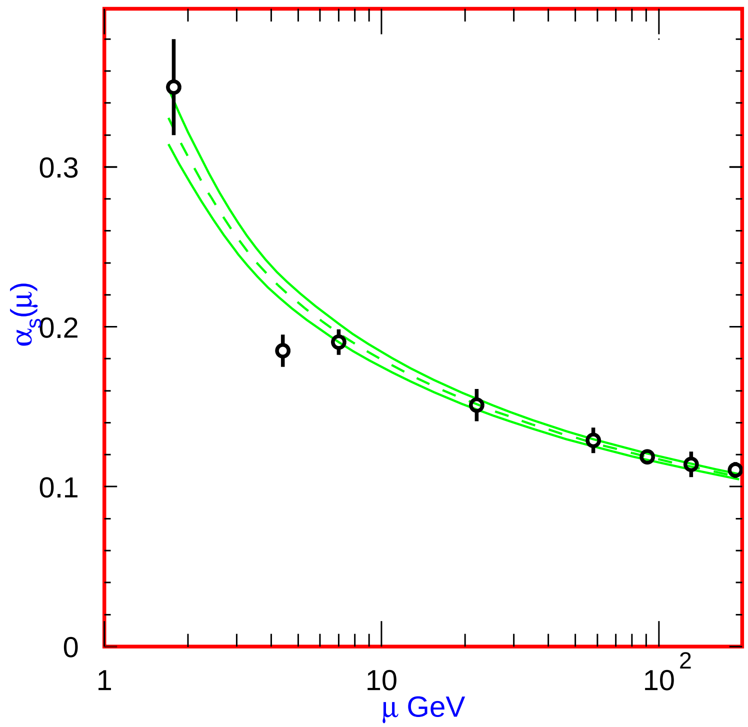
QCD prediction

$$\alpha_s(q^2) = \frac{\alpha_s(q_0^2)}{1 + \frac{\alpha_s(q_0^2)}{12\pi}(33 - 2N_f)\ln\left(\frac{q^2}{q_0^2}\right)}$$

Comparing to data



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  $\Rightarrow$ Energetically favorable to create a $q\bar{q}$ and bind them to the original quarks to form hadrons

  $\Rightarrow$ quarks cannot be observed in isolation

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  Ultimately it was the reason why QCD was accepted as the theory of strong interactions which unified the hadronic low energy physics with the very high-energy strong interaction effects.

6.4 Some tests of QCD: $e^+e^- \to jets$

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The cleanest data to do this comparison is to use data on

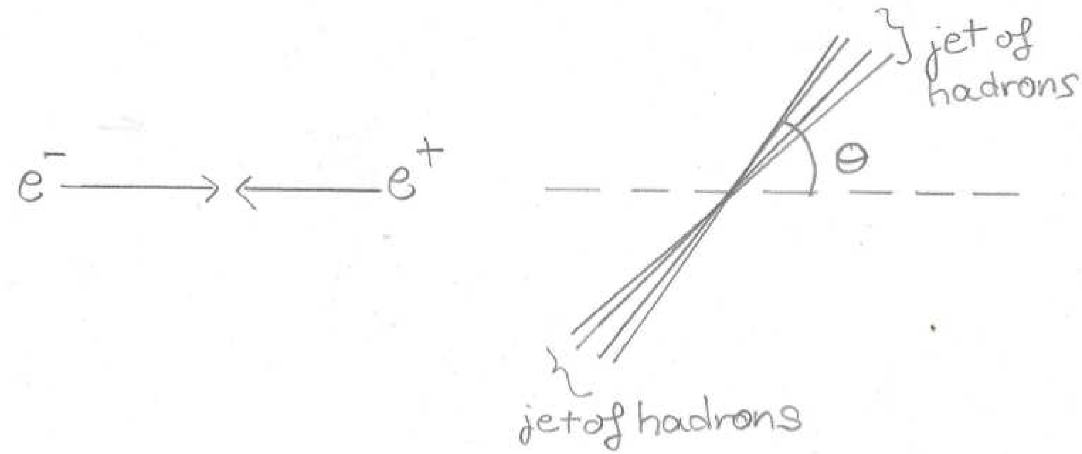
$$e^+e^- \rightarrow \text{ hadrons} \qquad \text{at large} \qquad s = (p_e^+ + p_e^-)^2$$

and see if it can be understood in terms of

$$e^+e^- \rightarrow \text{ quarks and gluons}$$

For example we can look at the process with two hadron jets in the final state. In COM

Initial Final



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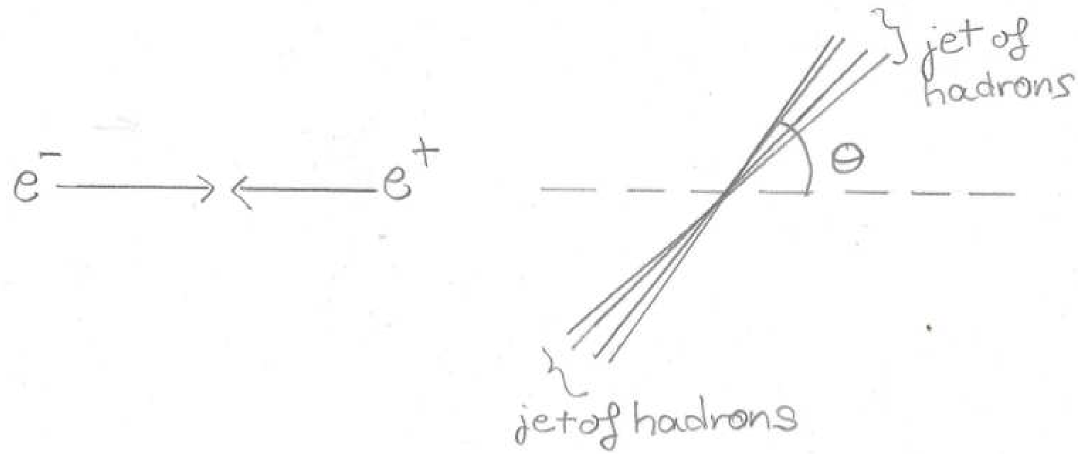

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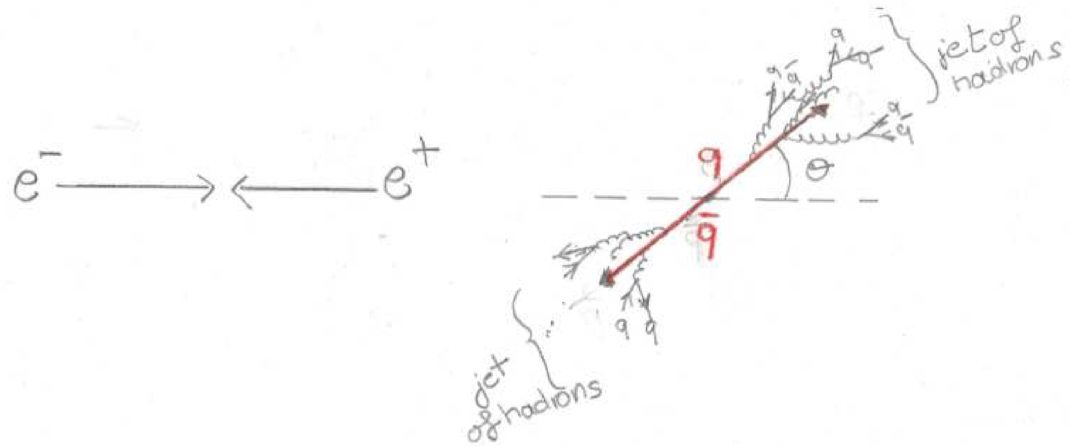

