Comparing to data

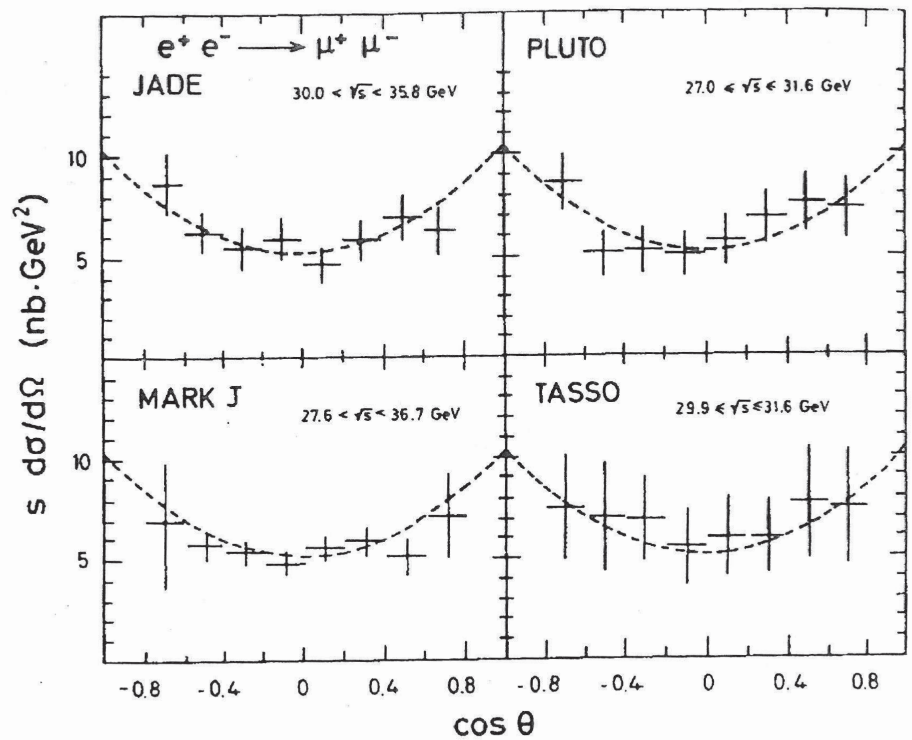


Fig. 9 : Differential cross section for the reaction $e^+e^- \to \mu^+\mu^-$. The dashed curves show the QED prediction. The data are corrected for radiative effects and hadronic vacuum polarization.

$\pi - \theta_2 < \theta < \pi - \theta_1$. The values are listed in Table 3 and show that more data are desirable.

group	A	reference
JADE	-0.05 ± 0.06	20
MARK J	-0.01 ± 0.06	21
PLUTO	0.07 ± 0.10	22
TASSO	-0.06 ± 0.08	23

Table 3 : Angular asymmetry of $\mu^+\mu^-$ pair production at $\sqrt{s}$ = 35 GeV (PLUTO $\sqrt{s}$ < 31.6 GeV).

NOTE:

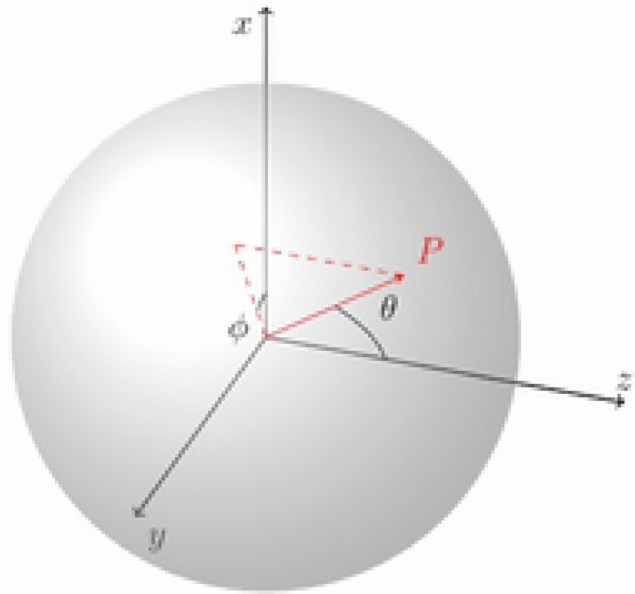
I have chosen the collision axis as $\hat{z}$

