Chapter 10

"Electroweak theory"

1) Weak interactions in isospin notation
2) Electroweak gauge theory
3) Spontaneous symmetry breaking
4) " " " of electroweak symmetry
5) the Higgs boson

1) Weak interactions in isospin notation

So far we have written some effective Lagrangians for the weak CC and NC interactions in terms of coupling of fermions to a $W^\pm$ and Z masive vector particles
For the 1s generation $(f \equiv \psi_f(x))$

$$\mathcal{L}_{weak} = -\frac{g_W}{\sqrt{2}}\Big\{\Big[\overbrace{\bar{e}\gamma^\mu\frac{(1-\gamma_5)}{2}\nu_e + \bar{d}\gamma^\mu\frac{(1-\gamma_5)}{2}u}^{J^{\mu -}_{CC}}\Big]W_\mu$$

$$+\Big[\overbrace{\bar{\nu}_e\gamma^\mu\frac{(1-\gamma_5)}{2}e + \bar{u}\gamma^\mu\frac{(1-\gamma_5)}{2}d}^{J^{\mu +}_{CC}}\Big]W^+_\mu$$

$$-g_Z\underbrace{\sum_{f=e\nu u d}\bar{f}\gamma^\mu\Big[C^f_L\frac{(1-\gamma_5)}{2}+C^f_R\frac{(1+\gamma_5)}{2}\Big]f}_{J^\mu_{NC}}\,Z_\mu$$

Data $\Rightarrow$

$$\frac{C^f_R}{Q^f} = -x \simeq 0.22\text{-}0.23 \quad \text{for } f=e,u,d \qquad C^\nu_R = 0$$

$$C^f_L = C^f_R + \frac{1}{2} \qquad f = u,\nu$$

$$= C^f_R - 1/2 \qquad f = d,e$$

and $\frac{M_W^2}{M_Z^2} \simeq \frac{g_W^2}{g_Z^2} \simeq 1-x$ and $\frac{e^2}{g_W^2} \simeq x$

We can write J^μ_{CC} and J^μ_{NC} in matrix notation

$$J^{\mu +}_{CC} = \bar\nu_L \gamma^\mu e_L + \bar u_L \gamma^\mu d_L = (\bar\nu_L, \bar e_L)\gamma^\mu \underbrace{\begin{pmatrix}0&1\\0&0\end{pmatrix}}_{\underset{T_+}{|||}} \begin{pmatrix}\nu_L\\e_L\end{pmatrix}$$

$$+ (\bar u_L \bar d_L)\gamma^\mu \begin{pmatrix}0&1\\0&0\end{pmatrix}\begin{pmatrix}u_L\\d_L\end{pmatrix}$$

$$J^{\mu -}_{CC} = \left(J^{\mu +}_{CC}\right)^\dagger = \bar e_L \gamma^\mu \nu_L + \bar d_L \gamma^\mu u_L = (\bar\nu_L \bar e_L)\gamma^\mu \overset{\overset{T_-}{|||}}{\begin{pmatrix}0&0\\1&0\end{pmatrix}}\begin{pmatrix}\nu_L\\e_L\end{pmatrix}$$

$$+(\bar u_L \bar d_L)\gamma^\mu\begin{pmatrix}0&0\\1&0\end{pmatrix}\begin{pmatrix}\nu_L\\e_L\end{pmatrix}$$

So $T_+ = \begin{pmatrix}0&1\\0&0\end{pmatrix} = \frac{1}{2}\left(\overset{\overset{\begin{pmatrix}0&1\\1&0\end{pmatrix}}{|||}}{\sigma_1} + i\overset{\overset{\begin{pmatrix}0&-i\\i&0\end{pmatrix}}{|||}}{\sigma_2}\right) \equiv T_1 + iT_2$

$T_- = \begin{pmatrix}0&0\\1&0\end{pmatrix} = \frac{1}{2}(\sigma_1 - i\sigma_2) \equiv T_1 - iT_2$

$T_i \equiv \frac{\sigma_i}{2}$ are the generators of SU(2) in doublet rep.

So if we define $L_L \equiv \begin{pmatrix}\nu_{e_L}\\e_L\end{pmatrix}$ $\quad Q_L \equiv \begin{pmatrix}u_L\\d_L\end{pmatrix}$

We can write

$$\mathcal{L}_{CC} = -\frac{g_W}{\sqrt{2}}\Big[\bar L_L\gamma^\mu(T_- W_\mu + T_+ W_\mu^+)L_L$$
$$+ \bar Q_L\gamma^\mu(T_- W_\mu + T_+ W_\mu^+)Q_L\Big\}$$

$$= -g_W\Big[\bar L_L \gamma^\mu(T_1 W^{(1)}_\mu + T_2 W^{(2)}_\mu)L_L$$
$$+ \bar Q_L\gamma^\mu(T_1 W^{(1)}_\mu + T_2 W^{(2)}_\mu)Q_L\Big]$$

with
$$W_\mu^{(1)} = \frac{1}{\sqrt{2}}(W_\mu + W_\mu^+) \qquad\qquad W_\mu = \frac{1}{\sqrt{2}}(W_\mu^{(1)} + iW_\mu^{(2)}]$$
$$\Rightarrow$$
$$W_\mu^{(2)} = \frac{i}{\sqrt{2}}(W_\mu - W_\mu^+) \qquad\qquad W_\mu^+ = \frac{1}{\sqrt{2}}(W_\mu^{(1)} - iW_\mu^{(2)})$$

For the NC

$$J_{NC}^\mu = \sum_{f=e,\nu,u,d} C_L^f\, \bar{f}_L \gamma^\mu f_L + C_R^f\, \bar{f}_R \gamma^\mu f_R$$

For $f \neq \nu$ (ie $Q_f \neq 0$) $C_R^f \neq 0$

If we put $Q_f = 0$ then

$$J_{NC}^\mu = \frac{1}{2}\bar{\nu}_L \gamma^\mu \nu_L - \frac{1}{2}\bar{e}\gamma^\mu e + \frac{1}{2}\bar{u}_L \gamma^\mu u_L - \frac{1}{2}\bar{d}_L \gamma^\mu d_L$$

$$= (\bar{\nu}_L\ \bar{e}_L)\gamma^\mu \underbrace{\frac{1}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}}_{\frac{\sigma_3}{2} \equiv T_3} \begin{pmatrix}\nu_L\\ e_L\end{pmatrix} + (\bar{u}_L \bar{d}_L)\gamma^\mu \frac{1}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}u_L\\ d_L\end{pmatrix}$$

← generator of SU(2)

$$= \bar{L}_L \gamma^\mu T_3 L_L + \bar{Q}_L \gamma^\mu T_3 Q_L$$

Now for the em interaction

$$\mathcal{L}_{em} = -e \underbrace{\sum_f Q_f \bar{f} \gamma^\mu f}_{\equiv J^\mu_{em}} A_\mu .$$

$$J^\mu{}_{em} = \sum_f Q_f \bar{f}\gamma^\mu f = \sum_f Q_f \left[\bar{f}_L \gamma^\mu f_L + \bar{f}_R \gamma^\mu f_R\right]$$