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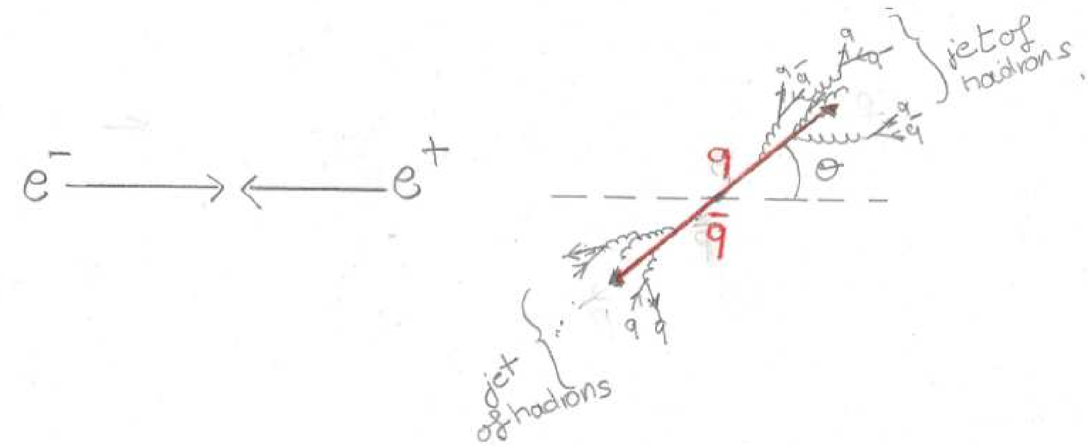

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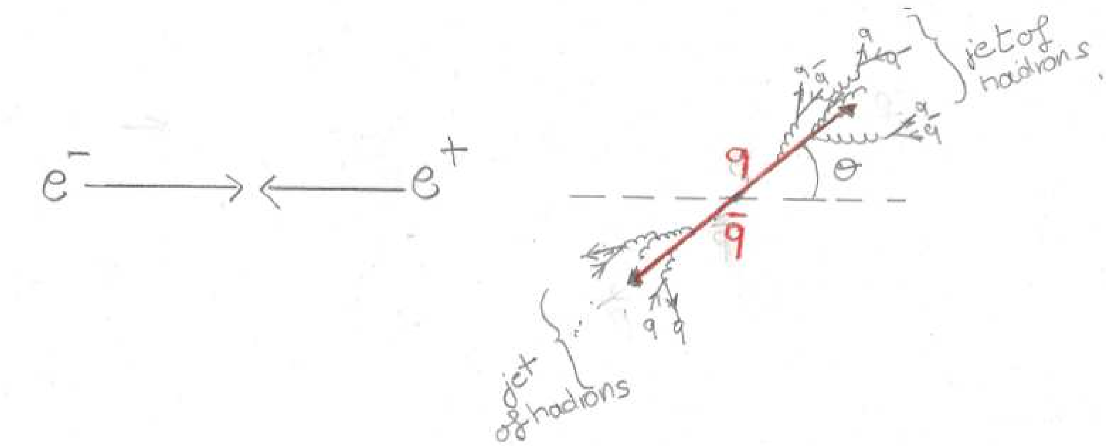
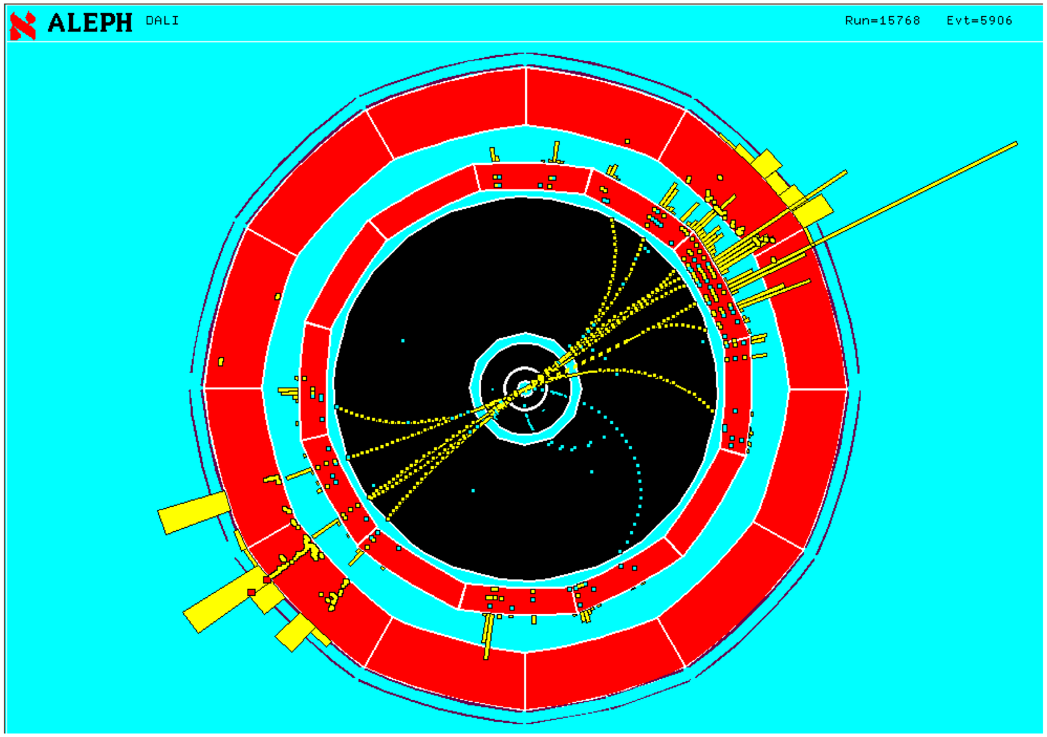


In the experiment



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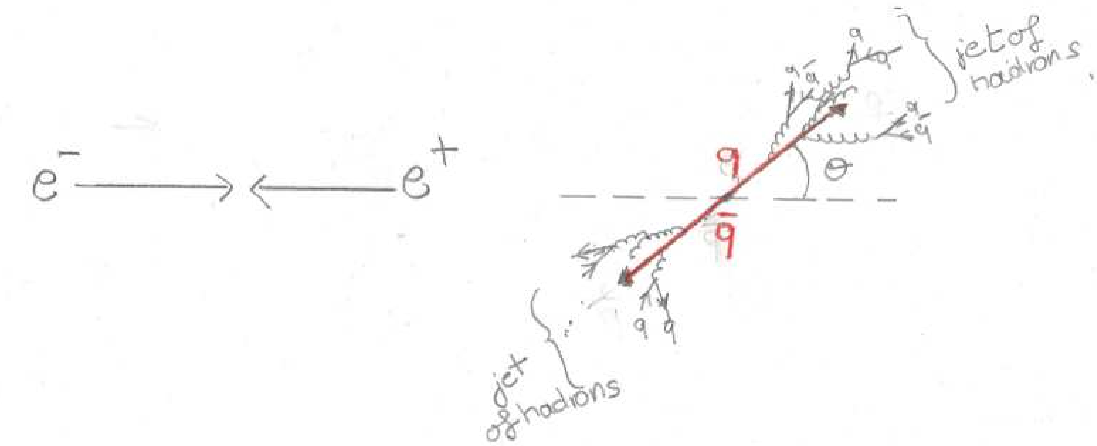

