(1)

(1)

- $\mathcal{P}\,\overline{\psi_a}\psi_b\mathcal{P}^{-1} = \mathcal{P}\,\overline{\psi_a}\mathcal{P}^{-1}\mathcal{P}\,\psi_b\mathcal{P}^{-1}$

$$= \overline{\psi_a}(x_P)\underbrace{\gamma^0\gamma^0}_{I}\psi_b(x_P) = \overline{\psi_a}(x_P)\psi_P(x_P)$$

- $\mathcal{C}\,\overline{\psi_a}\psi_b\mathcal{C}^{-1} = \mathcal{C}\,\overline{\psi_a}\mathcal{C}^{-1}\mathcal{C}\,\psi_b\mathcal{C}^{-1} = -\psi_a^T C^{-1}C\,\overline{\psi_b}^T$

$$= -\psi_a^T\,\overline{\psi_b}^T = \overline{\psi_b}\,\psi_a$$

(−) due to exchange of fermions

- $\mathcal{P}\,\overline{\psi_a}\gamma^5\psi_b\mathcal{P}^{-1} = \overline{\psi_a}\gamma^0\gamma^5\gamma^0\psi_b = -\overline{\psi_a}\gamma^5\gamma^0\gamma^0\psi_b = -\overline{\psi_a}\psi_b$

- $\mathcal{C}\,\overline{\psi_a}\gamma^5\mathcal{C}^{-1} = -\psi_a^T\underbrace{C^{-1}\gamma^5 C}_{\gamma^{5T}}\overline{\psi_b}^T = -\psi_a^T\gamma^{5T}\overline{\psi_b}^T = \overline{\psi_b}\gamma^5\psi_a$

again (−) due to fermion exchange

- $\mathcal{P}\,\overline{\psi_a}\gamma^\mu\psi_b\mathcal{P}^{-1} = \overline{\psi_a}\gamma^0\gamma^\mu\gamma^0\psi_b = P^\mu_{\ \nu}\ \overline{\psi_a}\gamma^\nu\psi_b$

$$P^\mu_{\ \nu} = \begin{pmatrix}1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1\end{pmatrix}$$

- $\mathcal{C}\,\overline{\psi_a}\gamma^\mu\psi\,\mathcal{C}^{-1} = -\psi_a^T\underbrace{C^{-1}\gamma^\mu C}_{-\gamma^{\mu T}}\overline{\psi_b}^T = +\psi_a^T\gamma^{\mu T}\overline{\psi_b}^T$

$$= -\overline{\psi}^b\gamma^\mu\psi_a$$

- $\mathcal{P}\,\overline{\psi_a}\gamma^\mu\gamma^5\psi_b\mathcal{P}^{-1} = \overline{\psi_a}\gamma^0\gamma^\mu\gamma^5\gamma^0\psi_b = -P^\mu_{\ \nu}\ \overline{\psi_a}\gamma^\mu\gamma^5\psi_b$

- $\mathcal{C}\,\overline{\psi_a}\gamma^\mu\gamma^5\psi_b\mathcal{C}^{-1} = -\psi_a^T\underbrace{C^{-1}\gamma^\mu\gamma^5 C}_{(\gamma^\mu\gamma^5)^T}\overline{\psi_b}^T = -\psi_a^T(\gamma^\mu\gamma^5)^T\overline{\psi_b}^T$

$$= +\overline{\psi_b}\gamma^\mu\gamma^5\psi_a$$

So using — transf properties of 4-vector under Parity

$$\mathcal{P} A^{\mu}(x) \mathcal{P}^{-1} = P^{\mu}_{\nu} A^{\nu}(x_p)$$

$$\mathcal{C} A^{\mu} \mathcal{C}^{-1} = -A^{\mu}$$

and same for Z^{μ} and for W^{μ} the second one

$$\mathcal{C} W^{\mu} \mathcal{C}^{-1} = -W^{\mu +}$$

$$\mathcal{L}_A = -B \bar{\psi}_a \gamma^{\mu} \psi_a A_{\mu}$$

- $$\mathcal{P} \mathcal{L}_A(x) \mathcal{P}^{-1} = -B \, \mathcal{P} \bar{\psi}_a(x) \gamma^{\mu} \psi_a(x) \mathcal{P}^{-1} \mathcal{P} A_{\mu}(x) \mathcal{P}^{-1}$$

$$= -B \bar{\psi}_a(x) \gamma^{\nu} \psi_a(x) \underbrace{P^{\mu}_{\nu} P^{\alpha}_{\mu}}_{\delta^{\alpha}_{\nu}} A_{\alpha}(x) = \mathcal{L}_A(x_p)$$