So in summary with the 7 chiral fermions of the 1st generation (ν_L e_L u_L d_L e_R u_R d_R) we can make two doublets of left-handed fermions and 3 righ-handed fermions with quantum #'s

	CC	$t_3^f = \langle T_3 \rangle$	Q_{em}^f	$\frac{Y^f}{2} \equiv Q^f - t_3^f$
$L_L = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$	SU(2) doublet	$\frac{1}{2}$ $-\frac{1}{2}$	0 -1	$-\frac{1}{2}$
$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	SU(2) doublet	$\frac{1}{2}$ $-\frac{1}{2}$	$\frac{2}{3}$ $-\frac{1}{3}$	$\frac{1}{6}$
e_R	SU(2) singlet	0	-1	-1
u_R	" "	0	$\frac{2}{3}$	$\frac{2}{3}$
d_R	" "	0	$-\frac{1}{3}$	$-\frac{1}{3}$

we call $Y^f \equiv$ hypercharge

So the combinations L_L, Q_L, e_R, u_R, d_R have well defined quantum # for $SU(2)_{\text{Left}}$

$\uparrow$ only for left-handed

and $U(1)_{\frac{Y}{2}}$

② Electroweak gauge theory

We have written the Lagrangians of weak cc and NC interactions together with em interactions of the fermions of one generation in terms of the interactions of 5 chiral combinations: Q_L, L_L, e_R, u_R, d_R which have well defined charges under $SU(2)_{Left} \times U(1)_{Y/2}$

So we can try to see how these interactions compare with what we would get from a gauge theory for those 5 combinations as matter content and $\underbrace{SU(2)_{Left} \times U(1)_{Y/2}}_{\parallel \; G_{EW}}$ gauge group

To build this lagrangian we start with the free Lagrangian for these 5 combinations

$$\mathcal{L}_{free} = \sum_{i=1}^{5} \bar{\Psi}_i \, i\gamma^\mu \partial_\mu \Psi_i$$

$\Psi_1 = L_L$

$\Psi_2 = Q_L$

$\Psi_3 = e_R$

$\Psi_4 = u_R$

$\Psi_5 = d_R$

under $SU(2)_{LEFT}$ Ψ_i transforms as

For $i = 1, 2$

$$\Psi_i'(x) = \Big[\underbrace{e^{-i\sum_{a=1}^{3} \overset{\text{parameter of transf}}{\alpha_a(x)} \, \overset{= \frac{\sigma_a}{2} \text{ generator}}{T_a}}}_{2\times 2 \text{ matrix}}\Big] \Psi_i$$

$$\bar{\Psi}_i'(x) = \bar{\Psi}_i \Big[e^{+i\sum_a \alpha_a(x) T_a}\Big]$$

$$T_a = \frac{\sigma_a}{2} \Rightarrow [T_a, T_b] = i\sum_c \epsilon_{abc} T_c$$

For $i=3,4,5$ Ψ_i is a righ-hande fermion ⇒ singlet of $SU(2)_{Left}$

$$\Rightarrow \Psi'_i(x) = \Psi_i(x) \Rightarrow \bar{\Psi}'_i(x) = \bar{\Psi}_i(x)$$

Under $U(1)_{Y/2}$ for $i=1..5$

$$\Psi'_i(x) = e^{-i\beta(x)\frac{Y_i}{2}}\Psi_i(x) \Rightarrow \bar{\Psi}_i(x) = \bar{\Psi}_i(x)\, e^{i\beta(x)\frac{Y_i}{2}}$$

In order to build a gauge invariant Lag we need to define a covariant derivative and for that we need 3 gauge bosons for $SU(2)_{Left}$ which we are going to call $W_\mu^{(a)}$ $a=1,2,3$ and one for $U(1)_{Y/2}$ which we call B_μ.

With this

$$\mathcal{L}_{ew}^{fermions} = \sum_{i=1}^{5} \bar{\Psi}_i\, \gamma^\mu [D_\mu]_i\, \Psi_i$$

Where for $i=1,2$

coupling constants

$$D^\mu\Psi_i = \left[\partial^\mu + ig\sum_{a=1}^{3} T_a W^{\mu(a)} + ig'\frac{Y_i}{2}B^\mu\right]\Psi_i$$

For $i=3,4,5$

$$D^\mu\Psi_i = \left[\partial^\mu + ig'\frac{Y_i}{2}B^\mu\right]\Psi_i$$

Let us define

$$W_\mu \equiv \frac{1}{\sqrt{2}}\left(W_\mu^{(1)} + i W_\mu^{(2)}\right) \equiv W_\mu^-$$

$$W_\mu^+ \equiv \frac{1}{\sqrt{2}}\left(W_\mu^{(1)} - i W_\mu^{(2)}\right) \equiv W_\mu^+$$

$$A_\mu = \cos\theta_W \, B_\mu + \sin\theta_W \, W_\mu^{(3)}$$

$$Z_\mu = -\sin\theta_W \, B_\mu + \cos\theta_W \, W_\mu^{(3)}$$

↑ as of now just a rotation of basis for the neutral bosons

We can rewrite $\mathcal{L}_{EW}$ in this basis and see what we get

1) Pieces with $W_\mu^{(1)}$ and $W_\mu^{(2)}$

$$-g \left\{ \bar{L}_L \gamma^\mu \left(T_1 W_\mu^{(1)} + T_2 W_\mu^{(2)}\right) L_L + \bar{Q}_L \gamma^\mu \left(T_1 W_\mu^{(1)} + T_2 W_\mu^{(2)}\right) Q_L \right.$$

$$= -\frac{g}{\sqrt{2}} \left\{ \left[\bar{L}_L \gamma^\mu T^+ L_L + \bar{Q}_L \gamma^\mu T^+ Q_L\right] W_\mu^+ + \left[\bar{L}_L \gamma^\mu T^- L_L + \bar{Q}_L \gamma^\mu T^- Q_L\right] W_\mu \right.$$

$$\equiv \mathcal{L}_{CC} \quad \text{with } g_W = g$$

2) Pieces with $W_\mu^{(3)}$ and B_μ from

$$-(\bar\nu_L\,\bar e_L)\,\gamma^\mu\left(g\,T_3\,W_\mu^{(3)} + g'\,\frac{Y^L}{2}\,B_\mu\right)\begin{pmatrix}\nu_L\\ e_L\end{pmatrix}$$

$$-(\bar u_L\,\bar d_L)\,\gamma^\mu\left(g\,T_3\,W_\mu^{(3)} + g'\,\frac{Y^Q}{2}\,B_\mu\right)\begin{pmatrix}u_L\\ d_L\end{pmatrix}$$

$$-\bar e_R\,\gamma^\mu\,\frac{Y^{e_R}}{2}B_\mu e_R - \bar u_R\,\gamma^\mu\,\frac{Y^{u_R}}{2}B_\mu u_R - \bar d_R\,\gamma^\mu\,\frac{Y^{d_R}}{2}B_\mu d_R$$