and the vertical in the page as $\hat{x}$

and the perpendicular to the page as

and θ is the angle in the page plane (

and the page is $\phi = 0$



In this system the coordinates of the position vector $\vec{r}$

$$x = r\,\sin\theta\,\cos\phi \qquad y = r\,\sin\theta\,\sin\phi \qquad z = r\,\cos\theta$$

The full space is covered by $0 \leq \theta \leq \pi$ and $0 \leq \phi \leq 2\pi$.

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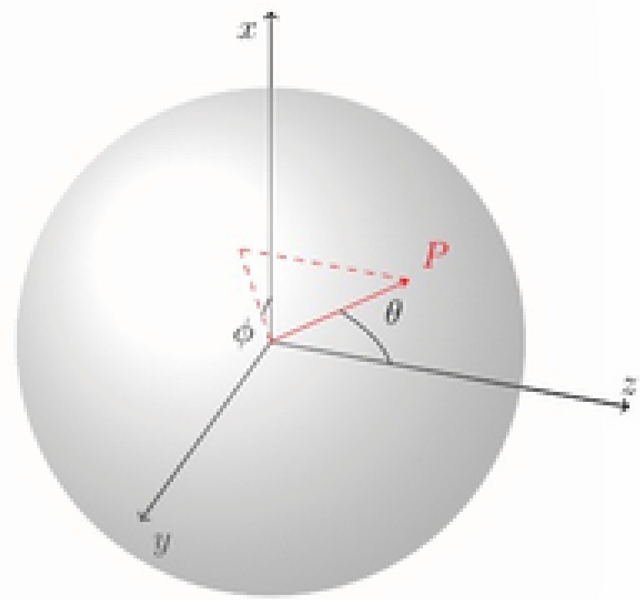

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So changing $\quad \vec{r}$ to $-\vec{r} \quad$ means $\quad$ changing $\quad (\theta, \phi)$ to $(\pi - \theta, \pi + \phi)$

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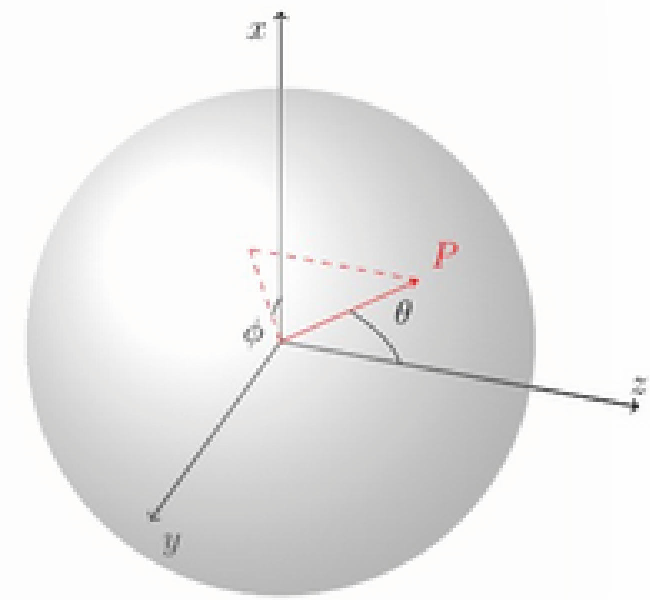

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So changing $\vec{r}$ to $-\vec{r}$ means changing (θ, ϕ) to $(\pi - \theta, \pi + \phi)$

But we know that the cross section cannot depend on ϕ because of symmetry about the collision axis. So

$$\frac{d\sigma}{d\Omega}(\theta) = \frac{d\sigma}{d\Omega}(\pi - \theta) \qquad \Leftrightarrow \qquad \frac{d\sigma}{d\Omega}(\vec{r}_{\mu^-}) = \frac{d\sigma}{d\Omega}(-\vec{r}_{\mu^-})$$

Remember that in chapter 4 I told you that electromagnetic interaction conserves Parity ($\mathcal{P}$) and Charge Conjugation ($\mathcal{C}$).

