Particle Physics: Assignment # 6

Due Thursday 03/07 before class

(1) Using the transformation properties of the fermion spinors given in class which I repeat here ($P = \gamma^0$ and $C = i\gamma^2\gamma^0$),

$$\psi_P(x) = P\,\psi(x_P) \qquad \overline{\psi}_P(x) = \overline{\psi}(x_P)\,P$$
$$\psi^C(x) = C\,\bar{\psi}^T(x) \qquad \overline{\psi}^C(x) = -\psi^T(x)C^{-1}$$

a) Derive the transformation properties under Parity and Charge Conjugation of the following four bilinears (a and b are two type of fermions)

$$1)\ \bar{\psi}_a(x)\psi_b(x) \qquad 2)\ \bar{\psi}_a(x)\gamma_5\psi_b(x)$$
$$3)\ \bar{\psi}_a(x)\gamma^\nu\psi_b(x) \qquad 4)\ \bar{\psi}_a(x)\gamma^\nu\gamma^5\psi_b(x)$$

Hint: Do not forget that there is a $(-)$ sign which you have to include when exchanging fermions.

b)With the resuts above check whether the following Lagrangiangs are invariant under Parity and Charge Conjugation

$$\mathcal{L}_A = -B\,\bar{\psi}_a(x)\gamma^\mu\psi_a(x)A_\mu(x)$$
$$\mathcal{L}_Z = -B\,\bar{\psi}_a(x)\gamma^\mu(1-\gamma^5)\psi_a(x)Z_\mu(x)$$
$$\mathcal{L}_{W1} = -D\,\bar{\psi}_a(x)\gamma^\mu\psi_b(x)W_\mu(x) + h.c.$$
$$\mathcal{L}_{W2} = -D\,\bar{\psi}_a(x)\gamma^\mu(1-\gamma^5)\psi_b(x)W_\mu(c) + h.c.$$

A^μ and Z^μ are real vector field, defined as odd under charge conjugation. while W^μ is a complex vector field which under charge conjugation changes to $-W^{\dagger\mu}$. B is a real constant (as required by reality of the Lagrangian). D is also a constant but can be complex.

What does this tell you about the Charge Conjugation and Parity properties of electromagnetic interactions?

(2) Draw the Feynman diagram for

$$e^+e^- \to \mu^+\mu^-$$

and using the Feynman rules given in class obtain the expression of the Feynman amplitude.

(3) Using the chiral representation of the 4-spinors given in homework 5, (neglect fermion masses) compute the Feynman amplitudes generated by QED for the process

$$e^+e^- \to \mu^+\mu^-$$

for the 16 helicity combinations in the COM. You should find that only some of the 16 helicity amplitudes are non-zero. Reason why (think of conservation of angular momentum). Compare with the results of question (2) of HW5.

Hint: define $\hat{z}$ axis as the collisions axis and define the $y = 0$ plane as the plane where the 4 particles are. Call θ the scattering angle of the μ. Remember you have to use the γ matrices in the chiral representation.