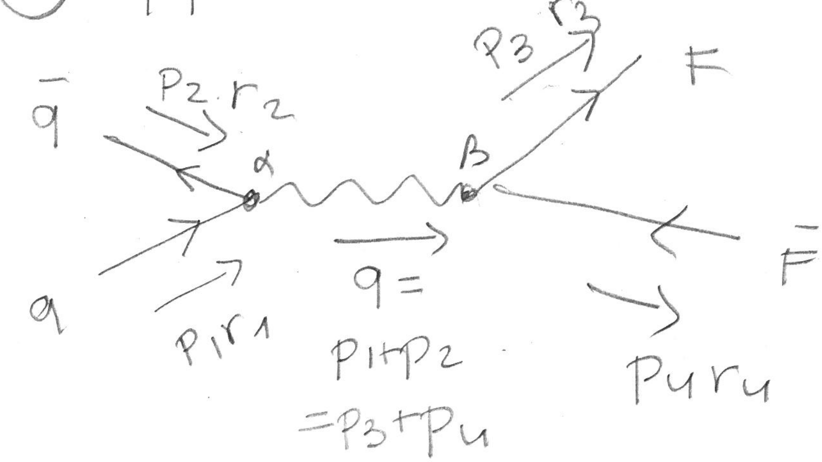
1

hw9

① $q\bar{q} \to F\bar{F}$ in QED



$$iM = \frac{ie^2}{q^2} \left[\bar{v}_q^{(2)} \gamma^\alpha u_q^{(1)}\right] \left[\bar{u}_F^{(3)} \gamma_\alpha v_F^{(4)}\right]$$

$q^2 = s$

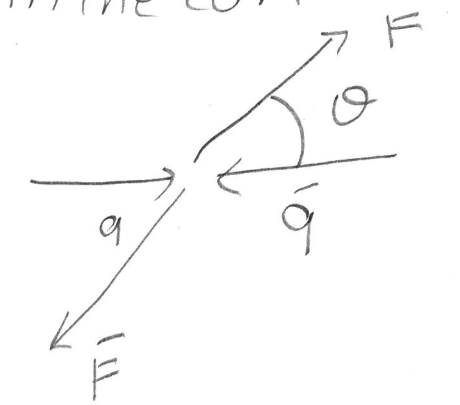
$$\overline{|M|^2} = \frac{e^4}{s^2} \frac{1}{4} \mathrm{Tr}\left[\not{p}_2 \gamma^\alpha \not{p}_1 \gamma^\beta\right] \mathrm{Tr}\left[(\not{p}_3 + m_F)\gamma_\alpha(\not{p}_4 - m_F)\gamma_\beta\right]$$

$$= \frac{e^4 4}{s^2}\left[p_2^\alpha p_1^\beta + p_1^\alpha p_2^\beta - g^{\alpha\beta} p_1 p_2\right]\left[p_{3\alpha} p_{4\beta} + p_{3\beta} p_{4\alpha} - g_{\alpha\beta} p_3 p_4 - m_F^2 g_{\alpha\beta}\right]$$

$$= \frac{8e^4}{s^2}\left[(p_1 p_3)(p_2 p_4) + (p_1 p_4)(p_2 p_3) + M_F^2 (p_1 p_2)\right]$$

$$\simeq \frac{2e^4}{s^2}(t^2 + u^2)$$

I am going to neglect m_F in the amplitude in the COM



$$t = -\frac{s}{2}(1-\cos\theta)$$

$$u = -\frac{s}{2}(1+\cos\theta)$$

$$|\overline{M}|^2 = e^4(1+\cos^2\theta) = 16\pi^2\alpha^2(1+\cos^2\theta)$$

$$\left.\frac{d\sigma}{d\Omega}\right|_{COM} = \frac{1}{64\pi^2 s}\frac{|P_f|}{|P_i|}|\overline{M}|^2$$

in this case $|P_i| = \frac{\sqrt{s}}{2}$ $\quad |P_f| = \frac{\sqrt{s-4M_F^2}}{2}$

