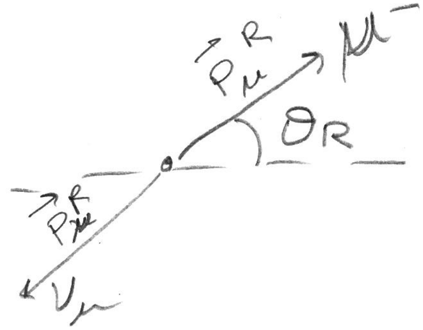
First let us write the inputs

(1) $m_{\pi^+} = 139.57 \text{ MeV}$

$m_\mu = 105.66 \text{ MeV}$

$m_{\nu_\mu} = 1 \text{ MeV}$

At $\pi^\pm$ rest frame



$\pi \leq \theta_R \leq 0$

and $\quad p_f \equiv |\vec{p}_\mu^{\,R}| = |\vec{p}_{\nu_\mu}^{\,R}| = \dfrac{1}{2m_\pi}\sqrt{(m_\pi^2 + (m_\mu + m_\nu)^2)(m_\pi^2 + (m_\mu - m_\nu)^2)}$

and $\quad E_\mu^R = \dfrac{m_\pi + m_\mu - m_\nu}{2m_\pi} \qquad E_\nu^R = \dfrac{m_\pi + m_\nu - m_\pi}{2m_\pi}$

To keep numerical precision is best to introduce numbers at the end

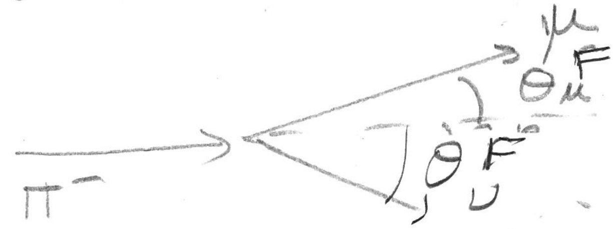
In π at flight frame where π flight with $\vec{\beta}_\pi = \beta_\pi \hat{x}$ $\left(\gamma_\pi = \dfrac{1}{\sqrt{1-\beta_\pi^2}}\right)$

the out going angle of μ and ν_μ (derived in class)

$$\tan\theta_\mu^F = \frac{\sin\theta_R}{\gamma_\pi\left(\cos\theta_R + \beta_\pi \dfrac{E_\mu^R}{p_f}\right)}$$

$$\tan\theta_{\nu_\mu}^F = \frac{-\sin\theta_R}{\gamma_\pi\left(-\cos\theta_R + \beta_\pi \dfrac{E_\nu^R}{p_f}\right)}$$

We want both ν_μ and μ produced forward wrt



$\Rightarrow \tan\theta_\mu^F > 0$ and $\tan\theta_\nu^F < 0$

$\sin\theta_R > 0$ always so the conditions above $\Rightarrow$

(a) $$\beta_\pi \frac{E_\mu^R}{p_f} + \cos\theta_R > 0 \Rightarrow \beta_\pi \frac{E_\mu^R}{p_f} > -\cos\theta_R \leq 1$$

(b) $$\beta_\pi \frac{E_\nu^R}{p_f} - \cos\theta_R > 0 \Rightarrow \beta_\pi \frac{E_\mu^R}{p_f} > \cos\theta_R = 1$$

strongest const

So strongest const is for $\nu_\mu \leftarrow \bullet \rightarrow \mu$ at π REST

So we need

$$\beta_\pi > \frac{p_f}{E_\mu^R}, \quad \text{and } \beta_\pi > \frac{p_f}{E_\nu^R}$$

Since $E_\nu^R < E_\mu^R$ because $m_\nu < m_\mu$

the strongest constraint is

$$\beta_\pi > \frac{p_f}{E_{\nu_R}} \Rightarrow \gamma_\pi = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-\frac{p_f^2}{E_\nu^{R2}}}} = \frac{E_\nu^R}{m_\nu}$$

$$\Rightarrow E_\pi = \gamma_\pi m_\pi > \frac{m_\pi}{m_\nu} E_\nu^R = \frac{1}{2m_\nu}(M_\pi^2 + M_\nu^2 - m_\mu^2)$$

$= 4.158$ GeV

(2) From above expression is clear that this is not possible if $m_\nu = 0$

So for $\pi^0 \to \gamma\gamma$ $(m_\gamma = 0)$ there is no energy of the π^0 which ensures both γ's forward

this is so because for $\theta_R = 0$ no matter how large the boost one cannot "flip" the momentum of the γ going backwards.

In the expressions this is seen because in this case $E_\gamma^R = P_\gamma^R \Rightarrow$ condition (b)

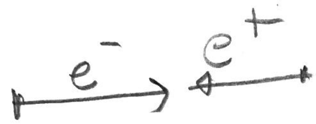
$\Rightarrow \beta_\pi \geq 1$ which is not possible

(3)

a) Let us neglect $m_{e^+} = m_{e^-} \simeq 0$ since both $E_{e^-} = 8$ GeV and $M_\Upsilon = 10.58$ GeV are much larger than $m_e \simeq 0.5$ MeV

(3.1)

Energy-momentum conser.