- $$\mathcal{C} \mathcal{L}_A \mathcal{C}^{-1} = -B \, \mathcal{C} \bar{\psi}_a \gamma^{\mu} \psi_a \mathcal{C}^{-1} \mathcal{C} A_{\mu} \mathcal{C}^{-1}$$

$$= (-)B \, \bar{\psi}_a \gamma^{\mu} \psi_a \, (-) A_{\mu} = \mathcal{L}_A$$

- $$\mathcal{P} \mathcal{L}_Z \mathcal{P}^{-1} = -B \, \mathcal{P} \bar{\psi}_a \gamma^{\mu}(1-\gamma^5) \psi_a \mathcal{P}^{-1} \mathcal{P} A_{\mu} \mathcal{P}^{-1}$$

$$= -B \bar{\psi}_a \gamma^{\mu}(1+\gamma_5) \psi_a \underbrace{P^{\mu}_{\nu} P^{\alpha}_{\mu}}_{\delta^{\alpha}_{\nu}} A_{\alpha}$$

$$= -B \bar{\psi}_a \gamma^{\mu}(1+\gamma_5) A_{\mu}$$

$$\neq \mathcal{L}_Z(x_p)$$

- $\mathcal{C} \mathcal{L}_Z \mathcal{C}^{-1} = -B \mathcal{C} \overline{\psi_a} \gamma^\mu (1-\gamma_5) \psi_a \mathcal{C}^{-1} \mathcal{C} Z_\mu \mathcal{C}^{-1}$
$$= -B \overline{\psi_a} \gamma^\mu (1+\gamma_5) \psi_a (-1)(-1) Z_\mu$$
$$= -B \overline{\psi_a} \gamma^\mu (1+\gamma_5) \psi_a Z_\mu \neq \mathcal{L}_Z$$

- $\mathcal{P} \mathcal{L}_{W_1} \mathcal{P}^{-1} = -D \mathcal{P} \overline{\psi_a} \gamma^\mu \psi_b \mathcal{P} \mathcal{P}^{-1} W_\mu \mathcal{P} + \text{h.c}$
$$= -D \overline{\psi_a} \gamma^\mu \psi_b P^\mu_\nu P^\alpha_\mu W_\alpha =$$
$$= -D \overline{\psi_a} \gamma^\nu \psi_b W_\nu = \mathcal{L}_{W_1}(x_P)$$

- $\mathcal{C} \mathcal{L}_{W_1} \mathcal{C}^{-1} = -D \mathcal{C} \overline{\psi_a} \gamma^\mu \psi_b \mathcal{C}^{-1} \mathcal{C} W_\mu \mathcal{C}^{-1}$
$$- D^* \mathcal{C} \overline{\psi_b} \gamma^\mu \psi_a \mathcal{C}^{-1} \mathcal{C} W_\mu^+ \mathcal{C}^{-1}$$
$$= -D \overline{\psi_b} \gamma^\mu \psi_a W_\mu^+ - D^* \overline{\psi_a} \gamma^\mu \psi_b W_\mu$$
$$= \mathcal{L}_{W_1} \text{ if } D \text{ is real}$$

- $\mathcal{P} \mathcal{L}_{W_2} \mathcal{P}^{-1} = -D \mathcal{P}^{-1} \overline{\psi_a} \gamma^\mu (1-\gamma_5) \psi_b \mathcal{P}^{-1} \mathcal{P} W_\mu \mathcal{P}^{-1} + \text{hc}$
$$= -D \overline{\psi_a} \gamma^\mu (1+\gamma_5) \psi_b P^\mu_\nu P^\alpha_\nu W_\alpha + \text{h c}$$
$$= -D \overline{\psi_a} \gamma^\nu (1+\gamma_5) \psi_b W_\nu + \text{h c} \neq \mathcal{L}_{W_2}(x_P)$$