So we predict

$$\frac{d\sigma}{d\Omega}(e^+\,e^- \to 2\ \text{jets})\bigg|_{\text{COM}} = \sum_q \frac{d\sigma}{d\Omega}(e^+\,e^- \to q\,\bar{q})\bigg|_{\text{COM}} \times \text{Prob}(\text{final } q \to \text{jet}) \times \text{Prob}(\text{final } \bar{q} \to \text{jet})$$

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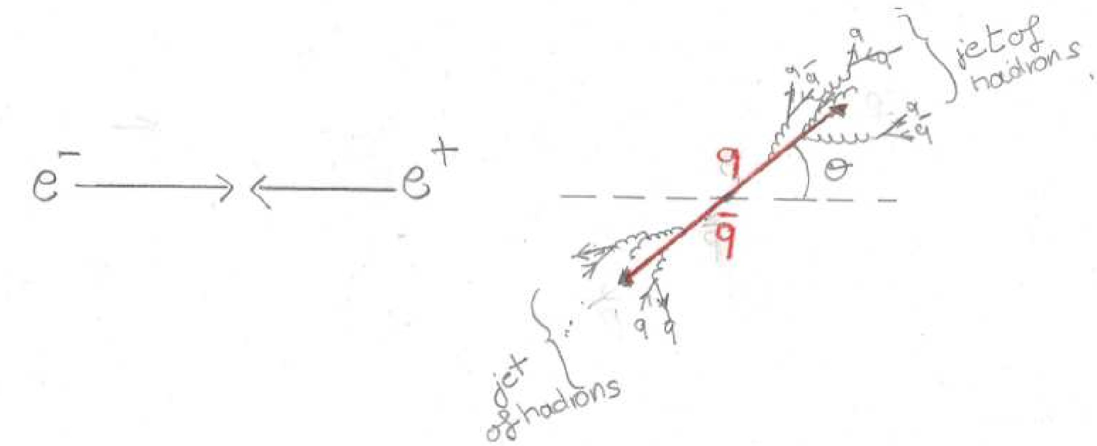

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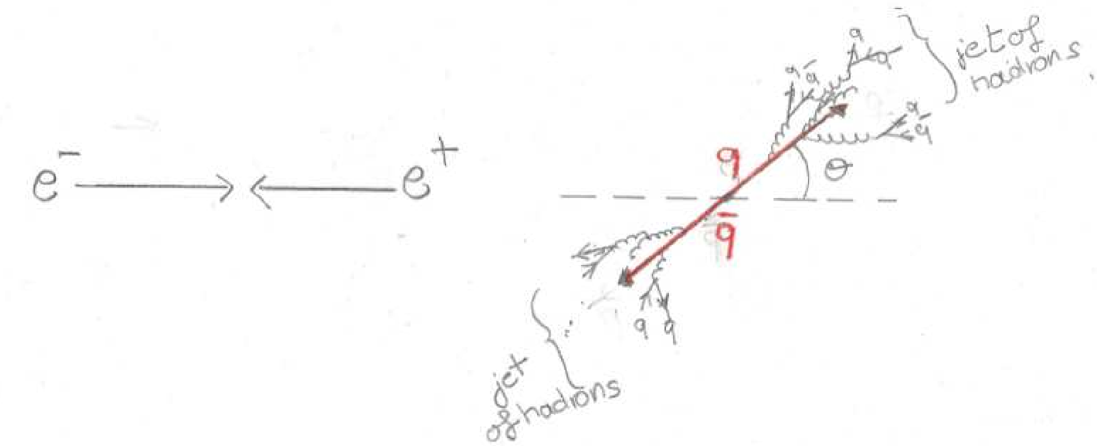

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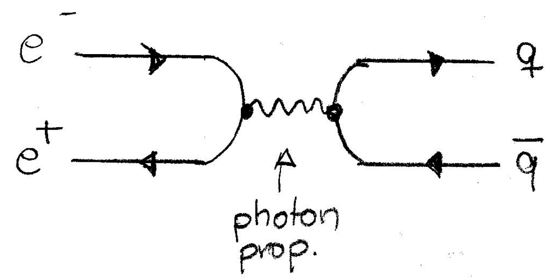
We compute the cross section for $e^+ e^- \to q\,\bar{q}$ from lowest order diagram in QED



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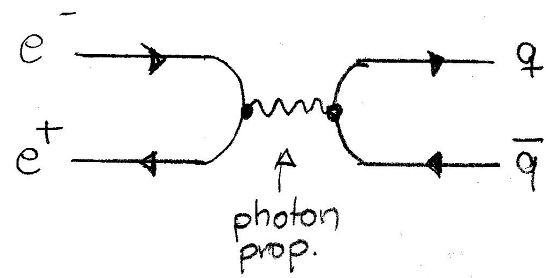
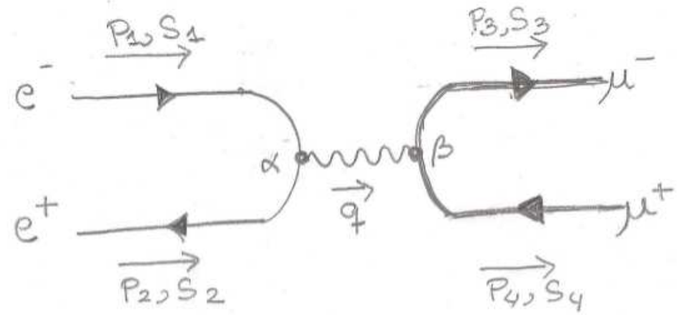


If quarks are fermions that amplitude is totally analogous to the one for



$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{COM}} = \frac{\alpha^2}{4\,s}\left[1 + \cos^2\theta\right]$$

up to the number of colors and the charge of the quarks.

