chapter 1

"Basics"

1) Introduction
2) Basic concepts of quantum physics for particles
3) the particle contents of the Standard Model
4) Natural units

1) Introductions

Particle physics addresses two fundamental questions:

a) What is matter made of?

b) How does it behave?

a) We know
- matter is made of tiny "chunks" ≡ particles
- they come in a small number of different types
- all particles of same type are "identical"

(≡ same intrinsic properties like mass, charge, ...)

b) II Interactions:

For these microscopic objects we can only obtain indirect information on their interactions from 3 sources

- scattering $A + B \to C + D + \cdots$

initial state

arrow represent 3-momentum

$A \longrightarrow$ $\longleftarrow B$

$E_A, \vec{p}_A$ $E_B, \vec{p}_B$

energy 3-momentum

final

$C\ E_C\ \vec{p}_C$

$D\ E_D, \vec{p}_D$

* Notation

3-vector

$\vec{v} \equiv \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}$

studying what particles come out and their energy and directions ($\equiv$ 3-momentum) we learn about the forces among these particles

- Decay $A \to B + C \ldots$

In relativistic quantum (not classical) systems it is possible that a state disintegrates into others

Initial

A

$E_A, \vec{P}_A$

Final

C $E_C, \vec{P}_C$

B $E_B, \vec{P}_C$

again, studying what particles are produced and their energy and 3-momentum we learn about their interactions

- Bound states made of these particles: studying the mass spectrum of bound states we learn about the interaction responsible for their bounding

How do we make progress :

- make a guess for some form of the interaction
  ≡ "build a model"
- compute prediction for something we can measure in these 3 probes (≡ observable)
  i.e cross section, decay lifetime
- compare expectation to data

To make these mathematical constructions we need a framework which must verify two guiding principles

- special relativity (as velocities of the particles are usually close to c) ⇒ chaper (2)
- quantum mechanics (as actions involved are comparable to $\hbar$) ⇒ next point

Macroscopic & slow object $(v \ll c)$ $\xrightarrow{\text{Small}}$ microscopic & slow

Framework ≡ classical mechanics — Quantum Mechanics

Fast ↓

Macroscopic & Fast

Framework Relativistic Mechanics

→ microscopic & Fast (particle physics)

Framework ≡ Quantum Field theory

Basics of QFT

- Particles are represented by "relativistic wave functions" $\equiv$ fields
- a physics process (scattering or decay) is a transition between an initial state consisting of some set of particles and a final state consisting of some set (same or different) of particles
- given an initial and final state we can only predict and measure the probability of such transition to occur

In this course we are going to omit the technicalities of QFT. But we are going to remind ourselves some basic concepts relevant to quantum relativistic systems next.

2) Basic concepts of QM for particles

I) The properties of a quantum system are completely specified by its state vector

- $|\varphi\rangle \equiv$ mathematical representation of physical state

  $(|\varphi\rangle^{+} \equiv \langle\varphi|)$

- $|\varphi\rangle$ is an element of the Hilbert space of physical states
- we chose the state vector to be normalized to 1 (as of today)

  $$\|\varphi\|^2 \equiv \langle\varphi|\varphi\rangle = 1$$

- If $|\varphi\rangle$ and $|\chi\rangle$ are vectors of 2 physical state
  $\Rightarrow$ linear combination $\lambda|\varphi\rangle + \beta|\chi\rangle$ is the vector of a physical state

II) Given states of vector $|\chi\rangle$ and $|\varphi\rangle$ there is a probability amplitude of finding $|\varphi\rangle$ in state $|\chi\rangle$

$$a(\varphi \to \chi) \equiv \langle\chi|\varphi\rangle$$

So the probability of measuring the state $|\varphi\rangle$ as $|\chi\rangle$ is

$$P(\varphi \to \chi) \equiv |a(\varphi \to \chi)|^2 = |\langle \chi | \varphi \rangle|^2$$

$\Rightarrow$ Vectors $|\varphi\rangle$ and $|\varphi'\rangle \equiv e^{i\alpha}|\varphi\rangle$ represent the same physical state because we only know how to measure probabilities and

$$P(\varphi' \to \chi) = |\langle \chi | \varphi' \rangle|^2 = \underbrace{|e^{i\alpha}|^2}_{=1} |\langle \chi | \varphi \rangle|^2 = P(\varphi \to \chi)$$

so we cannot distinguish $|\varphi\rangle$ and $|\varphi'\rangle$

III) For every physical property ($\equiv$ observable) A of the states (i.e energy, momentum, charge ...) there exist an associated hermitian operator which acts on the space of physical states $\hat{A}\ (\equiv \hat{A}^{+})$