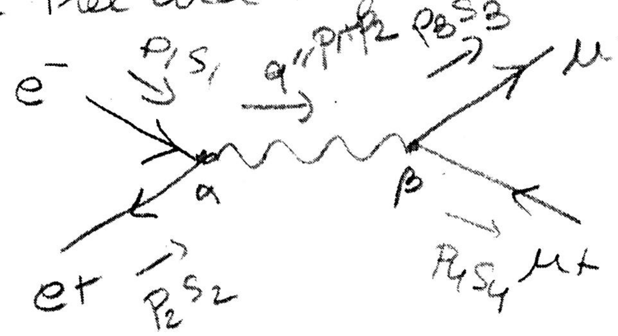
- $\mathcal{C} \mathcal{L}_{W_2} \mathcal{C}^{-1} = -D \mathcal{C}^{-1} \overline{\psi_a} \gamma^\mu (1-\gamma_5) \psi_b \mathcal{C}^{-1} \mathcal{C} W_\mu \mathcal{C}^{-1}$
$$- D^* \mathcal{C}^{-1} \overline{\psi_b} \gamma^\mu (1-\gamma_5) \psi_a \mathcal{C}^{-1} \mathcal{C} W_\mu^+ \mathcal{C}$$
$$= -D \overline{\psi_b} \gamma^\mu (1+\gamma_5) \psi_a W_\mu^+ - D^* \overline{\psi_a} \gamma^\mu (1+\gamma_5)^+ \psi_b W_\mu$$
$$\neq \mathcal{L}_{W_2} \text{ even if } D \text{ is real}$$

2) $e^- e^+ \to \mu^- \mu^+$

$p_1 \quad p_2 \quad p_3 \quad p_4$

$s_1 \quad s_2 \quad s_3 \quad s_4$

1 Tree level Feyman Diagram



From Feyman rule

$$M_{s_1 s_2 s_3 s_4} = \frac{e^2}{q^2}\left[\bar{v}_e^{s_2}(p_2)\,\gamma^\mu\, u_e^{s_1}(p_1)\right]\left[\bar{u}_m^{s_3}(p_3)\,\gamma_\mu\, v_m^{s_4}(p_4)\right]$$

the spinors I gave in hw2 setting $m = 0$

and in COM $\Rightarrow E_1 = E_2 = E_3 = E_4 \equiv E = \frac{\sqrt{s}}{2}$

choosing $\left.\begin{array}{l}\vec{p}_1 = (0,0,1)E \\ \vec{p}_2 = (0,0,-1)E\end{array}\right\}$ along $\hat{z}$ axis

$\left.\begin{array}{l}\vec{p}_3 = (\sin\theta, 0, \cos\theta)E \\ \vec{p}_4 = -(\sin\theta, 0, \cos\theta)E\end{array}\right\} \Rightarrow$ in $\hat{y} = 0$ plane

⑥

$$u_e^{+}(p_1) \simeq \sqrt{2E}\begin{pmatrix}0\\0\\1\\0\end{pmatrix} \qquad u_m^{+}(p_3) \simeq \sqrt{2E}\begin{pmatrix}0\\0\\\cos\frac{\theta}{2}\\\sin\frac{\theta}{2}\end{pmatrix}$$

$$u_e^{-}(p_1) \simeq \sqrt{2E}\begin{pmatrix}0\\1\\0\\0\end{pmatrix} \qquad u_m^{-}(p_3) \simeq \sqrt{2E}\begin{pmatrix}-\sin\theta/2\\\cos\theta/2\\0\\0\end{pmatrix}$$

$$v_e^{+}(p_2) \simeq \sqrt{2E}\begin{pmatrix}1\\0\\0\\0\end{pmatrix} \qquad u_m^{+}(p_4) \simeq \sqrt{2E}\begin{pmatrix}\cos\theta/2\\\sin\theta/2\\0\\0\end{pmatrix}$$