So

$$\frac{d\sigma}{d\Omega}(e^+ e^- \to 2 \text{ jets})\Big|_{\text{COM}} = \sum_q \frac{d\sigma}{d\Omega}(e^+ e^- \to q\,\bar{q})\Big|_{\text{COM}} = 3 \times \sum_q Q_q^2 \frac{d\sigma}{d\Omega}(e^+ e^- \to \mu^+ \mu^-)\Big|_{\text{COM}}$$

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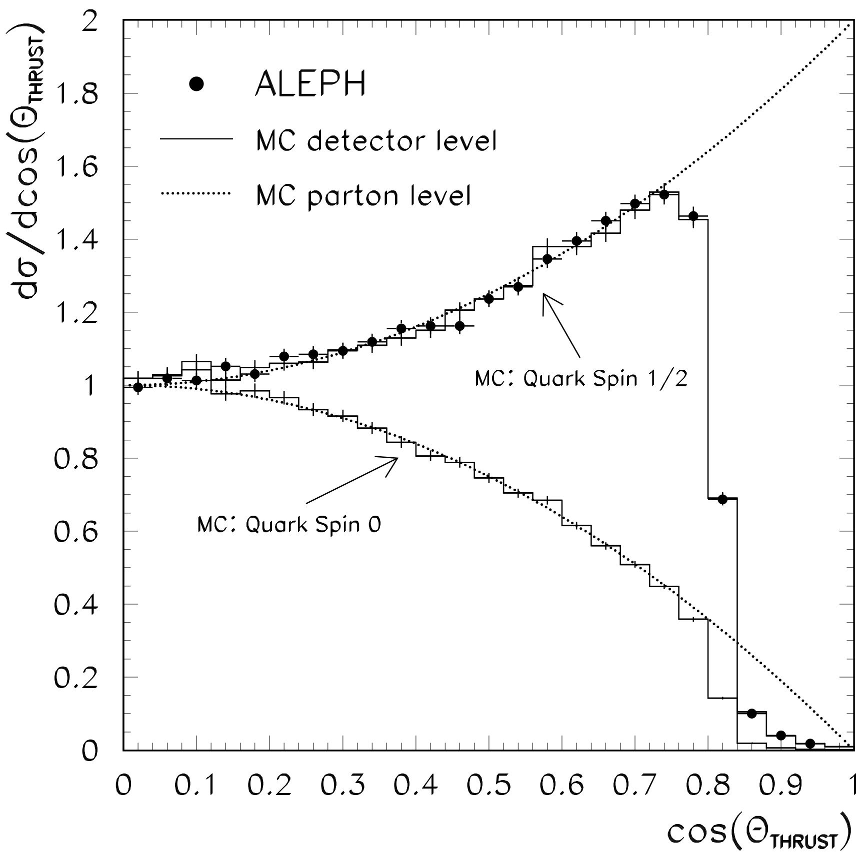
Comparing with data from ALEPH experiment



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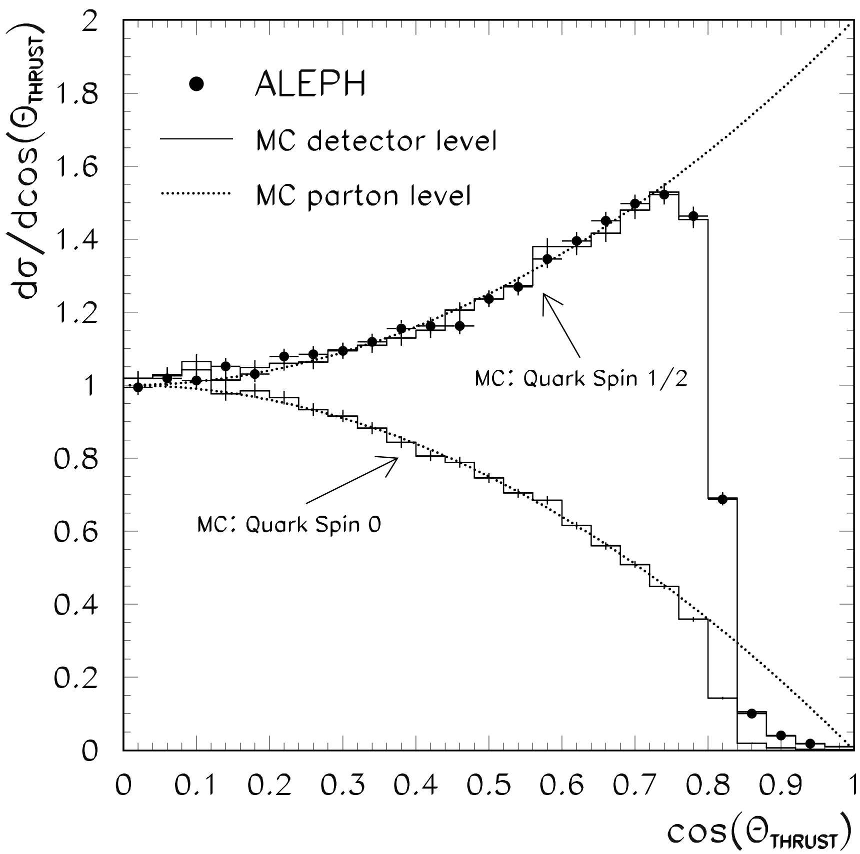


$\Theta_{\text{Thrust}} \equiv \theta_{\text{jet axis}}$

This confirms

– the picture of asymptotic free quarks

– that quarks are spin 1/2 particles like the muons

if quarks had spin 0

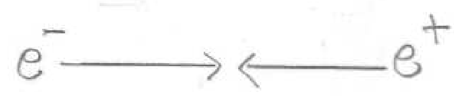
$\Rightarrow$ the amplitude would be proportional to $\sin\theta$

$\Rightarrow \frac{d\sigma}{d\Omega}(e^+ e^- \to 2 \text{ jets})\Big|_{\text{COM}} \propto \sin^2\theta = 1 - \cos^2\theta$

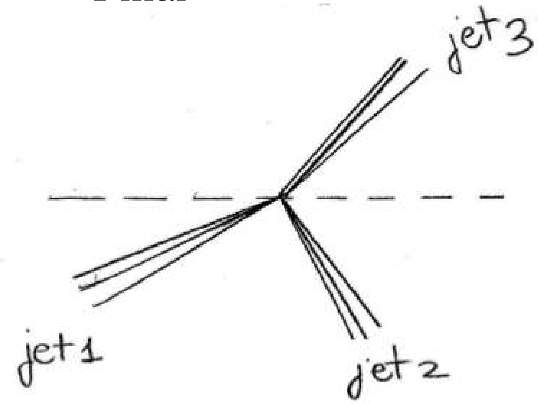
(dash-line)

Next we can look at the process with thee hadron jets in the final state. In COM

Initial



Final



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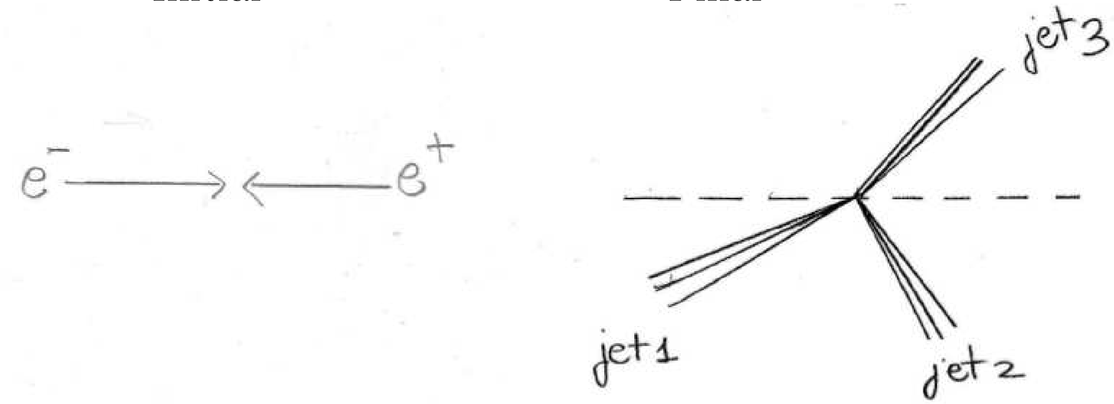

