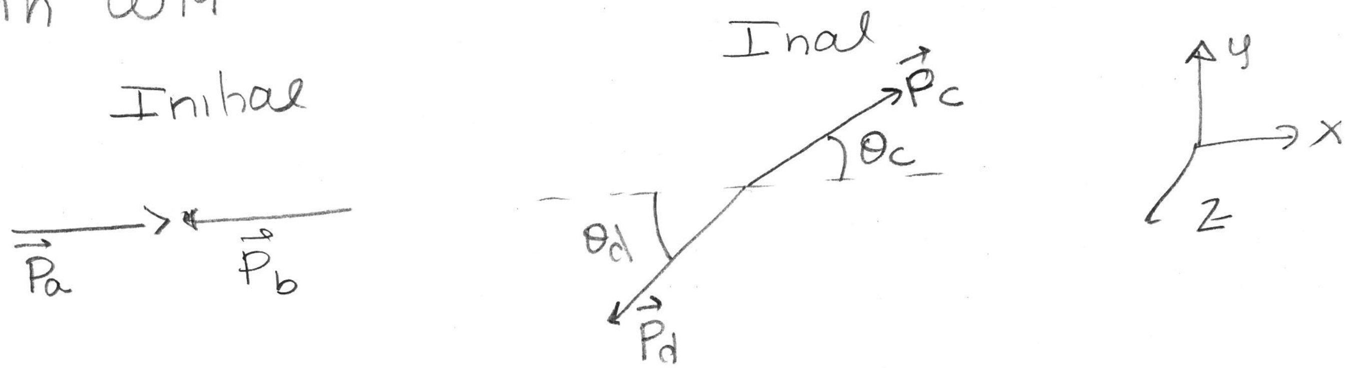


in AR

If $\cos\theta_{AR} > \dfrac{\beta E_b^{AR}}{P_f} \Rightarrow \tan\theta_b^{AF} < 0$

Example 2 : $a+b \to c+d$ (2 to 2 scattering)

- In COM



Definition of COM $\vec{p}_a + \vec{p}_b = 0 \Rightarrow |\vec{p}_a| = |\vec{p}_b| \equiv P_{in}$

conservation of energy-momentum $\Rightarrow \vec{p}_c + \vec{p}_d = \vec{p}_a + \vec{p}_b = 0$

$\Rightarrow \vec{p}_c = -\vec{p}_d \Rightarrow$ back to back

$|\vec{p}_c| = |\vec{p}|_d \equiv P_f$

Let us define the mandelstam variables

$$\underset{\text{4-vector}}{S} = (p_a + p_b)^2 = (p_c + p_d)^2 \equiv (E_a + E_b)^2 - |\vec{p}_a + \vec{p}_b|^2 = (E_c + E_d)^2 - |\vec{p}_c + \vec{p}_d|^2$$

$S \equiv$ norm of a 4-vector $\Rightarrow$ same value in any inertial ref frame

In COM $S = (E_a + E_b)^2 = (E_c + E_d)^2 \equiv$ energy available for collision

$$= \left(\sqrt{p_{in}^2 + m_a^2} + \sqrt{p_{in}^2 + m_b^2}\right) \to P_{in} = \frac{\sqrt{(S-(m_a+m_b)^2)(S-(m_a-m_b)^2)}}{2\sqrt{S}}$$

$$= \left(\sqrt{P_f^2 + m_c^2} + \sqrt{P_f^2 + m_d^2}\right) \to P_f = \frac{\sqrt{(S-(m_c+m_d)^2)(S-(m_c-m_d)^2)}}{2\sqrt{S}}$$

For $m_a, m_c, m_d, m_d \ll \sqrt{s}$ we get

$$P_{in} \simeq P_f \simeq \frac{\sqrt{s}}{2}$$

and
$$E_c = \frac{1}{2\sqrt{s}}(s - m_d^2 + m_c^2) \to \frac{\sqrt{s}}{2}$$
$$E_d = \frac{1}{2\sqrt{s}}(s - m_c^2 + m_d^2) \to \frac{\sqrt{s}}{2}$$

