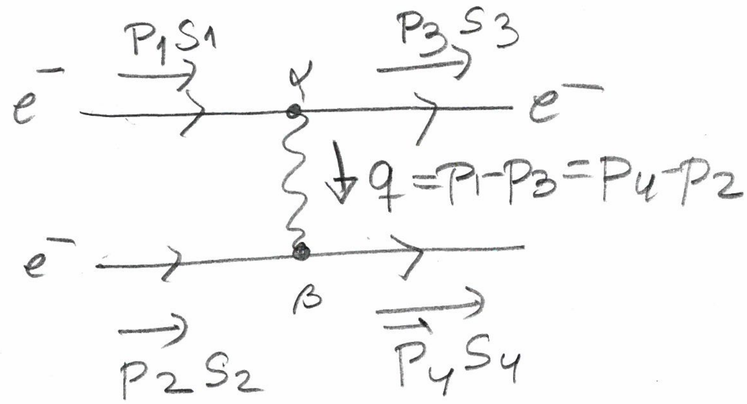
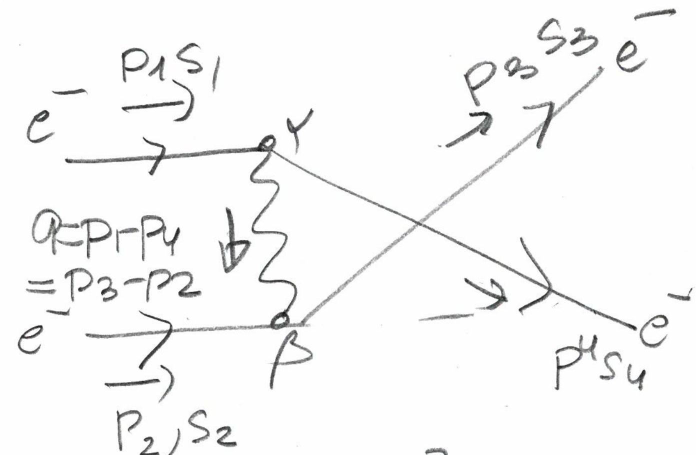
1 $e^- e^- \to e^- e^-$





(a) $\Rightarrow q^2 = t$

(b) $\Rightarrow q^2 = u$

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$$iM_a = \frac{e^2}{t} \bar{u}_3 \gamma^\alpha u_1 \, \bar{u}_4 \gamma_\alpha u_2$$

exchanged identical fermions

$$iM_b = (-) \frac{e^2}{u} \bar{u}_3 \gamma^\alpha u_2 \, \bar{u}_4 \gamma_\alpha u_1$$

$$\overline{|M|^2} = \frac{1}{4} \sum_{s_1 s_2 s_3 s_4} |M_a + M_b|^2 = \overline{|M_a|^2} + \overline{|M_b|^2} - 2\,\mathrm{Re}\,\overline{M_a \overline{M}_b^*}$$

$$\overline{|M_a|^2} = \frac{e^4}{4t^2} \mathrm{Tr}[\not{p}_3 \gamma^\alpha \not{p}_1 \gamma^\beta] \, \mathrm{Tr}[\not{p}_4 \gamma_\alpha \not{p}_2 \gamma_\beta]$$

$$= \frac{2e^4}{t^2} (u^2 + s^2)$$

$$\begin{pmatrix} a \leftrightarrow b \\ t \leftrightarrow u \end{pmatrix}$$

$$\overline{|M_b|^2} = \frac{2e^4}{u^2} (t^2 + s^2)$$

the interf.

$$-2\mathrm{Re}(\bar{\mathcal{M}}_a \bar{\mathcal{M}}_b^{*}) = -\frac{2e^4}{4tu}\,\mathrm{Tr}\overbrace{[\not p_3 \gamma^\alpha \not p_1 \gamma^\beta \not p_4 \gamma_\alpha \not p_2 \gamma_\beta]}^{-8s^2}$$