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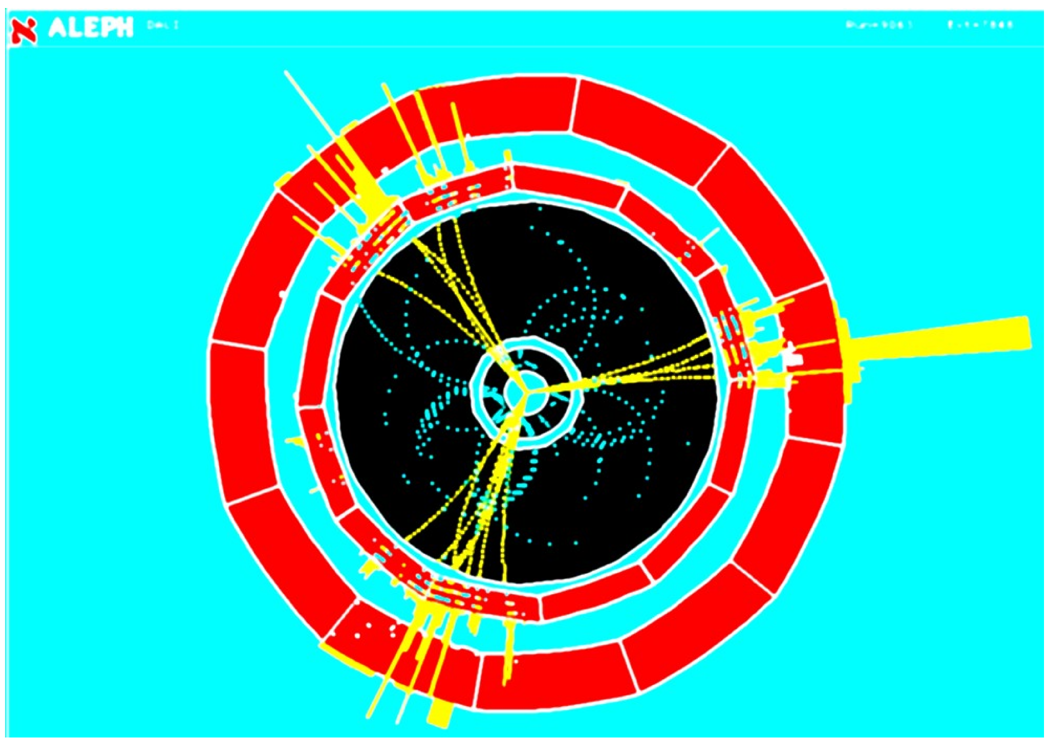

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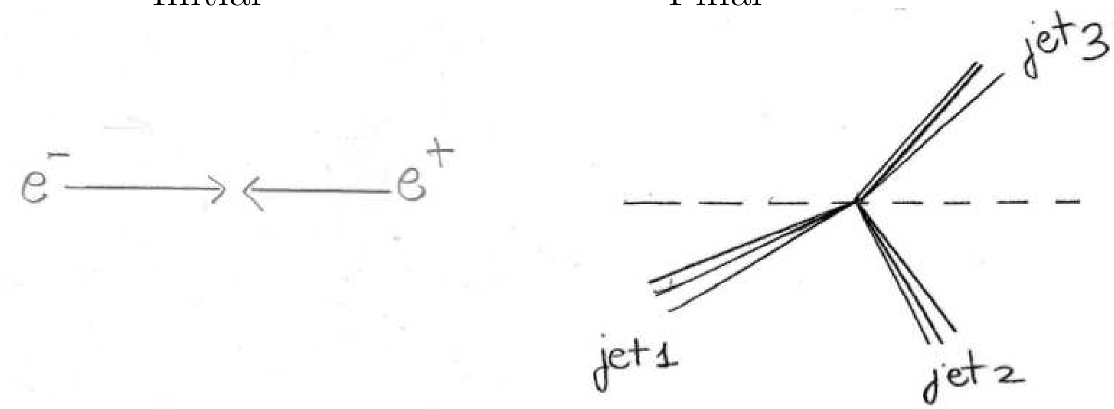
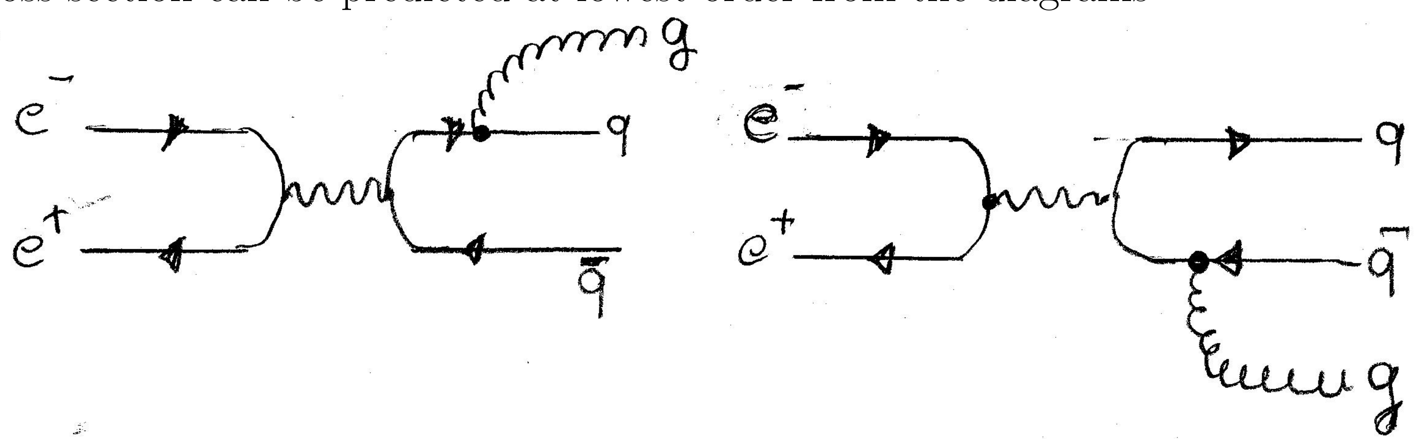


In QED+QCD the asymptotic free picture for this process is that the third jet comes from a gluon
So the cross section can be predicted at lowest order from the diagrams