Same as 2-body decay of a mass $M = \sqrt{s}$

So if we want to produce some final state with mass m_{fin} [in this case $m_{fin} = (m_c + m_d)$ but it works for any number of particles] we need

(take $m_a; m_b$ ~~very~~ $\ll \sqrt{s}$)

$$\sqrt{s} \geq M_{fin} \overset{\text{in COM}}{\Longrightarrow} E_a = E_b = \frac{m_{fin}}{2}$$

(otherwise P_f is imaginary)

Lets look at process in LAB frame where "b" is at rest

$E_b^{LAB} = m_b$

$\vec{P}_b^{LAB} = 0$

In LAB $s = (E_a^{LAB} + E_b^{LAB})^2 - (\vec{p}_a^{LAB} + \vec{p}_b^{LAB})^2 = \underbrace{E_a^{L\,2} - |\vec{p}_a|^2}_{m_a^2} + m_b^2 + 2E_a^L m_b$

$$\Rightarrow s > m_{fin}^2 \Rightarrow E_a^L > \frac{1}{2m_b}(m_{fin}^2 - m_a^2 - m_b^2) \underset{m_a, m_b \ll m_{fin}}{\simeq} \frac{m_{fin}^2}{2m_b}$$

3) Elementary parhcles of the SM

Matter is composed of fermions (spin $\frac{1}{2}$ parhcles) which we classify in two groups
- quarks (which are bounded inside nucleus)
- leptos which are not

they come in 3 generahons each one with 2 type of quarks and 2 type of leptons

		1st gen	2nd Gen	3rd Gen	Q	B	L
Quarks		up (u)	charm (c)	top (t)	$2/3$	$\frac{1}{3}$	0
		down (d)	strange (s)	bottom (b)	$-1/3$	$\frac{1}{3}$	0
Leptons		electron (e^-)	muon (μ^-)	tau (τ^-)	-1	0	1
		electron neutrino (ν_e)	muon neutrino (ν_μ)	tau neutrino (ν_τ)	0	0	1

Q ≡ electric charge in units of abs. value of electron charge "e"

B ≡ baryon # quantum #

L ≡ lepton # " #

$m_t > m_c > m_u$; $m_b > m_s > m_d$; $m_\tau > m_\mu > m_e$

B and Q are conserved in Nature as far as we tested

L conservation is an open . In SM L is conserved.

Special relativity + Quant mechanics ⇒

for each parhcle ~~there~~ must be an antiparhcle with same mass and spin but opposite Q, B, L

For example for 1st generation

		Q	L	B
antiquark	$\bar{u}$	$-2/3$	0	$-1/3$
"	$\bar{d}$	$1/3$	0	$-1/3$
positron	e^+	1	-1	0
electron antineutr	$\bar{\nu}$	0	-1	0

Quarks are never observed as free states. they are always in bound states of 2 types

- Mesons $\equiv (q\bar{q}') \Rightarrow B=0$ and spin is integer
  i.e $\pi^\pm, \pi^0$ (pions), $K^\pm K^0$ (kaons)

- Baryons $\equiv (qq'q'') \Rightarrow B=\frac{1}{2}$ and spin is half-integer

Interactions between these fermions are mediated by spin 1 particles $\equiv$ gauge bosons
($\equiv$ vector particles)

In SM there are 3 type of interactions

Interaction	Gauge boson	Q
electromagnetic	photon (γ)	0
strong	gluon (g)	0
weak	Z boson (Z)	0
	W " ($W^\pm$)	± 1

all have $B=L=0$

In addition the model contains a particle of spin zero (scalar) $\equiv$ Higgs boson (h)

In quantum mechanics we describe the state and evolution of a system by its wave function.

In relativistic quantum mechanics $\equiv$ Quantum Field theory the wave functions are substituted by fields which are operators acting on the Hilbert space of states.

the form and properties of the field representing a particle depends on its spin $\Longrightarrow$ chapter 2