$$=\sum_{f=\nu,e,u,d} -\bar f_L\,\gamma^\mu\left(g\,t_3^{f_L}\,W_\mu^{(3)} + g'\,\frac{Y^{f_L}}{2}\,B_\mu\right)f_L$$

$$-\bar f_R\,\gamma^\mu\,\frac{Y^{f_R}}{2}\,B_\mu f_R$$

$$=\sum_f -A_\mu\left[\bar f_L\,\gamma^\mu\left(g\,t_3^{f_L}\sin\theta_W + g'\,\frac{Y^{f_L}}{2}\cos\theta_W\right)f_L\right.$$

$$\left.+\,\bar f_R\,\gamma^\mu\,g'\,\frac{Y^{f_R}}{2}\cos\theta_W\,f_R\right]$$

$$-Z_\mu\left[\bar f_L\,\gamma^\mu\left(g\,t_3^{f_L}\cos\theta_W - g'\,\frac{Y^{f_L}}{2}\sin\theta_W\right)f_L\right.$$

$$\left.-\,\bar f_R\,\gamma^\mu\,g'\,\frac{Y^{f_R}}{2}\sin\theta_W\,f_R\right]$$

Let us define $e = g\sin\theta_W$. and $\tan\theta_W = \frac{g'}{g}$

$\Rightarrow$ piece with A^μ

$$-A_\mu\,e\left[\bar f_L\,\gamma^\mu\Big(\underbrace{t_3^{f_L} + \frac{Y^{f_L}}{2}}_{\equiv\, Q_f}\Big)f_L + \bar f_R\,\gamma^\mu\,\underset{\equiv\, Q^f}{\frac{Y^{f_R}}{2}}\,f_R\right.$$

$$= -A_\mu\,e\,Q^f\left(\bar f_L\gamma^\mu f_L + \bar f_R\gamma^\mu f_R\right) = -A_\mu e\,\bar f\gamma^\mu f = \mathcal{L}_{int,em}$$

the piece with Z_μ

$$-Z_\mu \frac{e}{\sin\theta_W\cos\theta_W}\Big[\bar{f}_L\gamma^\mu\left(t_3^{f_L}\cos^2\theta_W - \frac{Y^{f_L}}{2}\sin^2\theta_W\right)f_L$$

$$-\bar{f}_R\gamma^\mu \underbrace{\frac{Y^{f_R}}{2}}_{\equiv Q^f}\sin^2\theta_W f_R$$

now $t_3^{f_L}\cos^2\theta_W - \frac{Y^{f_L}}{2}\sin^2\theta_W$

$= t_3^{f_L}\cos^2\theta_W + (t_3^{f_L} - Q^f)\sin^2\theta_W$

$= t_3^{f_L} - Q^f\sin^2\theta_W$

So the terms with Z^μ

$$= -Z^\mu\frac{e}{\sin\theta_W\cos\theta_W}\Big[\bar{f}_L\gamma^\mu\overbrace{(t_3-\sin^2\theta_W Q^f)}^{\equiv C_L^f}f_L$$

$$-\bar{f}_R\gamma^\mu\underbrace{\sin^2\theta_W Q^f}_{\equiv C_R^f}f_R\Big]$$

$= \mathcal{L}_{NC}$ with

$$g_Z = \frac{e}{\sin\theta_W\cos\theta_W} = \frac{g}{\cos\theta_W}$$

and $x = \sin^2\theta_W \simeq 0.23$

$$\Rightarrow \frac{e^2}{g^2} = \sin^2\theta_W \simeq 0.23$$

In summary: starting with a gauge theory with gauge group $SU(2)_{Left} \times U(1)_{Y/2}$

We have been able to construct.

$$\mathcal{L}_{EW}^{fermion} = \mathcal{L}_{free}^{fermion} + \mathcal{L}_{CC} + \mathcal{L}_{NC} + \mathcal{L}_{em}^{int}$$

To do so we need to combine the T^3 part of $SU(2)_L$ and $U(1)_{Y/2}$ in two linear combinations with rotation angle θ_W with $\sin^2\theta_W \simeq 0.23$ and this rotation has to be related to the ratio of the coupling constants:

$$\tan\theta_W = \frac{g'}{g} \quad , \quad e = g\sin\theta_W = g'\cos\theta_W$$

the full Lagrangian would also contain the Lag for the gauge bosons

$$\mathcal{L}_{EW}^{TOT} = \sum_{i=1}^{5} \bar{\Psi}_i \gamma^\mu D_\mu \Psi_i - \frac{1}{4}\sum_{a=1}^{3} W_{\mu\nu}^{(a)} W^{\mu\nu(a)} - \frac{1}{4} B_{\mu\nu}B^{\mu\nu}$$

with $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$

$$W_{\mu\nu}^{(a)} = \partial_\mu W_\nu^{(a)} - \partial_\nu W_\mu^{(a)} - g\sum_{bc} \epsilon_{abc} W_\mu^{(b)} W_\nu^{(c)}$$

But notice:

- the four gauge bosons are massless as required by gauge invariance because a mass term $M_Z^2 Z^\mu Z_\mu$, $M_W^2 W_\mu W^{\mu +}$ would break gauge invariance

- the fermions are also massless because a fermion mass

$$m\bar{f}f = m(\bar{f}_R f_L + \bar{f}_L f_R) \Rightarrow \text{Not } SU(2)_L \text{ gauge inv.}$$

SU(2) Left singlet → $\bar{f}_R$

↑ f_L part of an SU(2)$_{\text{Left}}$ doublet

- We do not know what generate the rotation

$$(W_3^\mu, B^\mu) \longrightarrow (Z^\mu, A^\mu)$$

the solution to all this will arise from spontaneous breaking of the EW symmetry.

③ Spontaneous symmetry breaking

To give mass to the $W^\pm$ and Z and the fermions we need to break the gauge symmetry $SU(2)_L \times U(1)_Y$. We could break it "explicitly" just adding the mass terms to the $\mathcal{L}_{EW}$. But then we would lose all the bonuses of gauge theory, in particular the theory would be non renormalizable.

In order to break the symmetry in the mass spectrum of the particles but not in the Lagrangian we need to notice that particles are quantum excitations above a ground state. So if the ground state above which we define the quantum excitations breaks the EW symmetry, then the $\mathcal{L}_{EW}$ in terms of the quantum excitations (ie in terms of the fields for the particles will "apparently" not be gauge invariant. We call this form of breaking a symmetry ≡ "Spontaneous symmetry breaking" (SSB)

To introduce in our theory the possibility of SSB we need to add "something" to the theory which allows us to describe such non-trivial ground state. The minimum requirement would be to add a field with a potential which is minimized for a non-zero value of the field

Lorentz invariance ⇒ the ground state cannot have spin
⇒ the field must be scalar $(S=0)$

Let us take a complex scalar field

real

$$\phi(x) = \frac{1}{\sqrt{2}}\left(\phi_1(x) + i\,\phi_2(x)\right)$$

The most general Lag for this field ($\mathcal{L}$ must be real made with ϕ or $\partial\phi$ and dim E^4)