- If the eigenvalues of $\hat{A}$, a_n, are not degenerate

$\Rightarrow$ the eigenstates of $\hat{A}$ $|n\rangle$ (so $\underset{\text{operator}}{\hat{A}}|n\rangle = \underset{\text{number}}{a_n}|n\rangle$) can be taken as a basis of the space of physical states

- If the eigenvalues are degenerate we need another operator B verifying $[A,B]=AB-BA=0$ ($\equiv$ A and B are compatible)

so we can chose the basis as the eigenstates of both operators $|n_a, m_b\rangle$ with

$$A|n_a, m_b\rangle = a_{n_a}|n_a, m_b\rangle$$
$$B|n_a, m_b\rangle = b_{m_b}|n_a, m_b\rangle$$

- If (a_{n_a}, b_{m_b}) are still degenerate then we need to find another operator C / $[A,C]=[B,C]=0$ to characterize the basis.

- For states which are not eigenstates of A we define the "expectation value of A in $|\varphi\rangle$"

$$\langle\varphi|A|\varphi\rangle \equiv \langle A\rangle_\varphi$$

IV) the time evolution of state vector $|\varphi(t)\rangle$ is governed by the evolution Eq

$$i\hbar \frac{d|\varphi(t)\rangle}{dt} = H|\varphi(t)\rangle \qquad \left[-i\hbar \frac{d\langle\varphi(t)|}{dt} = \langle\varphi(t)|H\right]$$

$H \equiv$ hamiltonian operator which hermitian and represent the physical property of Energy

- if a time independent operator A verify
  $[A, H] = 0$

$$\Rightarrow \frac{d\langle A\rangle_\varphi}{dt} = \frac{d\langle\varphi|A|\varphi\rangle}{dt} = \left(\frac{d}{dt}\langle\varphi|\right) A|\varphi\rangle + \langle\varphi|A|\left(\frac{d|\varphi\rangle}{dt}\right)$$

$$= -i\hbar\langle\varphi|HA|\varphi\rangle + i\hbar\langle\varphi|AH|\varphi\rangle =$$

$$= -i\hbar\langle\varphi|[H, A]|\varphi\rangle = 0$$

$\Rightarrow$ the expectation value of A in φ is a constant

$\Rightarrow$ the eigenvalues of an operator A which commutes with Hamiltonian are constant in time

We call then the quantum #'s of the corresponding eigenstates.

If we call $|\vec{x}\rangle$ to eigenstates of position operator $\vec{X}$
and $|\vec{p}\rangle$ " " " 3-momentum " $\vec{P}$

$$\vec{X}|\vec{x}\rangle = \vec{x}|\vec{x}\rangle$$

$$\vec{P}|\vec{p}\rangle = \vec{p}|\vec{p}\rangle$$

We define the wave function for $|\varphi\rangle$ in position space

$$\varphi(x) \equiv \langle\vec{x}|\varphi\rangle$$

and the wave function of $|\varphi\rangle$ in momentum space

$$\tilde{\varphi}(p) \equiv \langle p|\varphi\rangle$$

- $|\varphi(x)|^2$ represents the probability density of finding φ at point $\vec{x}$
- $|\tilde{\varphi}(p)|^2$ represent the probability density of finding φ with 3 momentum $\vec{p}$

They verify

$$\tilde{\varphi}(p) \sim \int d^3x \, e^{\frac{-i\vec{p}\vec{x}}{\hbar}} \varphi(x)$$

$$\varphi(x) \sim \int d^3\vec{p} \, e^{\frac{i\vec{p}\vec{x}}{\hbar}} \tilde{\varphi}(p)$$

- A particle with well defined 3-momentum $\vec{p_0}$ ($|\vec{p_0}\rangle$) has wave function in momentum space

$$\tilde{\varphi}_{p_0}(p) \equiv \langle p|p_0\rangle \sim \delta^3(\vec{p}-\vec{p_0})$$

$\Rightarrow$ its wave function in position space

$$\langle x|p_0\rangle \equiv \varphi_{p_0}(\vec{x}) \sim e^{+i\vec{p_0}\vec{x}} \quad \equiv \text{plane wave}$$

[Remember $\int_{-\infty}^{\infty} dx \, \delta(x-x_0) f(x) = f(x_0)$]

- the equation for the time evolution of the wave function

$$i\hbar \frac{\partial \varphi(\vec{x},t)}{\partial t} = \langle x | H | \varphi \rangle$$

$\Rightarrow$ A particle of well defined 3-momentum $\vec{p}_0$ and energy E_0 (for a non relativistic free particle $E_0 = \frac{|\vec{p}_0|^2}{2M}$) verifies $H|\vec{p}_0\rangle \equiv E_0 |\vec{p}_0\rangle$

$$\Rightarrow i\hbar \frac{\partial \varphi_{p_0}(\vec{x})}{\partial t} = E_0 \langle x | p_0 \rangle = E_0\, \varphi_{p_0}(\vec{x})$$

$$\Rightarrow \varphi_{p_0}(\vec{x},t) \approx e^{-iE_0 t/\hbar}\, e^{\frac{i\vec{p}_0\cdot\vec{x}}{\hbar}}$$

Special relativity + quantum mechanics $\Rightarrow$
for each particle there must be an "antiparticle" with same mass and spin but opposite Q, B, L.