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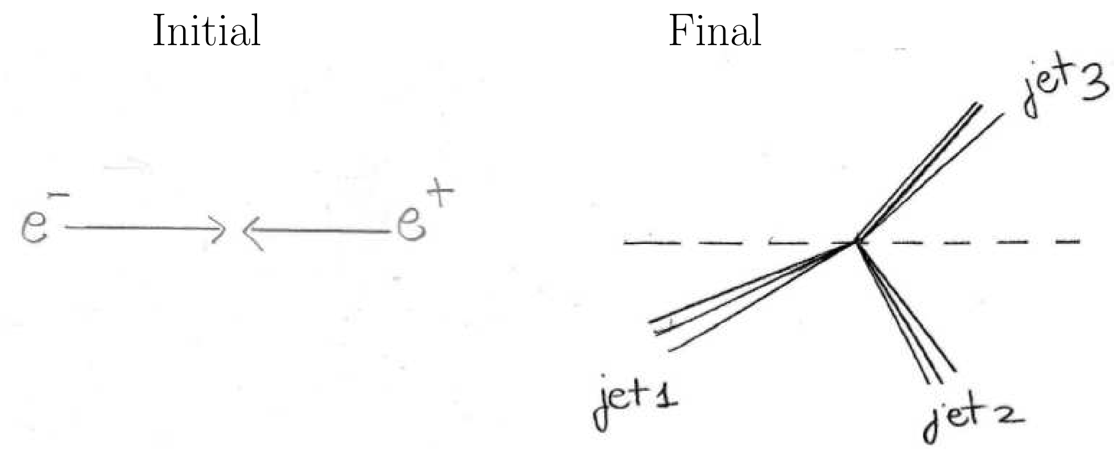

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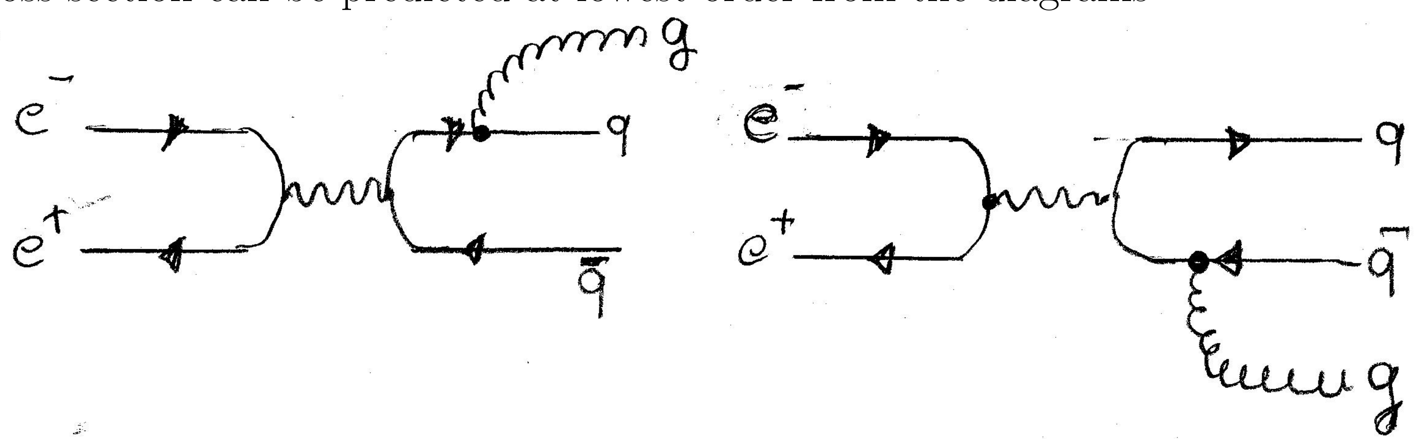

which involve the QCD vertex of quarks to gluons

$\Rightarrow$ comparing to data we can verify that gluons are vector particles and test gluon-q-$\bar{q}$ QCD vertex

$e^+ e^- \rightarrow 3\ jets$ first observed at the PETRA collider in DESY in Hamburg, Germany in the late 70's.

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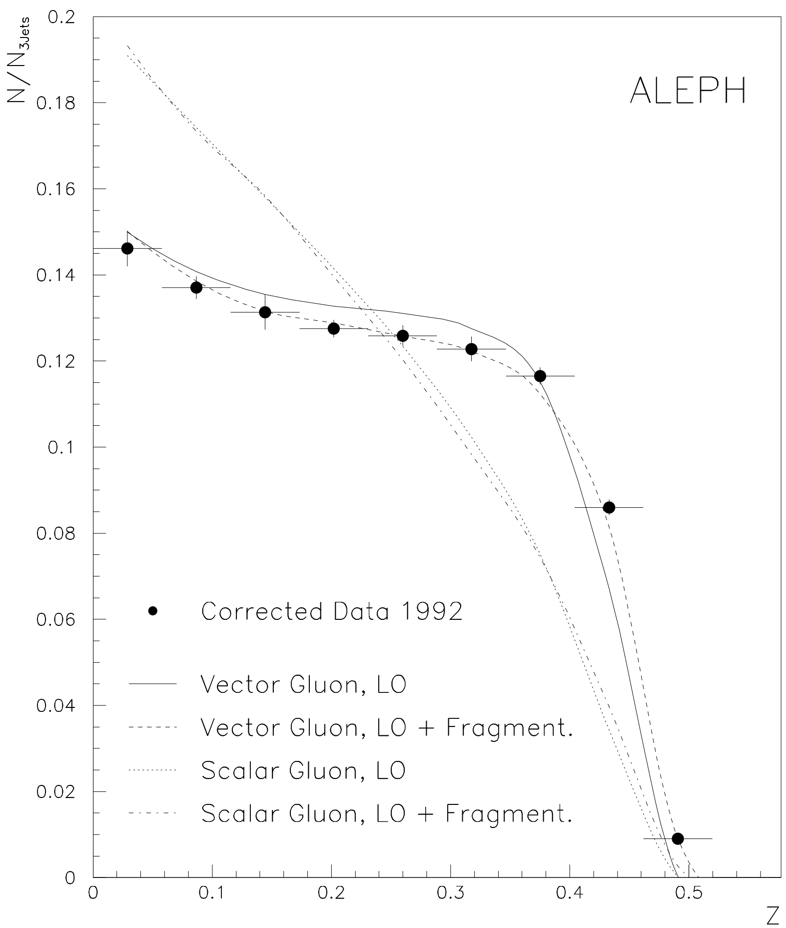
More precise data from ALEPH: Ordering $E_{jet,1} \leq E_{jet_2} \leq E_{jet_3}$

Study the $Z = \frac{1}{3}\left(\frac{2\,E_{jet_2}}{\sqrt{s}} - \frac{2\,E_{jet_3}}{\sqrt{s}}\right)$ distribution of events compared to QCD prediction