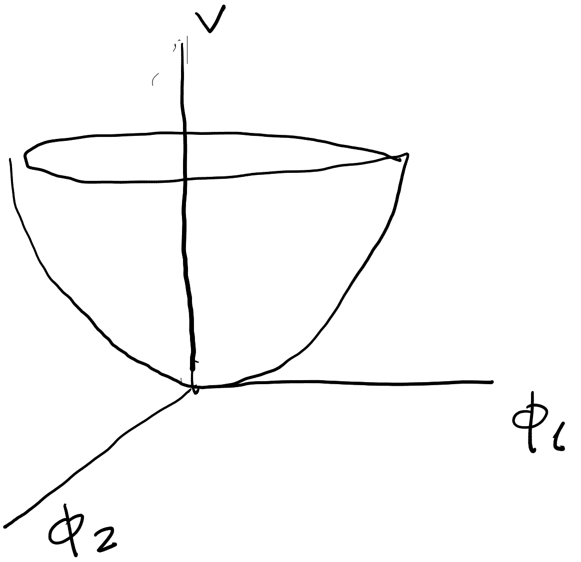
$$\mathcal{L}_\phi = (\partial_\mu\phi^*)(\partial^\mu\phi) \underbrace{-\mu^2\phi^*\phi - \lambda(\phi^*\phi)^2}_{\substack{|||\\ -V(\phi)}}$$

$\lambda > 0$ so potential is bounded for $|\phi| \to$ large

Notice that $\mathcal{L}_\phi$ is invariant under $\phi \to e^{i\alpha}\phi$ ($\equiv$ global $U(1)$)

If we plot $V(\phi) = \mu^2|\phi|^2 + \lambda|\phi|^4$ as functions of ϕ_1 and ϕ_2

- For $\mu^2 > 0$



Notice that minimum cond.

$$\frac{dV}{d|\phi|} = 2\mu^2|\phi| + 4\lambda|\phi|^3 = 0$$

$$\Rightarrow |\phi|_{min} = 0$$

$$\Rightarrow \phi_{1min} = \phi_{2min} = 0$$

and $V_{min} = 0$

- If $\mu^2<0$ $V(\phi) = -|\mu|^2 \cdot |\phi|^2 + \lambda |\phi|^4$

$$\frac{dV}{d|\phi|} = -2|\mu|^2|\phi| + 4\lambda|\phi|^3 = 0$$

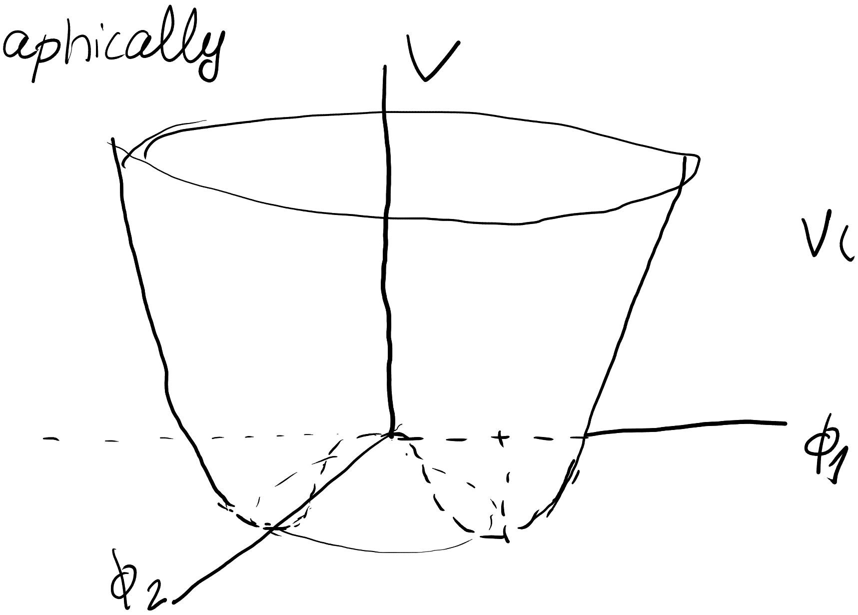
$$\Rightarrow 2|\phi|^2 = \frac{|\mu|^2}{\lambda} \equiv v^2 > 0$$

or $|\phi| = 0$

but $V\left(\frac{|\mu|^2}{2\lambda}\right) = -\frac{|\mu|^4}{4\lambda} < 0 = V(0) \Rightarrow 2|\phi|^2_{min} = \frac{|\mu|^2}{\lambda} \equiv v^2$

$|\phi| = 0$ local max

Graphically



$$V(\phi) = V_{min} + \lambda\left(|\phi|^2 - \frac{v^2}{2}\right)^2$$

Notice that there is a continuous of states ϕ all with $|\phi|_{min}$ all with V_{min}. They are

$$\phi = |\phi|_{min}\, e^{i\alpha} \quad \text{with } 0 \leq \alpha \leq 2\pi$$

$\Rightarrow$ they are related by a global $U(1)$ symmetry as it should because $\mathcal{L}$ has that symmetry

Now when we quantize this theory we need to fix which is the ground state above which we define our quantum excitations(ie our particles)

For example we can chose

$$\phi_0 = \frac{1}{\sqrt{2}} v + i 0 \qquad \left(\phi_{1min} = \frac{v}{\sqrt{2}} , \ \phi_{2min} = 0\right)$$

So our quantized field will be

$$\phi(x) \equiv \phi_0 + \tilde{\phi}(x)$$

particle field for a scalar particle

In terms of $\tilde{\phi}$

$$\mathcal{L}_\phi = (\partial_\mu \tilde{\phi})(\partial_\mu \tilde{\phi})^* - v^2 \lambda \left(\frac{v^2}{2} + |\tilde{\phi}|^2 + \frac{2v}{\sqrt{2}} \mathrm{Re}\,\tilde{\phi}\right)$$

$$+ \lambda \left(\frac{v^2}{2} + |\tilde{\phi}|^2 + 2\frac{v}{\sqrt{2}} \mathrm{Re}\,\tilde{\phi}\right)^2$$

notice this is not inv under $\tilde{\phi} \to e^{i\alpha} \tilde{\phi}$

$\Rightarrow$ the global U(1) symmetry has been "spontenously broken" by choosing to quantize above a given ϕ_0 which fixes the "direction" in the (ϕ_1, ϕ_2) plane for the ground state.