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In $e^- + e^+ \to \mu^- + \mu^+$ since the electromagnetic interaction vertex does not connect electrons and muons, if $\mathcal{P}$ and $\mathcal{C}$ are conserved, they have to be conserved independently in the initial electrons and in the final state muons.

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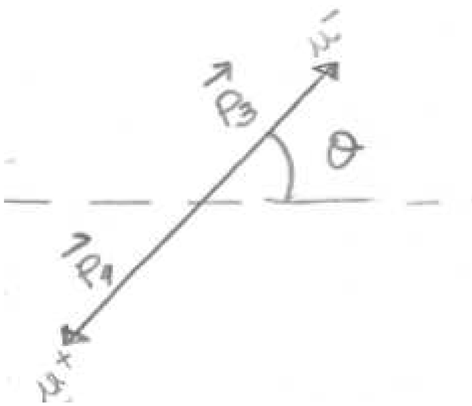
In particular looking at the final state muons



$\mathcal{P}$ Conservation: The number of muons produced in direction $\vec{r}$ must be same as produced in direction $-\vec{r}$. So

$$\frac{d\sigma}{d\Omega}(\vec{r}_{\mu^-}) = \frac{d\sigma}{d\Omega}(-\vec{r}_{\mu^-}) \quad \Rightarrow \quad \frac{d\sigma}{d\Omega}(\theta) = \frac{d\sigma}{d\Omega}(\pi - \theta)$$

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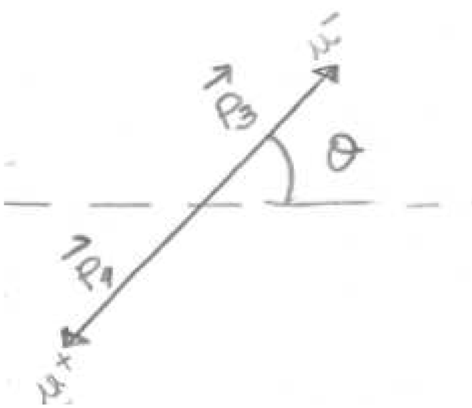

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$$\frac{d\sigma}{d\Omega}(\vec{r}_{\mu^-}) = \frac{d\sigma}{d\Omega}(-\vec{r}_{\mu^-}) \Rightarrow \frac{d\sigma}{d\Omega}(\theta) = \frac{d\sigma}{d\Omega}(\pi - \theta)$$

$\mathcal{C}$ Conservation: The number of μ^- produced in direction $\vec{r}$ must be the same as the number of μ^+ produced in that same direction.

But in the COM they are produced back to back ($\vec{r}_{\mu^+} = -\vec{r}_{\mu^-}$)

So again

$$\frac{d\sigma}{d\Omega}(\vec{r}_{\mu^-}) = \frac{d\sigma}{d\Omega}(\vec{r}_{\mu^+}) = \frac{d\sigma}{d\Omega}(-\vec{r}_{\mu^-}) \Rightarrow \quad \frac{d\sigma}{d\Omega}(\theta) = \frac{d\sigma}{d\Omega}(\pi - \theta)$$

Integrating over Ω we get the predicted total cross section

$$\begin{aligned}\sigma_{e^- + e^+ \to \mu^- + \mu^+} &= \int_0^{2\pi} d\phi \int_{-1}^{1} d(\cos\theta)\, \frac{d\sigma_{e^- + e^+ \to \mu^- + \mu^+}}{d\Omega}\bigg|_{\mathrm{COM}} \\ &= \frac{\alpha^2}{4\,s}\,(2\pi) \int_{-1}^{1} d(\cos\theta)\,(1+\cos^2\theta) = \frac{\alpha^2}{4\,s}\,(2\pi)\,\frac{8}{3} = \frac{4\pi\alpha^2}{3s}\end{aligned}$$