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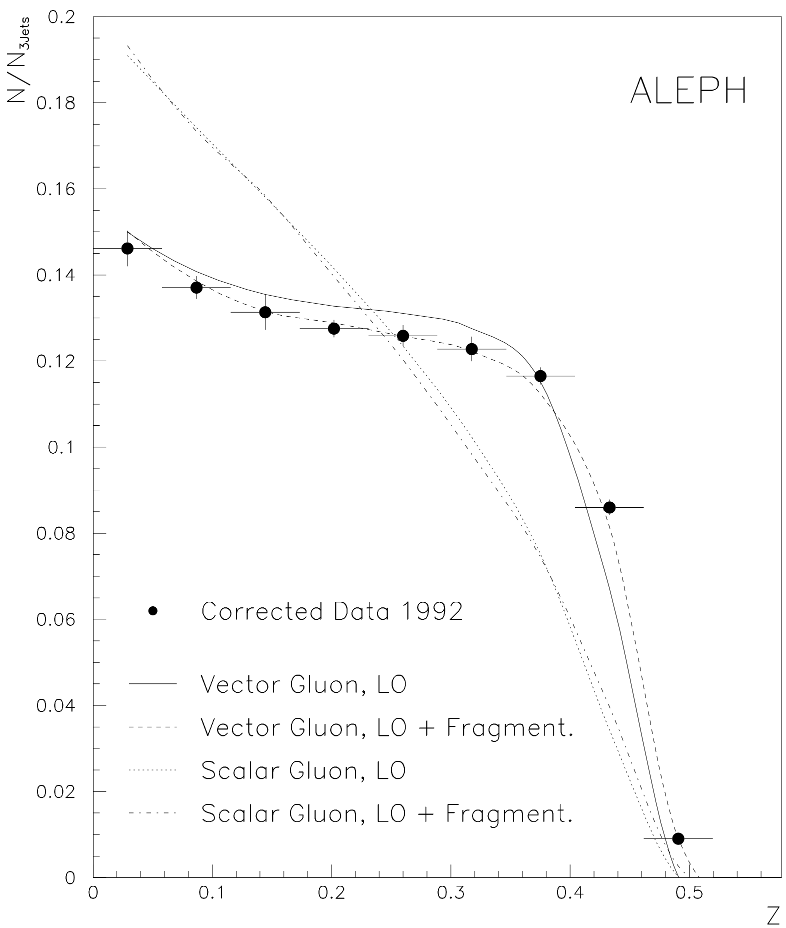

Z distribution would be different if gluon had spin=0

Data agrees with the QCD prediction

of a vector gluon-q-$\bar{q}$ coupling.

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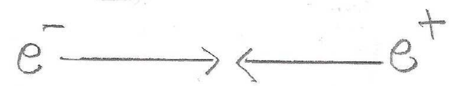
Initial



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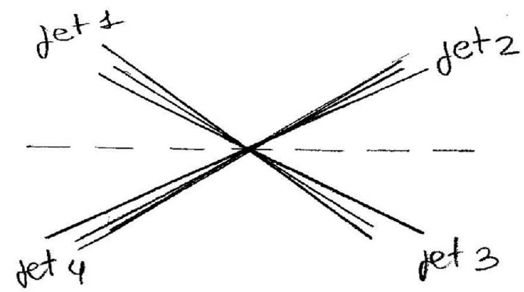

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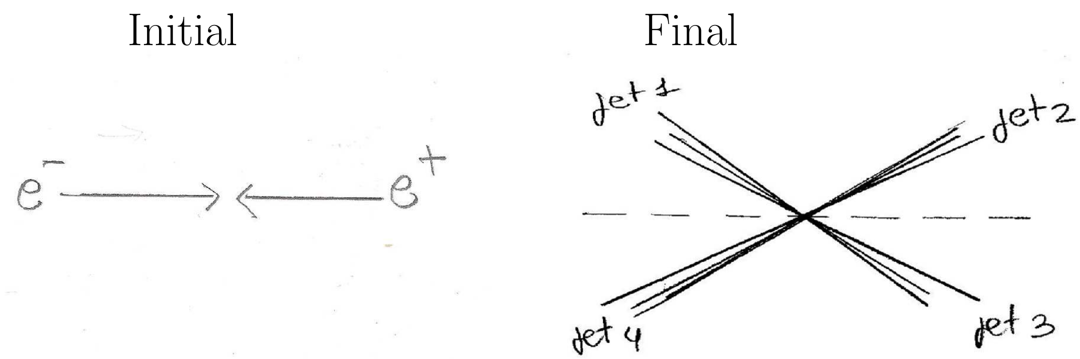

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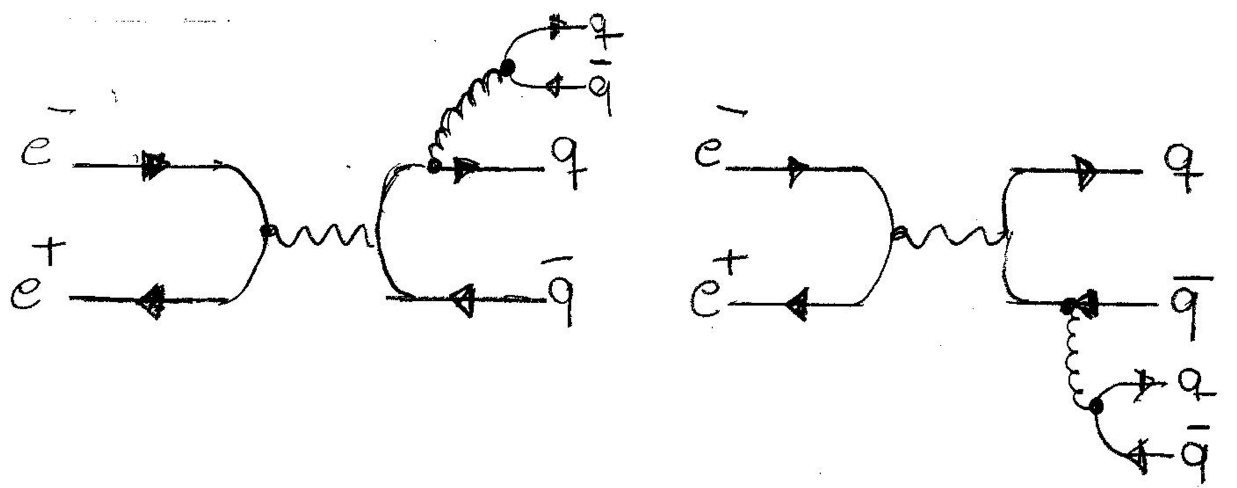

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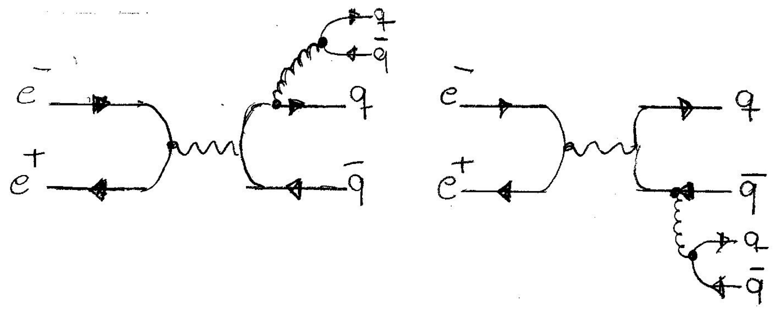