(a) $E_{e^-} + E_{e^+} = E_\Upsilon$

(b) $\vec{p}_{e^-} + \vec{p}_{e^+} = \vec{p}_\Upsilon \implies \xrightarrow{\vec{p}_\Upsilon}$ along collision axis $\hat{x}$ (or maybe $\xleftarrow{\vec{p}_\Upsilon}$)

$$|\vec{p}_{e}| - |\vec{p}_{e^+}| = (p_\Upsilon)_x$$

$$E_{e^-} - E_{e^+} = \sqrt{E_\Upsilon^2 - M_\Upsilon^2}$$

(a) in (b) $\implies E_{e^-} - E_{e^+} = \sqrt{(E_{e^-} + E_{e^+})^2 - M_\Upsilon^2}$

$$(E_{e^-} - E_{e^+})^2 = (E_{e^-} + E_{e^+})^2 - M_\Upsilon^2$$

$$\implies -2E_{e^-}E_{e^+} = 2E_{e^-}E_{e^+} - M_\Upsilon^2$$

$$\implies E_{e^+} = \frac{M_\Upsilon^2}{4E_{e^-}} = \frac{10.58^2}{4 \times 8}\ \text{GeV} = 3.498\ \text{GeV} \simeq 3.5\ \text{GeV}$$

(3.2)

(b) $(P_\Upsilon)_x = E_e - E_{e^+} = 4.5\ \text{GeV} > 0 \implies \Upsilon$ moving in direction of incoming e^-

$$E_\Upsilon = E_e + E_{e^-} = 11.5\ \text{GeV}$$

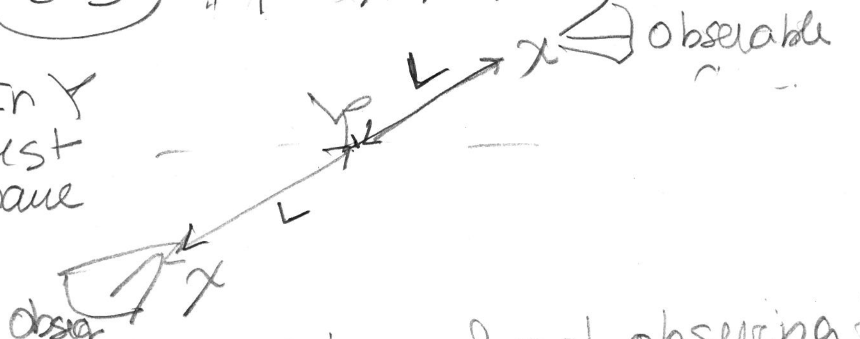
5

$$\gamma_Y = \frac{E_Y}{M_Y} = 1.087$$

$$\beta_Y = \frac{|\vec{P}_Y|}{E_Y} = \frac{4.5}{11.5} = 0.391$$

(3.3) $Y \to \chi\chi$ with $m_\chi = 4.5$ GeV

In Y rest frame



the condition of not observing this process which should have happen at least 200 times is that the χs' decay outside the detector.

Since detector is 8 m long the condition is that both χ's have traved a distance $d > 8\,m$ before decaying.

If τ_χ is the lifetime of τ_χ at rests
⇒ in the Y rest frame it travels

$$L = \gamma_\chi \beta_\chi \tau_\chi c$$ ← to get τ_χ in seconds

$$\text{with } \gamma_\chi = \frac{E_\chi}{m_\chi} = \frac{m_Y/2}{m_\chi} = \frac{10.58}{9} = 1.175$$

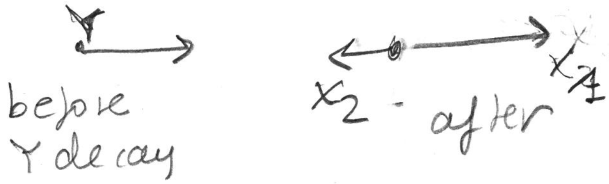
$$\Rightarrow \beta_\chi = 1 - \frac{1}{\gamma_\chi^2} = 0.526$$

So if we neglect the fact tha Υ is flying the condition is that

$$d = L_\chi = \gamma_\chi \beta_\chi \tau_\chi c > 8 \text{ m}$$

$$\tau_\chi > \frac{8}{3\times10^8 \times 1.176 \times 0.526} = 4.31\times10^{-8}\text{ s}$$

If we take into account that Υ is flying when it decays into χ's the distace travel by each χ before decaying depends on the direction of emission. The slowest will be the case of χ produced along the Υ direction but opposite



In this case

$$E_{\chi_2} = \gamma_\Upsilon\Big(\underbrace{E^R_{\chi_2}}_{=\frac{m_\Upsilon}{2}} - \beta_\Upsilon \underbrace{P^R_{\chi_2}}_{=\frac{1}{2}\sqrt{M_\Upsilon^2-4M_\chi^2}}\Big) = 1.087\left(\frac{m_\Upsilon}{2} - 0.391\sqrt{M_\Upsilon^2-4M_\chi^2}\right)$$

$$= 4.517 \text{ GeV}$$

So its γ and β factors are

$$\gamma_{\chi_2} = \frac{4.57}{4.5} = 1.015 \rightarrow \beta_{\chi_2} = 0.17$$

In LAB - χ_2 travels before decays

$$d_{\chi_2} = \beta_{\chi_2} \gamma_{\chi_2} \tau_\chi c$$

~~so~~ the condition $d_{\chi_2} > 8m$

$$\Rightarrow \tau_\chi > \frac{8}{3\times10^8 \times 1.015 \times 0.17} s = 1.55\times10^{-7} s$$

which is longer that the solution we got neglects the flight of the Υ because the back going χ has lower velocity and there for in needs more time to exit the detector and for the heavy χ's the time dilatation factors γ_χ are small

in both cases.