$$v_e^{-}(p_2) \simeq \sqrt{2E}\begin{pmatrix}0\\0\\0\\-1\end{pmatrix} \qquad u_m^{-}(p_4) \simeq \sqrt{2E}\begin{pmatrix}0\\0\\\sin\theta/2\\-\cos\theta/2\end{pmatrix}$$

$$\gamma^0 = \begin{pmatrix}0 & I\\ I & 0\end{pmatrix} \qquad \gamma^j = \begin{pmatrix}0 & \sigma^j\\ -\sigma^j & 0\end{pmatrix} \qquad \sigma^1 = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} \quad \sigma^2 = \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}$$

$$\sigma^3 = \begin{pmatrix}1 & \\ & -1\end{pmatrix}$$

So

$$\bar{v}_e^{+}(p_2) = (v_e^{+})^{\dagger}\gamma^0 = \sqrt{2E}\,(1\,0\,0\,0)\begin{pmatrix}0&0&1&0\\0&0&0&1\\1&0&0&0\\0&1&0&0\end{pmatrix} = (0\,0\,1\,0)$$

$$\bar{v}_e^{-}(p_2) = \sqrt{2E}\,(0\ -1\ 0\ 0)$$

$$\bar{u}_m^{+}(p_3) = \sqrt{2E}\left(\cos\frac{\theta}{2}\ \ \sin\frac{\theta}{2}\ \ 0\ \ 0\right)$$

$$\bar{u}_m^{-}(p_3) = \sqrt{2E}\left(0\ \ 0\ \ -\sin\frac{\theta}{2}\ \ \cos\frac{\theta}{2}\right)$$

$$\gamma^1 = \begin{pmatrix}0&0&0&1\\0&0&1&0\\0&-1&0&0\\-1&0&0&0\end{pmatrix} \qquad \gamma^2 = \begin{pmatrix}0&0&0&-i\\0&0&i&0\\0&i&0&0\\-i&0&0&0\end{pmatrix}$$

$$\gamma^3 = \begin{pmatrix}0&0&1&0\\0&0&0&-1\\-1&0&0&0\\0&1&0&0\end{pmatrix}$$

So

$$\bar{v}_e^+(p_2)\gamma^\mu u_e^+(p_1) = 2E\,(0\,0\,1\,0)\,\gamma^\mu\begin{pmatrix}0\\0\\1\\0\end{pmatrix} = 2E(\gamma^\mu)_{33}$$

but $\gamma^\mu_{33} = 0$ for all $\gamma^\mu \Rightarrow$

$$\Rightarrow \bar{v}_e^+(p_2)\gamma^\mu u_e^+(p_1) = 0$$

$$\bar{v}_e^-(p_2)\gamma^\mu u_e^-(p_1) = -2E(\gamma^\mu)_{22} = 0$$

$$\bar{v}_e^+(p_2)\gamma^\mu u_e^-(p_1) = 2E(\gamma^\mu)_{32} = \begin{pmatrix} 0 & \mu=0 \\ -1 & \mu=1 \\ i & \mu=2 \\ 0 & \mu=3 \end{pmatrix}$$

$$\bar{v}_e^-(p_2)\gamma^\mu u_e^+(p_1) = -2E(\gamma^\mu)_{23} = -\begin{pmatrix} 0 & \mu=0 \\ 1 & \mu=1 \\ i & \mu=2 \\ 0 & \mu=3 \end{pmatrix}$$

$$\bar{u}_m^+(p_3)\gamma^\mu v_m^+(p_4)$$

$$= 2E\left[\cos^2\tfrac{\theta}{2}\,\gamma^\mu_{11} + \sin^2\tfrac{\theta}{2}\,\gamma^\mu_{22} + \cos\tfrac{\theta}{2}\sin\tfrac{\theta}{2}\left(\gamma^\mu_{12} + \gamma^\mu_{21}\right)\right] = 0$$