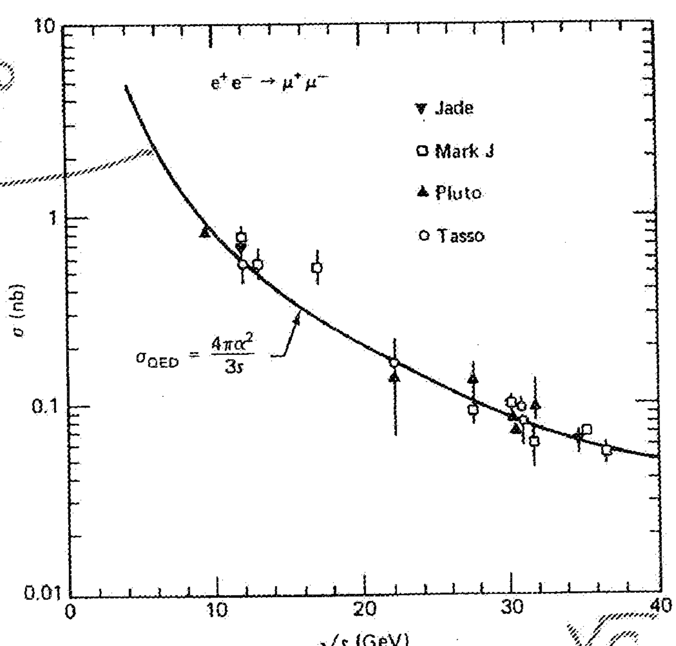
Integrating over Ω we get the predicted total cross section

$$\sigma_{e^- + e^+ \to \mu^- + \mu^+} = \int_0^{2\pi} d\phi \int_{-1}^{1} d(\cos\theta) \left. \frac{d\sigma_{e^- + e^+ \to \mu^- + \mu^+}}{d\Omega} \right|_{\mathrm{COM}}$$

$$= \frac{\alpha^2}{4\,s}(2\pi) \int_{1}^{1} d(\cos\theta)\,(1+\cos^2\theta) = \frac{\alpha^2}{4\,s}(2\pi)\,\frac{8}{3} = \frac{4\pi\alpha^2}{3s}$$

$$\sigma(e^+e^- \to \mu^+\mu^-) = \frac{4\pi\alpha^2}{3s}. \qquad (6.33)$$

A comparison of this result with PETRA data is shown in Fig. 6.6. The PETRA accelerator consists of a ring of magnets which simultaneously accelerate an electron and positron beam circulating in opposite directions. In selected spots, these beams are crossed, resulting in e^+e^- interactions with center-of-mass energy $\sqrt{s} = 2E_b$, where E_b is the energy of each beam. Equation (6.33) can be written in



$e^+e^- \to \mu^+\mu^-$

Fig. 6.6 The total cross section for $e^-e^+ \to \mu^-\mu^+$ measured at PETRA versus the center-of-mass energy.