We can always write instead of $\phi = \frac{1}{\sqrt{2}} v + \underset{\uparrow}{\tilde{\phi}}$

complex

$$\phi(x) = \frac{1}{\sqrt{2}}\,(v + h(x))\, e^{i\frac{\xi(x)}{v}}$$

→ real

$\mathcal{L}_\phi$ in terms of h and ξ

since $\partial_\mu \phi = \frac{1}{\sqrt{2}}\left(\partial_\mu h + i\frac{\partial_\mu \xi}{v}(v+h)\right) e^{i\frac{\xi}{v}}$

$$\mathcal{L}_\phi = \frac{1}{2}(\partial_\mu h)(\partial^\mu h) + \frac{1}{2}\partial_\mu \xi\, \partial^\mu \xi\left(1 + 2\frac{h}{v} + \frac{h^2}{v^2}\right)$$

$$- \frac{\mu^2}{2}(v+h)^2 - \frac{\lambda}{4}(v+h)^4$$

$$= \underbrace{\frac{1}{2}(\partial_\mu h)(\partial^\mu h) - h^2\overbrace{\left(+\frac{\mu^2}{2} + 6\frac{v^2}{4}\lambda\right)}^{-\mu^2 = |\mu|^2}} + \underbrace{(\partial^\mu \xi)\partial^\mu \xi)}_{\uparrow}$$

$$+ \partial^\nu \xi\, \partial_\mu \xi\, \frac{h}{v} + \partial^\mu \xi\, \partial_\mu \xi\, \frac{h^2}{v^2}$$

Free Lag for a neutral scalar particle with mass $m_h = \sqrt{2|\mu|^2} = \sqrt{2\lambda}\, v$

Lag for massless scalar
|||
Goldstone boson

interaction terms

notice that the linear pieces in h

$$-\frac{\mu^2}{2}(2vh) - \frac{\lambda}{4}\,4v^3 h = -\mu^2 vh + \mu^2 vh = 0$$

Let us repeat the same but lets take $\mathcal{L}_\phi$ to be gauge invariant under U(1) and ϕ charged under U(1) with charge $Q_\phi = -1$

$\Rightarrow$ under U(1) $\phi \to \phi\, e^{-i\alpha(x) Q_\phi}$

$$\mathcal{L}_\phi = (D_\mu \phi)^*(D_\mu \phi) - \mu^2 \phi^* \phi - \lambda (\phi^* \phi)^2$$

with $D_\mu \phi = \partial_\mu \phi - i\underline{e}\, A_\mu \phi$ U(1) gauge boson

$\nwarrow$ vector field

the introduction of A_μ does not change the potential because $V(\phi) = \mu^2 |\phi|^2 + \lambda |\phi|^4$ is GI

So again for $\mu^2 < 0$ we have the same continuos of ground state

And again we can chose to quantize about a specific one

$$\phi_0 = \frac{v}{\sqrt{2}}(1 + i0)$$

and the quantized field will be $\phi = \frac{v + h(x)}{\sqrt{2}}\, e^{\frac{i\xi(x)}{v}}$

$$\Rightarrow D_\mu \phi = \frac{1}{\sqrt{2}}\left[\partial_\mu h + \frac{i(v+h)}{v}\partial_\mu \xi - ieA_\mu(v+h)\right] e^{\frac{i\xi(x)}{v}}$$

But notice that GI ⇒ physics is the same if we use A_μ or $A'_\mu = A_\mu - \frac{1}{e}\partial_\mu \xi$

and in terms of A'_μ

$$D_\mu \phi = \frac{1}{\sqrt{2}}\left(\partial_\mu h - ie\, A'_\mu (v+h)\right) e^{i\xi/v}$$

$$\Rightarrow (D_\mu \phi)^*(D_\mu \phi) = \frac{1}{2}\left[(\partial_\mu h)\partial^\mu h) + e^2 A'_\mu A'^\mu (v+h)^2\right]$$

⇒ ξ disappears from $\mathcal{L}_\phi$.

We say that the would be goldstone boson has been "eaten" by A_μ

In this gauge

$$\mathcal{L}_{\phi,A} = \mathcal{L}_\phi - \frac{1}{4} F_{\mu\nu}F^{\mu\nu}$$

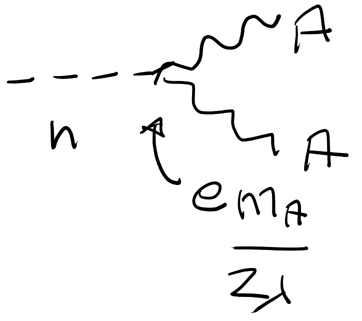
$$= \frac{1}{2}(\partial_\mu h)(\partial^\mu h) - \overbrace{\left(\frac{\mu^2}{2} + \frac{6}{4}\lambda v^2\right)}^{-\mu^2 = |\mu|^2} h^2 \rightarrow$$ Lag for real scalar of mass $m_h = \sqrt{2|\mu|^2} = \sqrt{2}\lambda v$

$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}e^2 v^2 A^\mu A_\mu \leftarrow$$ Lag for real vector with mass

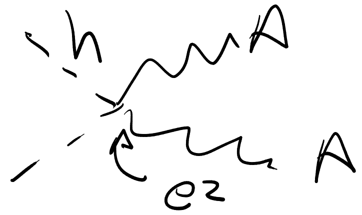
$$m_A^2 = e^2 v^2$$

$+ \ldots$

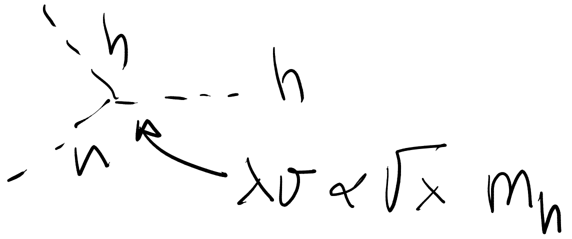
$$-\underbrace{e v}_{\overset{|||}{e\frac{m_A}{2\lambda}}} A^\mu A_\mu h \longrightarrow \text{vertex}$$



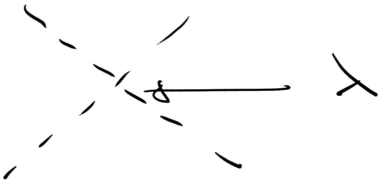
$$+\frac{e^2}{2} A^\mu A_\mu h^2 \longrightarrow \text{vertex}$$



$$-\lambda v h^3 \longrightarrow \text{vertex}$$



$$-\frac{\lambda}{4} h^4 \longrightarrow \text{vertex}$$



So after expressing $\mathcal{L}$ in terms of the particle fields $h(x)$ and $A_\mu(x) \equiv$ quantum excitations above the specific Φ_0 we have generated a mass for the gauge boson to which the Φ couples to. The generated mass is proportional to the vev of the scalar and the gauge coupling constant

counting degrees of freedom

- before breaking : massless vector (2) + complex scalar (2)
- after " : massive vector (3) + real scalar (1)

④ Spontaneous breaking of electroweak symmetry

- SSB ≡ breaking of symmetry by choice of ground state above which the theory is quantized
- Simplest implementation ≡ add a complex scalar with a potential which is minimum at a non-zero value of the scalar
- Symmetry ⇒ there is a degenerate ground state
- After chosing one of these ground states as the state above which we define the quantum excitations (≡ particles) the Lag written in terms of the particle states is apparently not symmetric
- the spectrum of particles contains a massive real scalar and massless scalars (≡ Goldstone bosons)
- If the symmetry was local (≡ gauge) and the complex scalar is charged under the gauge symmetry the spectrum of particles after SSB contains a massive real scalar and the gauge boson adquire a mass

to go from $\mathcal{L}_{EW}$ → real world with $M_Z \neq 0$
$M_W \neq 0$ but $M_\gamma = 0$

we need to break

$$SU(2)_{Left} \times U(1)_{Y/2} \longrightarrow U(1)_{em}$$

- To break $SU(2)_L \Rightarrow \phi$ must not be a $SU(2)_L$ singlet. the lowest representation is that Φ is a doublet

$$\Phi = \begin{pmatrix} \phi_3 + i\phi_4 \\ \phi_1 + i\phi_2 \end{pmatrix} \qquad \phi_i \text{ all real}$$