$$\bar{u}_m^-(p_3)\gamma^\mu v_m^-(p_4) =$$

$$= 2E\left[\sin^2\tfrac{\theta}{2}\,\gamma^\mu_{33} + \cos^2\tfrac{\theta}{2}\,\gamma^\mu_{44} + \cos\tfrac{\theta}{2}\sin\tfrac{\theta}{2}\,\gamma^\mu_{43} + \gamma^\mu_{34}\right]$$

$$= 0$$

$$\bar{u}_m^{+}(p_3)\,\gamma^\mu v_m^{-}(p_4)$$

$$= 2E\left[\cos\frac{\theta}{2}\sin\frac{\theta}{2}\left(\gamma^\mu_{13} - \gamma^\mu_{24}\right) + \sin^2\frac{\theta}{2}\,\gamma^\mu_{23} - \cos^2\frac{\theta}{2}\,\gamma^\mu_{14}\right]$$

$$= 2E\begin{pmatrix} 0 & \mu=0 \\ \sin^2\frac{\theta}{2} - \cos^2\frac{\theta}{2} & \mu=1 \\ i\left(\sin^2\frac{\theta}{2} + \cos^2\frac{\theta}{2}\right) & \mu=2 \\ 2\cos\frac{\theta}{2}\sin\frac{\theta}{2} & \mu=3 \end{pmatrix}$$

$$= 2E\begin{pmatrix} 0 & \mu=0 \\ \cos\theta & \mu=1 \\ i & \mu=2 \\ \sin\theta & \mu=3 \end{pmatrix}$$

$$\bar{u}_m^{-}(p_3)\,\gamma^\mu v_m^{+}(p_4) =$$

$$2E\left[+\sin\frac{\theta}{2}\cos\frac{\theta}{2}\left(-\gamma^\mu_{31} + \gamma^\mu_{42}\right) - \sin^2\frac{\theta}{2}\,\gamma^\mu_{32} + \cos^2\frac{\theta}{2}\,\gamma^\mu_{41}\right]$$

$$= 2E\begin{pmatrix} 0 & \mu=0 \\ \sin^2\frac{\theta}{2} - \cos^2\frac{\theta}{2} & \mu=1 \\ i\left(-\sin^2\frac{\theta}{2} - \cos^2\frac{\theta}{2}\right) & \mu=2 \\ 2\sin\frac{\theta}{2}\cos\frac{\theta}{2} & \end{pmatrix} = 2E\begin{pmatrix} 0 & \mu=0 \\ \cos\theta & \mu=1 \\ -i & \mu=2 \\ \sin\theta & \mu=3 \end{pmatrix}$$

So the only non-zero amplitud :

$$\mathcal{M}_{+-+-} = \frac{4E^2}{\underset{\underset{4E^2}{\parallel}}{q^2}} e^2 \begin{pmatrix} 0 & 1 & -i & 0 \end{pmatrix} \begin{pmatrix} 0 \\ \cos\theta \\ i \\ \sin\theta \end{pmatrix}$$

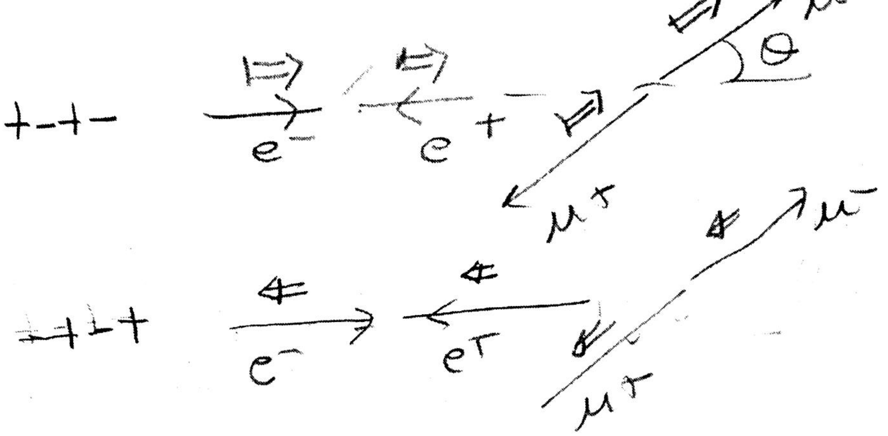
$$= e^2(\cos\theta + 1)$$

$$\mathcal{M}_{+--+} = e^2 \begin{pmatrix} 0 & 1 & -i & 0 \end{pmatrix} \begin{pmatrix} 0 \\ \cos\theta \\ -i \\ \sin\theta \end{pmatrix} = e^2(\cos\theta - 1)$$