Comparing to data

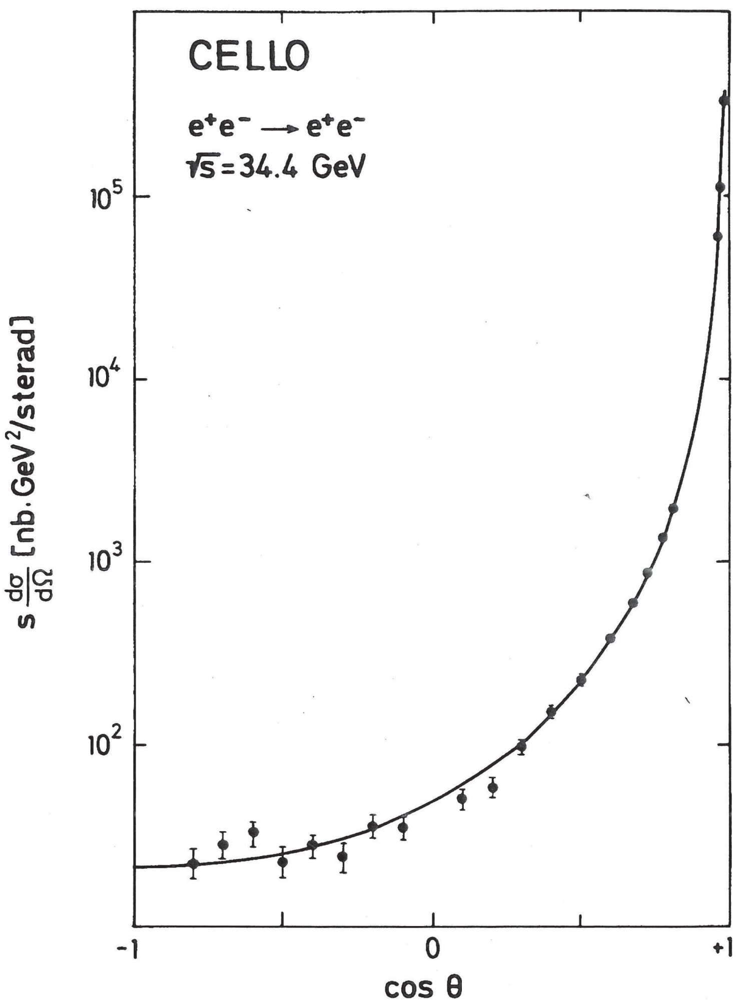


Fig.1

Comparing to data

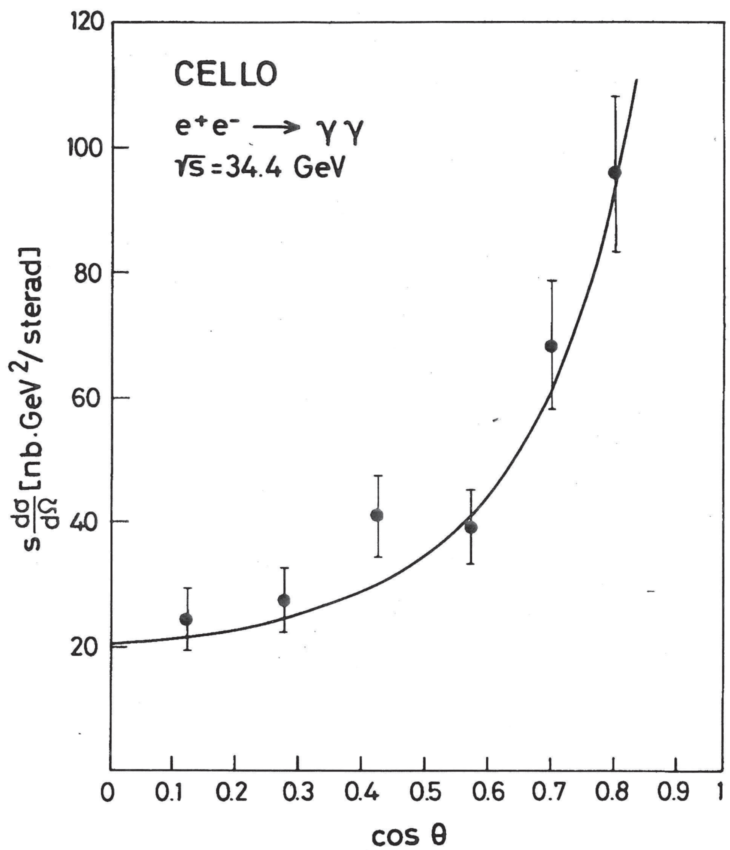


Fig. 3

Notice here plotting only from $\cos\theta = 0$ ($\theta = \frac{\pi}{2}$) to $\cos\theta = 1$ ($\theta = 0$)

The other half, $-1 \leq \cos\theta \leq 0$, has to be the mirror copy of this.