$$\left.\frac{d\sigma}{d\Omega}\right|_{COM} = \frac{\alpha^2}{4s}\sqrt{1-\frac{4M_F^2}{s}}\,(1+\cos^2\theta)$$

$$\sigma = \int_0^{2\pi} d\varphi \int_{-1}^{1} d\cos\theta \,\frac{d\sigma}{d\Omega} = \frac{\pi\alpha^2}{2s}\sqrt{1-\frac{4M_F^2}{s}}$$

$$\times \underbrace{\int_{-1}^{1}(1+\cos^2\theta)}_{2+\frac{2}{3}=\frac{8}{3}} = \quad = \frac{4\pi\alpha^2}{3s}\sqrt{1-\frac{4M_F^2}{s}}$$

(2) in pp or $p\bar{p}$ collision at COM S

the CS could be obtained by the convolution of the $q\bar{q}$ CS in (1) with the quark and antiquark distribution functions.

$$\sigma(A+B \to F\bar{F}X)(s)$$

$$= \sum_{ab} C_{ab} \int_0^1 dx_a \int_0^1 dx_b \left[f_{a/A}(x_a) f_{b/B}(x_a) + f_{b/A}(x_a) f_{a/B}(x_b) \right] \sigma(a+b \to F\bar{F})(\hat{s} = x_a x_b s)$$

since $\hat{s}$ must be $\geq 4M_F^2 \Rightarrow \tau > \tau_{min} = \frac{4M_F^2}{s}$

it is convenient to use as variables x_a, $\tau = x_a x_b$

$$\Rightarrow dx_a dx_b = d\tau \frac{dx_a}{x_a}$$

since $x_b < 1 \Rightarrow x_a > \tau_a$ (to set the integral limits)

$$\sigma(A+B \to F\bar{F}X)(s)$$

$$= \underbrace{\sum_{ab} C_{ab}}_{\frac{1}{3}\sum_{ud}} \int_{\underbrace{\tau_{min}}_{4M_F^2/s}}^{1} d\tau \int_{\tau}^{1} \frac{dx_a}{x_a} \left[f_{a/A}(x_a) f_{b/B}\left(\frac{\tau}{x_a}\right) + f_{b/A}(x_a) f_{a/B}\left(\frac{\tau}{x_a}\right) \right] \frac{4\pi\alpha^2}{3s} \frac{1}{\tau} \sqrt{1 - \frac{\tau_{min}}{\tau}}$$

puting some units

for $s = 7\,TeV$ $\quad \frac{4\pi\alpha^2}{3s} = \frac{4\pi}{137^2}\frac{1}{3(7}\frac{(197\,fm \times MeV)^2}{TeV)^2}$

$$= 0.18 \times 10^{-12}\, fm^2 = 1.8 \times 10^{-39}\, cm^2$$

$$fm^2 = (10^{-13}\,cm)^2$$

$$= 1.8 \times 10^{-6}\, nb \equiv \sigma_0 \qquad nb = 10^{-33}\,cm^2$$

For $p\bar{p} \to F\bar{F}X$

using $f_{\bar{a}/\bar{p}} = f_{a/p}$

$$\sigma(p\bar{p} \to F\bar{F}X) =$$

$$= \frac{\sigma_0}{3}\int_{\tau_{min}}^{1} d\tau\, \frac{1}{\tau}\sqrt{1-\frac{\tau_{min}}{\tau}}\int_{\tau}^{1} 2\Big[(u_v(x_a) + u_s(x_a))(u_v(\tfrac{\tau}{x_a}) + u_s(\tfrac{\tau}{x_a}))$$

$$2[(d_v(x_a) + d_s(x_a))(d_v(\tfrac{\tau}{x_a}) + d_s(\tfrac{\tau}{x_a}))$$

$$+2\, u_s(x_a)\, u_s(\tau/x_a) + 2\, d_s(x_a)\, d_s(\tau/x_a)\Big]\frac{dx_a}{x_a}$$