- We want $U(1)_{em}$ to remain unbroken

$\Rightarrow \Phi$ must have some component with $Q_{\phi_i} \neq 0$

since $Q_{\phi_i} = \frac{Y^\phi}{2} + \underset{\pm 1/2}{t_3^f} \Rightarrow \frac{Y^\phi}{2} = \mp \frac{1}{2}$

the usual choice is $\frac{Y^\phi}{2} = \frac{1}{2}$

$\Rightarrow \phi_1 + i\phi_2$ has $Q = 0$

the Lag for Φ

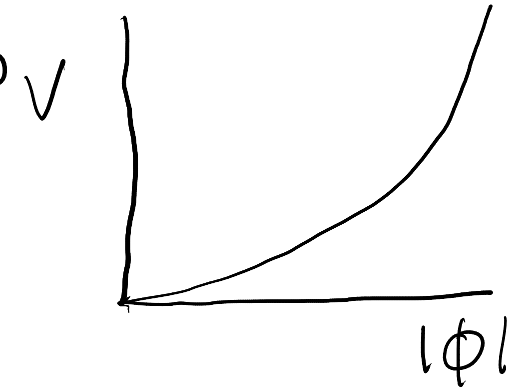
$$\mathcal{L}_\phi = (D^\mu\phi)^+(D_\mu\phi) - [\mu^2\phi^+\phi + \lambda(\phi^+\phi)^2]$$

with $D^\mu\phi = (\partial_\mu + ig\sum_{a=1}^{3} T^a W_\mu^{(a)} + i\frac{g'}{2}B_\mu)\Phi$

$$= \left\{\begin{bmatrix}\partial_\mu & 0\\ 0 & \partial_\mu\end{bmatrix} + \frac{i}{2}\begin{bmatrix} gW_\mu^3 + g'B_\mu & g(W_\mu^{(1)} - iW_\mu^{(2)}) \\ g(W_\mu^{(1)} + iW_\mu^{(2)}) & -gW_\mu^{(3)} + g'B_\mu\end{bmatrix}\right\}\begin{pmatrix}\phi_3 + i\phi_2\\ \phi_1 + i\phi_2\end{pmatrix}$$

If we plot V as function of $|\phi|^2 = \phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2$

For $\mu^2 > 0$



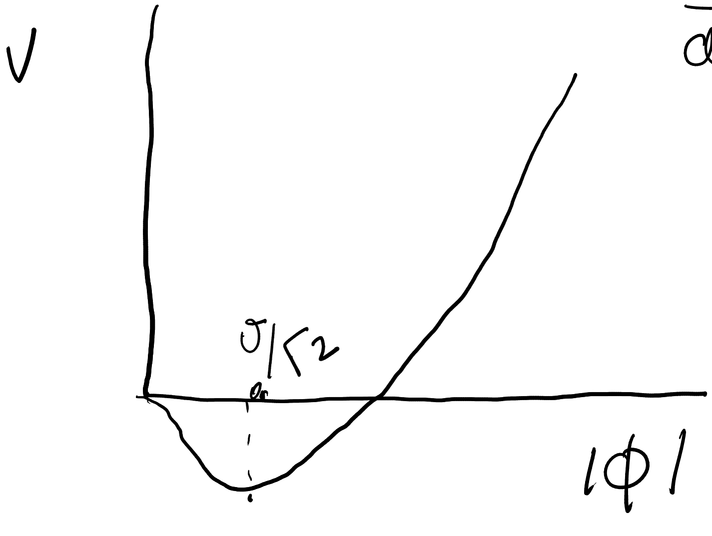
$$\frac{dV}{d|\phi|} = |\phi|(2\mu^2 + 4\lambda|\phi|^2) = 0$$

$$\Rightarrow |\phi|_{min} = 0$$

$$\Rightarrow \phi_i = 0 \quad i = 1, 2, 3, 4$$

For $\mu^2 < 0$

$$\frac{dV}{d|\phi|} = 0 \begin{cases} |\phi| = 0 \\ |\phi|^2_{min} = -\frac{\mu^2}{2\lambda} = \frac{v^2}{2} \end{cases}$$



So again there is a continuous of minima of states with

$$|\Phi|^2 = \phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2 = \frac{v^2}{2}$$

We spontaneously break the symmetry when we chose one of these states as the ground states of our quantum theory.

Since we want that the ground state has no em charge we must chose $\phi_{3min} = \phi_{4min} = 0$

We can chose any state with $\phi_{2min}^2 + \phi_{1min}^2 = \frac{v}{2} = -\frac{\mu^2}{2\lambda}$

ie $\Phi_0 = \begin{pmatrix} 0 + i0 \\ \frac{v}{\sqrt{2}} + i0 \end{pmatrix}$

So the quantum field

$$\phi(x) = \frac{1}{\sqrt{2}} \begin{bmatrix} h_3(x) + i h_4(x) \\ v + h_1(x) + i h_2(x) \end{bmatrix}$$

h_i = real scalars

we can always write it like this

$$\equiv \frac{1}{\sqrt{2}} \underbrace{e^{i \sum_a T_a \xi^a(x)}}_{=U} \overbrace{\begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}}^{\Phi_u}$$

$T_a = \frac{\sigma_a}{2}$

3 real scalar

real scalar

So $D_\mu \Phi = \frac{1}{\sqrt{2}}\left\{ \partial_\mu\left[U\begin{pmatrix}0\\v+h\end{pmatrix}\right] + \left(ig\sum_a T_a W^a_\mu + i\frac{g'}{2}B_\mu\right) U\begin{pmatrix}0\\v+h\end{pmatrix}\right\}$

$$= \frac{1}{\sqrt{2}}\left\{U\partial_\mu\begin{pmatrix}0\\v+h\end{pmatrix} + \partial_\mu U \begin{pmatrix}0\\v+h\end{pmatrix} + \frac{ig'}{2}B_\mu U\begin{pmatrix}0\\v+h\end{pmatrix} + ig\sum_a T_a W^a_\mu U \begin{pmatrix}0\\v+h\end{pmatrix}\right\}$$

If we define

$$U ig\sum_a T^a W'^a_\mu = ig\sum T^a W^a_\mu + \partial_\mu U \quad (*)$$

Then $D_\mu \Phi = \frac{1}{\sqrt{2}} U\left\{\partial_\mu\begin{pmatrix}0\\v+h\end{pmatrix} + \left(ig\sum_a T_a W'^a_\mu + i\frac{g'}{2}B_\mu\right)\begin{pmatrix}0\\v+h\end{pmatrix}\right\}$

$$= U D_\mu \Phi_U$$

$$(D_\mu\Phi)^\dagger(D^\mu\Phi) = (D'_\mu \Phi_U)^\dagger \underbrace{U^\dagger U}_{I} D'^\mu \Phi_U$$

$$= \left[D'^\mu \begin{pmatrix}0\\v+h\end{pmatrix}\right]^\dagger \left[D^{\mu'} \begin{pmatrix}0\\v+h\end{pmatrix}\right]$$

