1. The states of definite mass are the eigenstates of Hamiltonian in rest frame.

the eigenvalue equation is

determinant

$$H - \lambda I = 0 \Rightarrow \begin{vmatrix} m_{\mu\mu}-\lambda & m_{\mu\tau} \\ m_{\mu\tau} & m_{\tau\tau}-\lambda \end{vmatrix} = 0$$

$$(m_{\mu\mu}-\lambda)(m_{\tau\tau}-\lambda) - m_{\mu\tau}^2 = 0$$

$$\lambda^2 - \lambda(m_{\mu\mu}+m_{\tau\tau}) + m_{\mu\mu}m_{\tau\tau} - m_{\mu\tau}^2 = 0$$

$$\lambda = \frac{m_{\mu\mu}+m_{\tau\tau} \pm \sqrt{(m_{\mu\mu}+m_{\tau\tau})^2 + 4m_{\mu\tau}^2 - 4m_{\mu\mu}m_{\tau\tau}}}{2}$$

$$'' m_{1,2} \qquad\qquad '' (m_{\mu\mu}-m_{\tau\tau})^2$$

$$m_{1,2} = \frac{m_{\mu\mu}+m_{\tau\tau}}{2} \pm \sqrt{\left[\frac{m_{\mu\mu}-m_{\tau\tau}}{2}\right]^2 + m_{\mu\tau}^2}$$

the eigenstate $\nu_1 = \cos\theta\,|\nu_\mu\rangle + \sin\theta\,|\nu_\tau\rangle$

$$\Rightarrow \begin{pmatrix} m_{\mu\mu}-m_1 & m_{\mu\tau} \\ m_{\mu\tau} & m_{\tau\tau}-m_1 \end{pmatrix} \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} = 0$$

$$(m_{\mu\mu}-m_1)\cos\theta + m_{\mu\tau}\sin\theta = 0$$

$$m_{\mu\tau}\sin\theta + (m_{\tau\tau}-m_1)\cos\theta = 0$$

$$\tan\theta = -\frac{m_{\mu\mu}-m_1}{m_{\mu\tau}}$$

$$\tan 2\theta = \frac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta} = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$= -\frac{2(m_{\mu\mu}-m_1)m_{\mu\tau}}{m_{\mu\tau}^2 - (m_{\mu\mu}-m_1)^2} = \frac{2m_{\mu\tau}}{m_{\mu\mu}-m_{\tau\tau}}$$

2) $|\varphi(t=0)\rangle = |\nu_\mu\rangle = \cos\theta|\nu_1\rangle - \sin\theta|\nu_2\rangle$

after a time "t" the state has evolved to

$$|\varphi(t)\rangle = \cos\theta\, e^{-iE_1t}|\nu_1\rangle - \sin\theta\, e^{-iE_2t}|\nu_2\rangle$$

so the amplitude to detect $|\varphi(t)\rangle$ as $|\nu_\mu\rangle$ is

$$a(\varphi\to\nu_\mu) \equiv \langle\nu_\mu|\varphi(t)\rangle = \cos^2\theta\, e^{-iE_1t}\underbrace{\langle\nu_1|\nu_1\rangle}_{=1}$$
$$+\sin^2\theta\, e^{-iE_2t}\underbrace{\langle\nu_2|\nu_2\rangle}_{=1}$$
$$-\sin\theta\cos\theta\left[e^{-iE_1t}\cancelto{0}{\langle\nu_2|\nu_1\rangle} + e^{-iE_2t}\cancelto{0}{\langle\nu_1|\nu_2\rangle}\right]$$

But $|\nu_1\rangle$ and $|\nu_2\rangle$ are orthogonal and are assumed normalized to 1

$$a(\varphi\to\nu_\mu) = \sin^2\theta\, e^{-iE_2t} + \cos^2\theta\, e^{-iE_1t}$$

(3) the probability

$$P(\varphi(t) \to \nu_\mu) = |a(\varphi(t) \to \nu_\mu)|^2$$

$$= \cos^4\theta + \sin^4\theta + 2\sin^2\theta\cos^2\theta \,\mathrm{Re}\, e^{-i(E_1-E_2)t}$$

$$= \cos^4\theta + \sin^4\theta + 2\sin^2\theta\cos^2\theta \underbrace{\cos(E_1-E_2)t}_{1-\sin^2\frac{\Delta E t}{2}}$$

$$= (\cos^2\theta + \sin^2\theta)^2 - 2\sin^2\theta\cos^2\theta \sin^2 \Delta E t/2$$

$$= 1 - \frac{1}{2}\sin^2 2\theta \sin^2\frac{\Delta E t}{2}$$

(4) $\Delta E = E_1 - E_2 = \sqrt{|\vec{p}|^2 + m_1^2} - \sqrt{|\vec{p}|^2 + m_2^2}$

$$\simeq |\vec{p}| + \frac{m_1^2}{2|\vec{p}|} - |\vec{p}| - \frac{m_2^2}{2|\vec{p}|}$$

$$\Rightarrow \frac{\Delta E t}{2} \overset{t = L/c}{\simeq} \frac{\Delta E L}{2|\vec{p}|} = \frac{\Delta m^2 L}{4|\vec{p}|} = \frac{\Delta m^2}{eV^2} \frac{L}{km} \frac{GeV}{|\vec{p}|} \times F$$

$$F = \frac{1}{4} \frac{eV^2}{GeV} (km) \times \frac{1}{192\, fm\, MeV}$$

$$= \frac{1}{4} \times \frac{10^{-9}}{192} \frac{eV\, km}{MeV\, fm} \overset{?}{=} 10^{12} \frac{10^3}{4 \times 197} = 1.27$$

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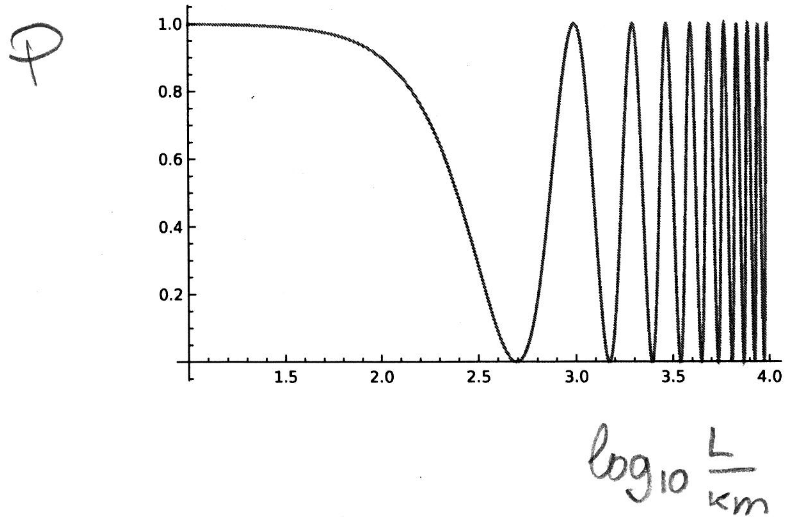

6 Notice that the the wavelength of oscillation

$$\frac{\lambda}{km} = \frac{1}{1.27} \frac{eV^2}{\Delta m^2} \frac{|\vec{p}|}{GeV} \pi \simeq 10^3 \, km \ll 10^4 \, km$$

so near the upcoming direction $\sim 10^4$ km within the resolution one can measure L (and $|\vec{p}|$) there will be many oscillations $\Rightarrow$ what one is really observing is

$$\langle P \rangle = \langle \sin^2 \pi \frac{L}{\lambda} \rangle \simeq \frac{1}{2}$$