$$u_v(x_a)\, u_v(\tfrac{\tau}{x_a}) + d_v(x_a)\, d_v(\tfrac{\tau}{x_a})$$

$$= (1+\tfrac{1}{4})\, 36\, (1-x_a)^2 (1-\tfrac{\tau}{x_a})^2 = 45\,\frac{1}{x_a^2}(1-x_a)^2(x_a-\tau)^2$$

$$u_v(x_a)\, u_s(\tfrac{\tau}{x_a}) + u_v(\tfrac{\tau}{x_a})\, u_s(x_a) + d_v(x_a)\, d_s(\tfrac{\tau}{x_a})$$

$$+ d_v(\tfrac{\tau}{x_a})\, d_s(x_a) = (1+\tfrac{1}{2})\,\tfrac{6}{4}(1-x_a)^2(1-\tfrac{\tau}{x_a})^3\frac{x_a}{\tau}$$

$$+ (1+\tfrac{1}{2})\,\tfrac{6}{4}(1-\tfrac{\tau}{x_a})^2 (1-x_a)^3\frac{1}{x_a}$$

$$= \frac{9}{4}\left[(1-x_a)^2 (x_a-\tau)^3 \frac{1}{\tau x_a^2} + (x_a-\tau a)^2 \frac{(1-x_a)^3}{x_a^3} \right]$$

$$u_s(x_a)\, u_s\left(\frac{\tau}{x_a}\right) + d_s(x_a)\, d_s\left(\frac{\tau}{x_a}\right)$$

$$= \frac{2}{16} (1-x_a)^3 \left(1-\frac{\tau}{x_a}\right)^3 \frac{1}{x_a} \frac{x_a}{\tau}$$

$$= \frac{1}{8} (1-x_a)^3 (x_a-\tau)^3 \frac{1}{\tau x_a^3}$$

So using the partial integrals in the homework we find that

$\sigma(p\bar{p} \to F\bar{F}X)$

$$= \frac{\sigma_0}{3} \int_{\tau_{min}}^{1} d\tau \, \frac{1}{\tau} \sqrt{1-\frac{\tau_{min}}{\tau}} \left[90\, I_1(\tau) + 9\, I_2(\tau) + \frac{1}{2\tau} I_3(\tau) \right]$$

$$= \frac{\sigma_0}{3} \left(90\, J_1 + 9\, J_2 + \frac{1}{2} J_3 \right)$$

with

m_f	J_1	J_2	J_3
100 GeV	7.64	235	1984
1000 GeV	0.11	0.21	18

For $\sigma(pp \to F\bar{F}X)$

$$= \frac{\sigma_0}{3} \int_{\tau_{min}}^{1} d\tau \, \frac{1}{\tau} \sqrt{1 - \frac{\tau_{min}}{\tau}} \int_{\tau}^{1} [u_v(x_a) + u_s(x_a)] \, u_s\left(\frac{\tau}{x_a}\right)$$

$$+ [d_v(x_a) + d_s(x_a)] \, d_s\left(\frac{\tau}{x_a}\right)$$

$$= \frac{\sigma_0}{3} \int_{\tau_{min}}^{1} d\tau \, \frac{1}{\tau} \sqrt{1 - \frac{\tau_{min}}{\tau}} \left[\frac{9}{4} I_2(\tau) + \frac{1}{8\tau} I_3(\tau) \right]$$

$\frac{1}{6}$

$$\Rightarrow \sigma(p\bar{p} \to F\bar{F}X) = \frac{\sigma_0}{3} \begin{cases} 3800 & M_F = 100 \text{ GeV} \\ 21 & M_F = 1000 \text{ GeV} \end{cases}$$

$$\Rightarrow \sigma(pp \to F\bar{F}X) = \frac{\sigma_0}{3} \begin{cases} 780 & m_H = 100 \text{ GeV} \\ 2.7 & m_H = 1000 \text{ GeV} \end{cases}$$

So $\sigma(pp \to F\bar{F}X) < \sigma(p\bar{p} \to F\bar{F}X)$ as expected because of valence quarks in both proton and antiproton in $p\bar{p}$ vs valence x sea in pp and the difference becomes larger the heavier m_H $\Rightarrow$ larger x needed