But the condition (*) is just a SU(2) gauge transformation

To see this explicitly we can expand a 1st order

$$U^{-1}\left(\frac{1}{ig}\right)(*) = \sum_a T^a W_\mu'^a = U^{-1}\sum_a T^a W_\mu^a U - \frac{1}{g}U^{-1}\partial_\mu U$$

$$\simeq \left(1-\sum_b \frac{\xi^b}{v}T^b\right)\sum_a T^a\left(1+i\sum_b \frac{\xi^b T^b}{v}\right)W_\mu^a - \frac{1}{g}\, i\sum_c \frac{\partial_\mu \xi^c}{v}T^c$$

1st order in ξ ↓

$$\simeq \sum_a T^a W_\mu^a - \frac{i}{v}\sum_{ab}\underbrace{[T^a, T^b]}_{\sum_c i\epsilon^{abc}T^c}\xi^b W_\mu^a + \frac{1}{gv}\sum_c \partial_\mu \xi^c T^c$$

multiplying by T^d and using $T^a T^d = \delta^{ad}$

$$W_\mu'^d = W_\mu^d + \frac{1}{gv}\partial_\mu \xi^d + \sum_{ab}\epsilon_{abd}\,\xi^b W_\mu^{(a)}$$

this is exactly the gauge transformation for the gauge bosons of SU(2)

Gauge invariance ⇒ physics described by $W_\mu^{(a)}$ and $W_\mu^{(a)\prime}$ must be same

So in the "!" gauge ≡ unitary gauge

$$(D_\mu \Phi)^\dagger (D^\mu \Phi) = (D_\mu \Phi_U)^\dagger (D_\mu \Phi_U)$$

$$= \frac{1}{2}\left\{\begin{pmatrix} 0 \\ \partial_\mu h \end{pmatrix} + \frac{i}{2}\begin{bmatrix} -g W_\mu^{(3)} + g' B_\mu & \overbrace{g(W_\mu^{(1)} - i W_\mu^{(2)})}^{= \sqrt{2} W_\mu^+} \\ \underbrace{g(W_\mu^{(1)} + i W_\mu^{(2)})}_{= \sqrt{2} W_\mu} & -g W_\mu^{(3)} + g' B_\mu \end{bmatrix}\begin{pmatrix} 0 \\ v+h \end{pmatrix}\right\}^\dagger$$

$$\left\{\begin{pmatrix} 0 \\ \partial^\mu h \end{pmatrix} + \frac{i}{2}\begin{bmatrix} g W^{\mu(3)} + g' B^\mu & g(W^{\mu(1)} - i W^{\mu(2)}) \\ g(W^{\mu(1)} + i W^{\mu(2)}) & -g W^{\mu(3)} + g' B^\mu \end{bmatrix}\begin{pmatrix} 0 \\ v+h \end{pmatrix}\right\}$$

$$= \frac{1}{2}(\partial_\mu h)(\partial^\mu h) + \left[\frac{g}{2}(v+h)\right]^2 W_\mu^+ W^\mu$$

$$+ \frac{1}{8}\left[g' B_\mu - g W_\mu^{(3)}\right]\left[g' B^\mu - g W^{\mu(3)}\right](v+h)^2$$

(the ξ^a dissapear)

If we define $\tan \theta_W = \dfrac{g'}{g}$

then $$-g' B_\mu + g W_\mu^{(3)} = \frac{g}{\cos\theta_W}\left(-\sin\theta_W B_\mu + \cos\theta_W W_\mu^{(1)}\right)$$

$$= \frac{g}{\cos\theta_W} Z_\mu$$

So if we define $M_W = \frac{1}{2} g v$

$$M_Z = \frac{1}{2}\frac{g}{\cos\theta_W} = \frac{1}{2} g_Z v = \frac{M_W}{\cos\theta_W}$$

then

$$(D_\mu \Phi_v)^\dagger (D_\mu \Phi_v) = \frac{1}{2}(\partial^\mu h)(\partial_\mu h) +$$

$$+ M_W^2 W_\mu^+ W^\mu \left(1+\frac{h}{v}\right)^2 + \frac{1}{2} M_Z^2 Z_\mu Z^\mu \left(1+\frac{h}{v}\right)^2$$

So in this gauge W_μ and the combination $Z_\mu = \cos\theta_W W_\mu^{(3)} - \sin\theta_W B_\mu$ with $\tan\theta_W = \frac{g'}{g}$ acquire a mass and the orthogonal combination $A_\mu = \cos\theta_W B_\mu + \sin\theta_W W_\mu^{(3)}$ remains massless

M_W and M_Z verify that

$$\rho = \frac{g_Z^2 M_W^2}{g_W^2 M_Z^2} = 1$$

Also

$$\left.\begin{aligned} &\sin^2\theta_W \sim 0.23 \\ &e = \sqrt{\frac{4\pi}{137}} \\ &M_W \sim 80 \text{ GeV} \end{aligned}\right\} \Rightarrow v = \frac{2M_W}{g} = \frac{2M_W \sin\theta_W}{e} = 246 \text{ GeV}$$

Counting degrees of freedom.

- Before SSB 1 complex scalar doublet $= 4$ } 12
  $(B^\mu, W^{\mu(a)})$ 4 massless vectors $2\times 4 = 8$
- After SSB 1 real scalar 1 } 12
  3 massive vectors 3×3
  1 massless " 2

⑤ the Higgs Boson

We have wullen the $SU(2)_L \times U(1)_{Y/2}$ gauge invariant Lag for ϕ

$$\mathcal{L}\phi = (D^\mu \phi^\dagger)(D_\mu \phi) - (\mu^2 \phi^\dagger \phi + \lambda(\phi^\dagger \phi)^2)$$

We spontaneously break the symmetry by choosing ground state $\phi_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$

with $v = \frac{-\mu^2}{\lambda}$

So the quantum field in.