$$\mathcal{M}_{-++-} = e^2 \begin{pmatrix} 0 & 1 & i & 0 \end{pmatrix} \begin{pmatrix} 0 \\ \cos\theta \\ i \\ \sin\theta \end{pmatrix} = e^2(\cos\theta - 1)$$

$$\mathcal{M}_{-+-+} = e^2 \begin{pmatrix} 0 & 1 & i & 0 \end{pmatrix} \begin{pmatrix} 0 \\ \cos\theta \\ -i \\ \sin\theta \end{pmatrix} = e^2(\cos\theta + 1)$$

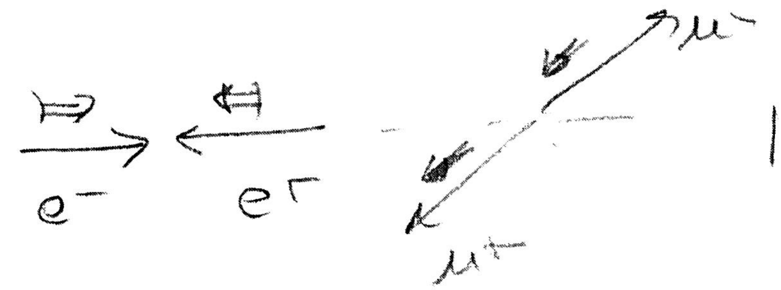
So the only non zero amplitudes correspond



$$|\mathcal{M}_{+-+-}|^2 = e^4(1+\cos\theta)^2$$

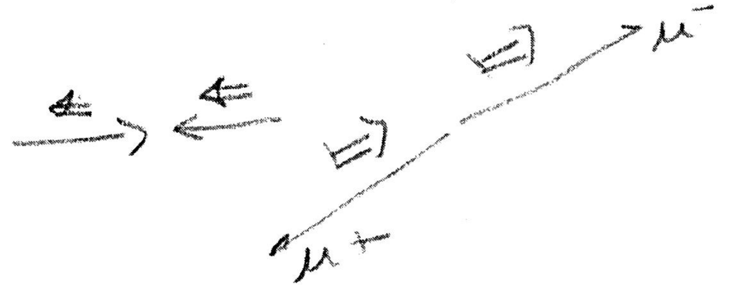
$$|\mathcal{M}_{-+-+}|^2 = e^4(1-\cos\theta)^2$$

$+ - - +$



$$|\mathcal{M}_{+--+}|^2 = e^2(1-\cos\theta)^2$$

$- + + -$



$$|\mathcal{M}_{-++-}|^2 = e^2(1+\cos\theta)^2$$

So $$\overline{|\mathcal{M}|^2} = \frac{1}{4}\sum |\mathcal{M}_{s_1 s_2 s_3 s_4}|^2 = \frac{e^2}{4}\, 2\left[(1-\cos\theta)^2 + (1+\cos\theta)^2\right]$$

$$= e^2(1+\cos^2\theta)$$ as we got in class

Comparing with the scalar case the non zero combinations where thos with $s_1 = s_2$ and $s_3 = s_4$ as in that case $\vec{J}^{\,init} = 0 = \vec{J}^{\,final}$

which is what requires the intermediate partica being a scalar for any $\theta \Rightarrow$ amplitude independent of θ

~~For~~ QED the photon has $\vec{J} = 1$ (vector)

$\Rightarrow s_1 = -s_2$ and $s_3 = -s_4$ to have $J_z^{init} = \pm 1$

and $J_{z'}^{final} = \pm 1$
$\Leftarrow$ depend on θ

the amplitudes obtained correspond to the projection of the $\vec{J}_{final}$ over the $\hat{z}$ axis