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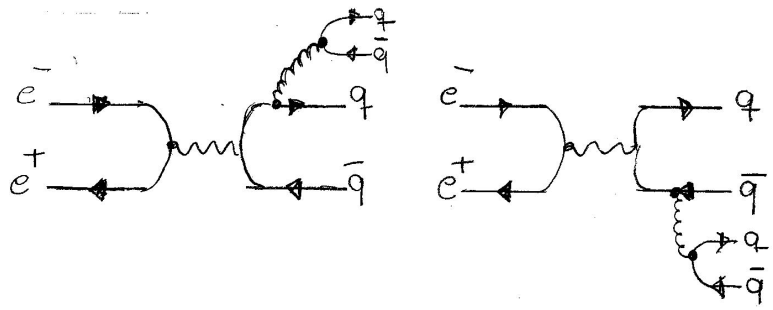
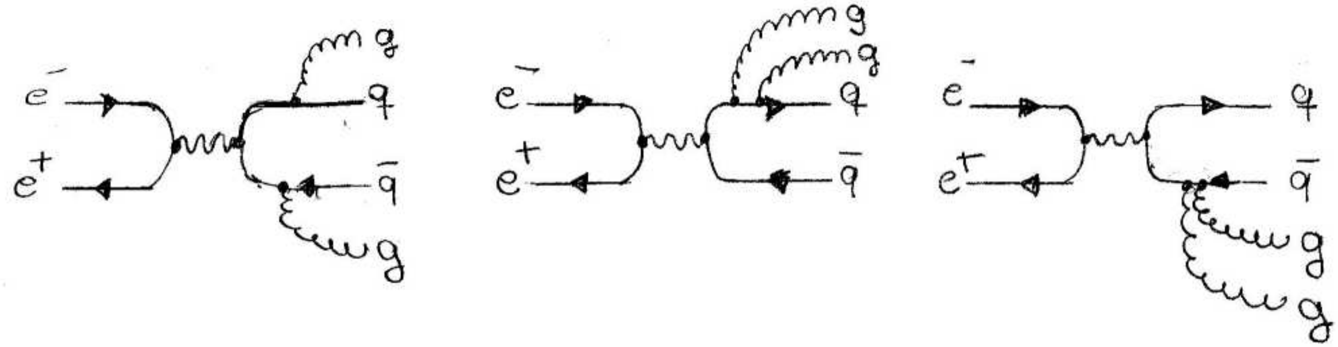
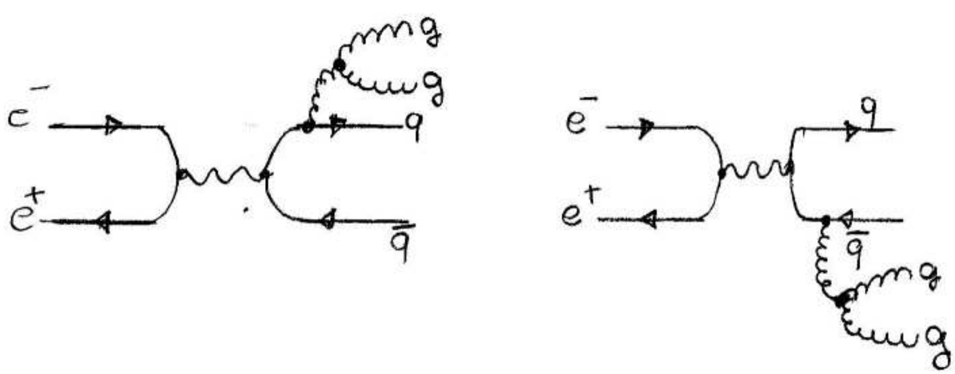


or 2 quarks and 2 gluons and there are very different diagrams contributing





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$\Rightarrow$ test of symmetry group $SU(N)_{\text{color}}$

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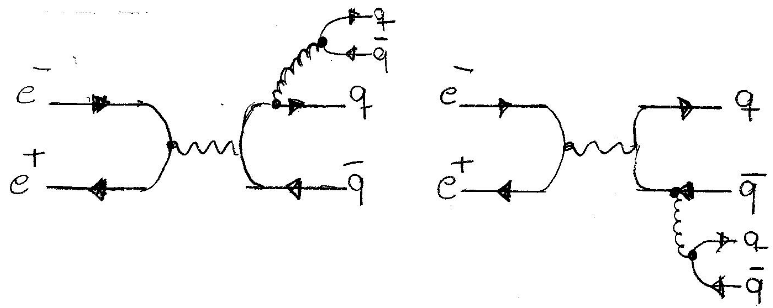

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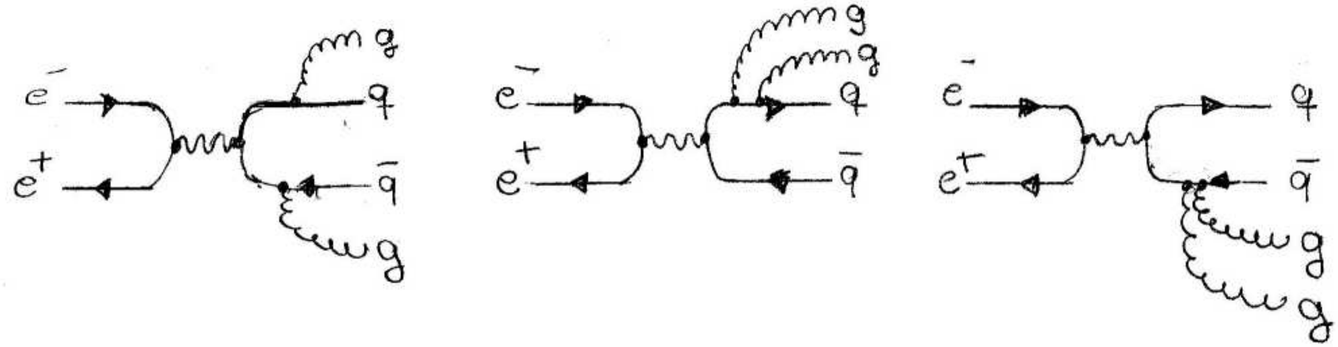

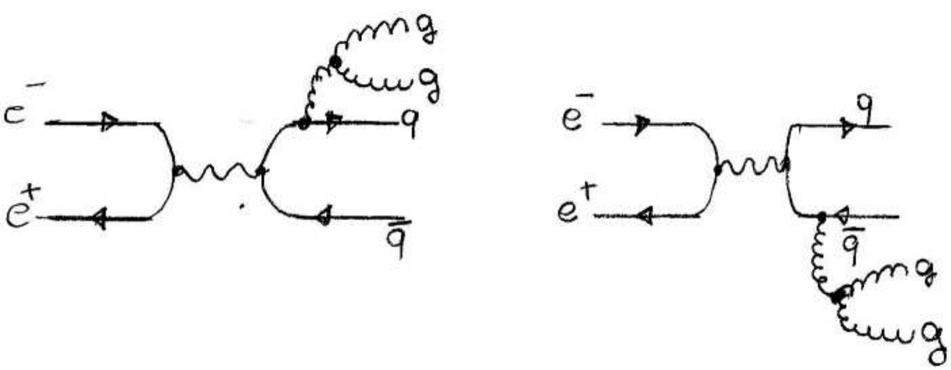

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Fit to data:

$C_A \equiv N = 2.9 \pm 0.6$ and $C_F \equiv \frac{N^2-1}{2N} = 1.35 \pm 0.27$

In agreement with QCD prediction :

$$C_A = 3,\ C_F = \tfrac{4}{3} = 1.33$$