I.e. for 1st generation

		Q	L	B
antiquark	$\bar{u}$	$-2/3$	0	$-1/3$
antiquark	$\bar{d}$	$1/3$	0	$-1/3$
positron	e^+	$+1$	-1	0
electron antineutrino	$\bar{\nu}_e$	0	-1	0

Quarks are never observed as free states. They are always in bound states of 2 types

- mesons $\equiv$ bound state $(q\bar{q}) \Rightarrow B=0$ and
  spin is integer (chapter 3)
  example: pions $(\pi^\pm, \pi^0)$, kaons $(K^\pm, K^0)$

- baryons $\equiv$ bound states of 3 quarks $(q q' q'') \Rightarrow B=1$
  spin is half-integer
  examples: proton (p), neutron (n)

Interactions between these fermions are mediated by <u>spin 1 particles</u> ≡ gauge bosons
≡ vectors

In the SM there are 3 type of interactions

Interaction	Gauge boson	Q
electromagnetic	photon (γ)	0
strong	gluon (g)	0
weak	Z boson	0
	$W^{\pm}$ "	± 1

All have $B = L = 0$

In addition model contains a particle of spin zero (scalar) ≡ Higgs boson (h)

We will describe the state and evolution of these particles in terms of "relativistic wave-functions"

The form the particle wave function depends on its spin ⇒ chapter 4

4 Natural units

In physics there are 3 independent dimension full quantities, i.e

Quantity	Dimension	Units in IS
Mass	$[M]$	kg (kilogram)
Length	$[L]$	m (meter)
Time	$[T]$	s (seconds)

The dimension of any other quantity is expressed as powers of these 3

			units in IS
i.e	velocity	$[v] = \left[\frac{L}{T}\right]$	m/s
	Energy	$[E] = \left[M \frac{L^2}{T^2}\right]$	$kg \frac{m^2}{s^2} \equiv J$ (Joules)
	Action	$[S] \equiv [ET] = \left[M \frac{L^2}{T}\right]$	$kg \frac{m^2}{s} \equiv J.s$

In particle physics we need to use a much smaller unit for energy. We use the electron volt

$$eV \equiv 1.602 \times 10^{-19}\ J$$

and their multiples

$KeV \equiv 10^3 eV$, $MeV \equiv 10^6 eV$, $GeV \equiv 10^9 eV$, $TeV \equiv 10^{12} eV$

...

Two universal constants

- speed of light $c \overset{IS}{\equiv} 2.999 \times 10^{8}$ m/s

- reduced Planck constant $\hbar = 1.055 \times 10^{-34}$ J s
$= 6.582 \times 10^{-16}$ eV.s

Most particles move with velocities close to c
and for quantum systems actions are order $\hbar$
So it is convenient to define a system of units
$\equiv$ natural units in which velocities are given
in units of c and action in units of $\hbar$

$\Rightarrow$ In NU
speed of light $= 1$
Planck reduced constant $= 1$

$\Rightarrow$ In NU we can express any quantity in units
of a single dimension full quantity
We use energy and express any quantity in
units of powers of eV

Quantitatively

$$\hbar \overset{IS}{=} 6.58\times10^{-16}\,\text{eV s} \overset{\text{in NU}}{\equiv} 1 \Rightarrow 1\,\text{s} \overset{\text{in NU}}{=} 1.52\times10^{15}\,\text{eV}^{-1}$$

$$\hbar c = 1.97\times10^{-7}\,\text{eV m} \equiv 1 \Rightarrow 1\,\text{m} = 5.08\times10^{6}\,\text{eV}^{-1}$$

$\Rightarrow$ we can express time and length in units of eV^{-1}

$$\frac{\hbar}{c^2} \overset{IS}{=} 0.117\times10^{-50}\,\text{kg s} \Rightarrow 1\,\text{kg} = 8.52\times10^{50}\,\text{s} = 5.6\times10^{35}\,\text{eV}$$

$\Rightarrow$ we can express mass in units of eV

i.e. proton mass $= 1.673\times10^{-27}\,\text{kg} = 938\,\text{MeV}$

From now on we will use NU and we will not write "c's" nor "$\hbar$'s" in our equations. So we are implicitly giving velocities in units of c, actions in units of $\hbar$ and when we want to change a time from eV^{-1} to s or a length from eV^{-1} to m or a mass from eV to kg we just use the relations above.