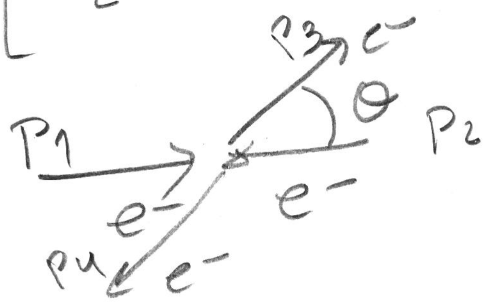
$$= +\frac{4s^2}{tu}$$

2 identical particles in final state

So
$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}\,\frac{1}{2}\,|\bar{\mathcal{M}}|^2$$

$$= \frac{e^4}{64\pi^2 s}\left[\frac{s^2+u^2}{t^2} + \frac{s^2+t^2}{u^2} + \frac{2s^2}{tu}\right]$$

in COM

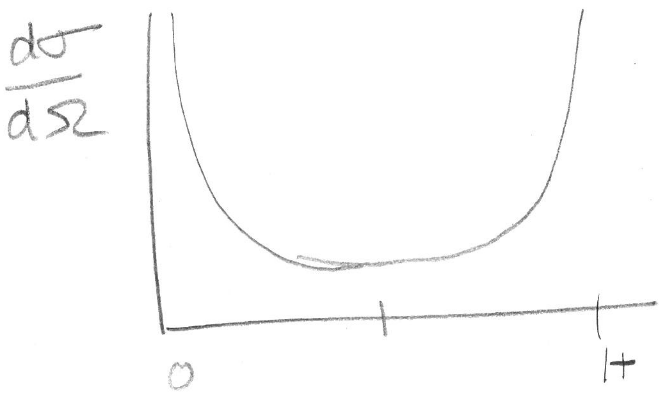


$$t = -\frac{s}{2}(1-\cos\theta) = -s\sin^2\theta/2 \quad u = -\frac{s}{2}(1+\cos\theta) = -s\cos^2\frac{\theta}{2}$$

$$\left.\frac{d\sigma}{d\Omega}\right|_{COM} = \frac{e^4}{64\pi^2 s}\underbrace{\left[\frac{1+\sin^4\theta/2}{\cos^4\theta/2} + \frac{1+\cos^4\theta/2}{\sin^4\theta/2} + \frac{1}{\sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2}}\right]}_{F(\theta)}$$

2 this cs becomes ∞ at $\theta = 0, \pi$ so and it must be symmetr when $\theta \Longleftrightarrow \pi - \theta$ since $e^- \Longleftrightarrow e^-$

identical



numerically using $\alpha = \frac{e^4}{4\pi} \approx \frac{1}{137}$ $\quad s = (50+50)^2 = (100\,\text{GeV})^2$

1 in NU

$$\frac{e^4}{64\pi^2 s} = \frac{\alpha^2}{4s} = 13.3 \times 10^{-10}\,\text{GeV}^{-2} \times (197\,\text{MeV fm})^2$$

$$= 5.16 \times 10^{-11}\,\text{fm}^2 = 5.16 \times 10^{-41}\,\text{m}^2$$

$$= 0.516\ \text{pb}$$

$$F(\theta) = \begin{cases} \theta = 0 & \infty \\ \theta = 10 & 3.5 \times 10^4 \\ \theta = 45 & 98 \\ \theta = 90 & 18 \\ \theta = 137 & 98 \\ \theta = 170 & 3.5 \times 10^4 \\ \theta = 180 & \infty \end{cases}$$

(4)

(3)

$$\frac{N_{e^-}}{\text{sr min}}(\theta) = \mathcal{L} \times \frac{d\sigma}{d\Omega}(\theta) \times \text{time}$$

$$\mathcal{L} = \frac{N^2 \times f}{A} = \frac{(10^{11})^2}{400 \times (10^{-6} m)^2} \frac{1}{25 \times 10^{-9} s} = 10^{39} m^{-2} s^{-1}$$

2 per collision

1 minute

$$\frac{N_{e^-}(\theta)}{\text{sr min}} = 2 \times 5.16 \times 10^{-41} m^2 \times 10^{39} m^{-2} s^{-1} \times 60 s \; F(\theta)$$

$$= 6.2 \times F(\theta) = \begin{cases} 1.12 & \text{for } \theta = 90^\circ \\ 2.1 \times 10^9 & \text{for } \theta = 1^\circ \end{cases}$$

At $\theta = 1$ deg we are close to $\theta = 0$ for which the CS becomes ∞ due to ∞ range of em interaction