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + \underset{\uparrow \text{ real scalar}}{h(x)} \end{pmatrix} e^{-i \sum_a T_a \xi^a}$$

and using gauge invariance we have elimated the would be gauge bosons $\xi^a(x)$ and got

$$(D^\mu \phi)^\dagger (D_\mu \phi) = \frac{1}{2} (\partial^\mu h)(\partial_\mu h) + M_W^2 W_\mu W^{\mu+} (1 + \frac{h}{v})^2$$
$$+ \frac{1}{2} M_Z^2 Z^\mu Z^\mu (1 + \frac{h}{v})^2$$

the potential in terms of h

$$V(\Phi) = \mu^2 \frac{1}{2}(v+h)^2 + \frac{\lambda}{4}(v+h)^4$$

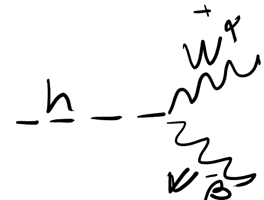
$$= h(\underbrace{\mu^2 v + \lambda v^3}_{0}) + h^2(\underbrace{\frac{\mu^2}{2} + 6\lambda v^2}_{\overset{|||}{-\mu^2}}) + \lambda h^3 v + \frac{\lambda}{4}h^4$$

$$+ \underbrace{\frac{1}{2}\mu^2 v^2 + \frac{\lambda}{4}v^4}_{} \equiv -\frac{\lambda}{4}v^4 \quad \equiv V_{min}$$

So altogether

$\mathcal{L}_h = \frac{1}{2}[(\partial_\mu h)^2 + 2\mu^2 h^2] \to$ massive real scalar with mass $m_h = \sqrt{-2\mu^2} = \sqrt{2\lambda}\, v$

$+ h W^+_\mu W^\mu \frac{2M_W^2}{v} \equiv g M_W \longrightarrow$ vertex



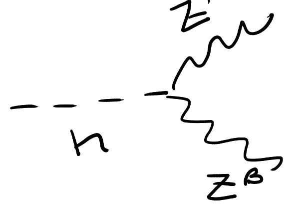
$$-i\frac{2M_W^2}{v} g^{\alpha\beta} = -i g M_W g^{\alpha\beta}$$

$+ h^2 W^+_\mu W^{\mu +} \frac{M_W^2}{v^2} \longrightarrow$ vertex

$\overset{|||}{\frac{1}{4}g^2}$

$$-i\frac{g}{2} g^{\alpha\beta}$$

$+ h Z^\mu Z_\mu \frac{M_Z^2}{v} \equiv 2\frac{g}{c_W} M_Z \longrightarrow$ vertex



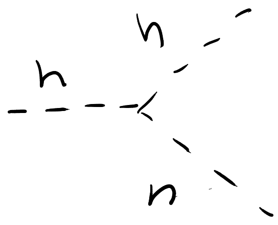
$$-i\frac{2M_Z^2}{v} g^{\alpha\beta} = -i\boxed{\frac{g}{c_W}} M_Z g^{\alpha\beta}$$

$$\frac{g}{c_W} \equiv g_Z$$

$+ hh Z^\mu Z_\mu \frac{M_Z^2}{2v^2} \longrightarrow$ vertex

$$-i\frac{g_Z^2}{2} g^{\alpha\beta}$$

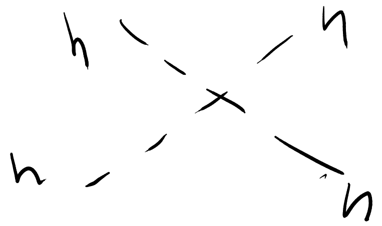
$- h^3 \lambda v \longrightarrow$ vertex



$= -i\, 6 \lambda v$

$= -i\, 3 \sqrt{\frac{\lambda}{2}}\, m_h$

$- \frac{\lambda}{4} h^4 \longrightarrow$ vertex



$- i\, 6 \lambda$

How about fermion masses?

We saw that we could not construct a fermion mass because

$m_f \bar{f} f = m_f (\bar{f}_L f_R + \bar{f}_R f_L)$ not $SU(2)_L$ GI

But now we have another $SU(2)$ doubel Φ

so we can buil

$$\mathcal{L}_{Yuk} = -\lambda_e \overset{\text{doublet}}{\bar{L}_L} \overset{\text{doublet}}{\Phi} e_R - \lambda_d \bar{Q}_L \Phi d_R - \lambda_u \bar{Q}_L (i \sigma_2 \Phi) u_R + h.c$$

which is GI

In unitary gauge

$$\lambda_e \bar{L}_L \Phi e_R + h c = \frac{\lambda_e}{\sqrt{2}} (\bar{\nu}_{e_L}\, \bar{e}_L) \begin{pmatrix} 0 \\ v+h \end{pmatrix} e_R + hc$$

$$= \underbrace{\frac{\lambda_e}{\sqrt{2}}}_{\equiv m_e} v (\bar{e}_L e_R + h.c) + \underbrace{\frac{\lambda_e}{\sqrt{2}}}_{\equiv m_e/v} h (\bar{e}_L e_R + hc)$$

In the same way

$$\lambda_d \, \bar{Q}_L \, \phi \, d_R + h.c = \frac{\lambda_d v}{\sqrt{2}} \bar{d} d + \frac{\lambda_d}{\sqrt{2}} \bar{d} d h$$

$$\begin{pmatrix} v+h \\ 0 \end{pmatrix}$$

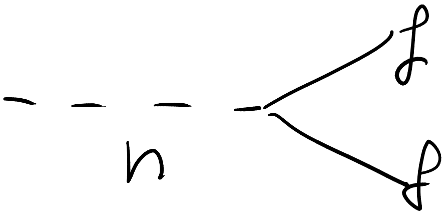
$$= m_d \bar{d} d + \frac{m_d}{v} \bar{d} d h$$

$$(\bar{u}_L, \bar{d}_L) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

$$\lambda_u \, \bar{Q}_L \, (i\sigma_2 \phi) \, u_R + h.c = \frac{\lambda_u}{\sqrt{2}} v \bar{u} u + \frac{\lambda_u}{\sqrt{2}} \bar{u} u h$$

$$= m_u \bar{u} u + \frac{m_u}{v} \bar{u} u h$$

So we have generated mass for the fermions and couplings of the fermions to the Higgs



$$i \frac{m_f}{v}$$

In 3 generations λ_u, λ_d, λ_e are 3×3 matrices and when diagonalized $\Rightarrow V_{CKM}$

So in this model all couplings of the Higgs boson $h(x)$ to any particle are proportional to the mass of the particle. The only unknown is its mass because λ is undetermined

As we have seen at tree level

$$M_W = \frac{1}{2} g v$$

$$M_Z = \frac{1}{2} g \frac{v}{\cos\theta_W}$$

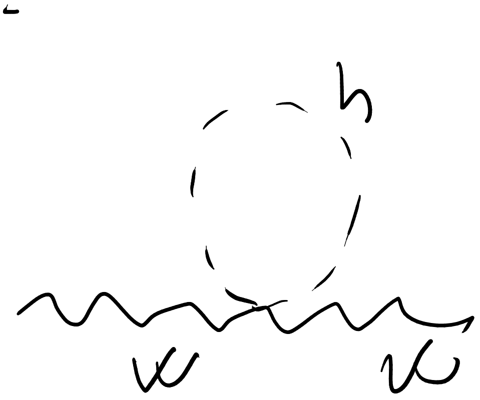
$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 M_W^2}$$

$$\alpha = \frac{e^2}{4\pi} = \frac{g^2 \sin^2\theta_W}{4\pi}$$

$$C_L^f = \pm\frac{1}{2} - Q_f \sin^2\theta_W .$$

Redundant ways to measure 3 independent quantities. So we can check loop contributions which depend on

i.e



correct the relation between M_Z and M_W and could be determined ⇒ indirect information on m_H