HW4

(1) $n \to p + e^-$

$S_n = \frac{1}{2}$

$S_p = \frac{1}{2}$

$S_e = \frac{1}{2}$

a)

$$\vec{S}_n = \vec{S}_p + \vec{S}_e + \vec{L}_{ep}$$

S_{ep} can take values from $|S_p - S_e| = 0$ to $S_p + S_e = 1$.

$\ell = 0, 1, 2, 3$

So the total final angular momentum can take values ℓ or from $|1-\ell|$ to $(1+\ell)$ but it will certainly be an integer while S_n is half integer $\Rightarrow$ angular momentum cannot be conserved

b) If we add ν

$$\vec{S}_n = \underbrace{\overbrace{\vec{S}_p + \vec{S}_e}^{}_{\vec{S}_{pe}} + \vec{S}_\nu}^{\vec{S}_{pe\nu}} + \vec{L}_{pe\nu}$$

$S_{pe} = 0, 1$

$S_{pe\nu} = S_\nu$ or from $|1 - S_\nu| \leq S_{pe\nu} \leq |1 + S_\nu|$

$\Rightarrow S_n \to$ $|S_\nu - \ell| \leq S_n \leq |S_\nu + \ell|$

 $||1 - S_\nu - \ell| \leq S_n \leq |1 + S_\nu + \ell|$

So $\left| C^{0,2,2}_{0\,0\,0} \right|^2 = \frac{1}{5}$

$\left| C^{2\,2\,2}_{000} \right|^2 = \frac{2}{7}$ $\qquad \frac{1}{5} + \frac{2}{7} + \frac{18}{35} = \frac{7+10+18}{35} = 1$

$\left| C^{4,2,2}_{0,0,0} \right|^2 = \frac{18}{35}$

(3) Both are mesons so they are $(q\bar{q}')$ state

$\vec{S}_{Meson} = \vec{S}_q + \vec{S}_{\bar{q}'} + \vec{L}_{qq'}$

So $S_{q\bar{q}'} = \left|\frac{1}{2} - \frac{1}{2}\right|$ to $\left(\frac{1}{2}+\frac{1}{2}\right)$ ie 0, 1

$a_0 \Rightarrow S_{meson} = 0 \rightarrow \begin{cases} S_{qq'} = 0 & l_{qq'} = 0 \\ S_{qq'} = 1 & l_{qq'} = 1 \end{cases}$

$a_1 \Rightarrow S_{meson} = 1 \begin{cases} S_{qq'} = 0 & l_{qq'} = 1 \\ S_{qq'} = 1 & l_{qq'} = 0, 1, 2 \end{cases}$

- $P_{meson} = (-)^{1+l_{qq'}}$ $\begin{cases} \rightarrow a_0 \Rightarrow l_{qq'} = 1 \Rightarrow S_{qq'} = 1 \\ \rightarrow a_1 \Rightarrow l_{qq'} = 1 \Rightarrow S_{qq'} = 0, 1 \end{cases}$

- $C_{meson} = (-)^{S_{qq'} + l_{qq'}} \Rightarrow \boxed{\begin{cases} a^0 \Rightarrow S_{qq} = 1, l_{qq} = 1 \quad \text{OK} \\ a^1 \Rightarrow l_{qq'} = S_{qq'} = 1 \end{cases}}$

So $|C^{0,2,2}_{000}|^2 = \frac{1}{5}$

$|C^{222}_{000}|^2 = \frac{2}{7}$ $\qquad \frac{1}{5} + \frac{2}{7} + \frac{18}{35} = \frac{7+10+18}{35} = 1$

$|C^{4,2,2}_{0,0,0}|^2 = \frac{18}{35}$

3) $\eta \to \gamma \rho_0$ with $S_\gamma = 1$ $\quad S_\rho = 1$

$C_\gamma = -1 \quad C_{\rho_0} = -1$

$\ell_{\gamma\rho} = 1$

- conservation of angular momentum

$$\vec{S}_\eta = \underbrace{\vec{S}_\gamma + \vec{S}_{\rho_0}}_{\vec{S}_{\gamma\rho}} + \vec{L}_{\gamma\rho}$$

composing the q# for $|\vec{S}_{\gamma\rho}|^2$ $\quad S_{\gamma\rho}$ can take val

$0 = |S_\gamma - S_\rho| \leq S_{\gamma\rho} \leq S_\gamma + S_\rho = 2 \Rightarrow S_{\gamma\rho} = 0, 1, 2$

composing with $\vec{L}_{\gamma\rho} \Rightarrow$

$0 \leq S_{\gamma\rho} + \ell_{\gamma\rho} \leq S_\eta \leq S_{\gamma\rho} + \ell_{\gamma\rho} \leq 4 =$

so conservation of angular momentum in decay

$\Rightarrow S_\eta = 0, 1, 2, 3, 4,$

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But em decay $\Rightarrow$ P and C consered

$$P_\eta = P_\rho \times P_\gamma \times (-)^{\ell_{\rho\gamma}} = (-1)(-1)(-1) = -1$$

$$C_\eta = C_{\rho^0} \times C_\gamma = (-1)(-1) = +1$$

We know tha η is a meson $\Rightarrow$ bound state $q\bar{q}$

$\Rightarrow$ its spin, C, and P are related to the spin and orbital angular momentum of its components

$$\vec{S}_\eta = \underbrace{\vec{S}_q + \vec{S}_{\bar{q}}}_{\vec{S}_{q\bar{q}}} + \vec{L}_{q\bar{q}}$$

$\Rightarrow$

with $0 \leq |S_q - S_{\bar{q}}| \leq S_{q\bar{q}} \leq S_q + S_{\bar{q}} \leq L$ ($S_q = \frac{1}{2}$, $S_{\bar{q}} = \frac{1}{2}$)

$\Rightarrow S_{q\bar{q}} = 0, 1$

$$|\ell_{q\bar{q}} - \cancel{\ell_{q\bar{q}}}| \leq S_\eta \leq S_{q\bar{q}} + \ell_{q\bar{q}} \Rightarrow \begin{cases} S_{q\bar{q}} = 0 & \ell_{q\bar{q}} = 0, 1, 2, 3 \\ S_{q\bar{q}} = 1 & \ell_{q\bar{q}} = 0, 1, 2, 3, 4 \end{cases}$$

Possible

$\Rightarrow$ its parity $P_\eta = P_q \times P_{\bar{q}} + (-)^{\ell_{q\bar{q}}} = (-1)^{\ell_{q\bar{q}}+1}$

so if we know that $P_\eta = -1 \Rightarrow \ell_{q\bar{q}}$ is even

$\Rightarrow$ its charge conj $C_\eta = (-)^{\ell_{q\bar{q}} + S_{q\bar{q}}}$

so if we know that $C_\eta = 1 \Rightarrow S_{q\bar{q}}$ is even

$\Rightarrow S_{q\bar{q}} = 0$

$\Rightarrow \ell_{q\bar{q}} = 0, 2$ possible

So altogether η can have $S_\eta = 0, 2$