# Chapter 2

## "Relativistic Kinematics"

1) Lorentz transformations : implications
2) Four vector notation
3) Energy- momentum 4-vector
4) Examples

Griffits chapter 3

# 1 Lorentz transformation: implications

Relativity principle ≡ same laws of physics apply in any inertial system (inertial ≡ moving at constant velocity) ⇒ light (em wave) must travel at same speed in any inertial system

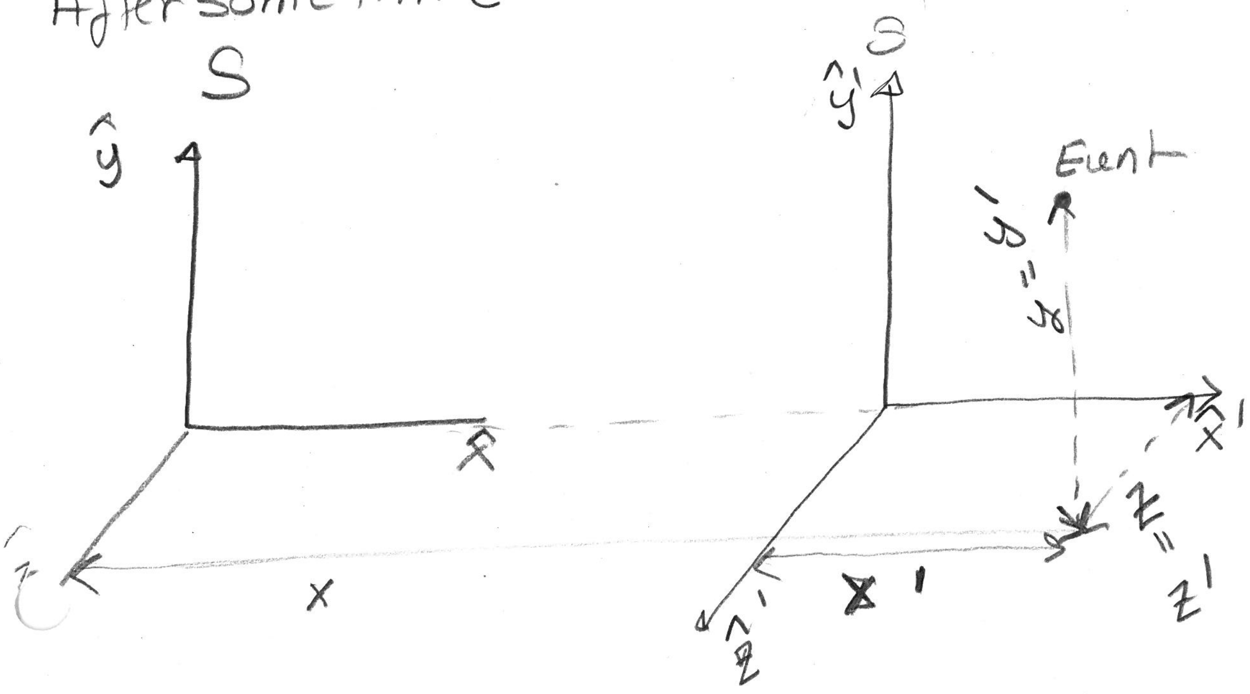
Take two systems of coordinates

- S with spatial coordinates $(x, y, z)$ and time coordinate $t$
- S' " " " $(x', y', z')$ " " " $t'$

S' is moving wrt S at velocity $\vec{v} = v\hat{x}$

At $t = t' = 0 \Rightarrow x = x' = y = y' = z = z' = 0$

After some time




an event occurs at $(x, y, z)$ and time $t$ in S

In S' is seen at $(x', y', z')$ at time $t'$

with

$$x' = \gamma(x - vt) \overset{\text{NU}}{=} \gamma(x - \beta t)$$
$$y' = y \quad = y$$
$$z' = z \quad = z$$
$$t' = \gamma\left(t - \frac{v}{c^2}x\right) = \gamma(t - \beta x)$$

with

$$\beta = \frac{v}{c} < 1$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \geq 1$$

when $\beta \to 1 \Rightarrow \gamma \to \infty$

or equivalently the inverse

$$x = \gamma(x' + \beta t')$$
$$y = y'$$
$$z = z'$$
$$t = \gamma(t' + \beta x')$$

check:

$$x \stackrel{?}{=} \gamma(\gamma(x - \beta t) + \beta(\gamma t - \beta x))$$
$$= x$$

Most ubiqutuous consequences in particle physic

- <u>time dilatation</u>: if at rest ($S'$) a particle has a lifetime $\tau \equiv (t'_2 - t'_1)$, in a system in which is moving with velocity $\beta$ ($S$) it lives

$$t_2 - t_1 = \Delta t = \gamma[\tau + \beta\underbrace{(x'_2 - x'_1)}_{\substack{\parallel \\ 0 \text{ (at rest in } S')}}] = \gamma\tau > \tau$$

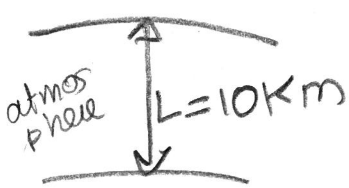
For ultrarelativistic particle $\beta \to 1 \Rightarrow \gamma \gg 1$

$$\Rightarrow \Delta t \gg \tau$$

So in S the particle travels

$$d = \beta \Delta t = \beta \gamma \tau \gg \beta \tau \leftarrow \text{naive non-relativistic estimate.}$$

Example muon $\mu^-$ was discovered at Earth surface when it was produced on top of atmosphere by collision of cosmic rays.
So it had to cross the atmosphere before decaying




At rest (S') its lifetime

$$\tau = 2.2 \times 10^{-6}\,s$$

$\Rightarrow$ "naively" ($\equiv$ non relativistic) one would estimate that it can travel at most

$$d = v \cdot \tau < c \cdot \tau = 660\ m \ll 10\ km$$

But in the Earth system (S) the muon is travelling at speed $v$ $\Rightarrow$ lives a time $\Delta t = \gamma \tau$
$\Rightarrow$ it can travel a distance

$$L = v \Delta t = \gamma v \tau = \frac{\overbrace{v \quad \tau}^{660\ m}}{\underbrace{\sqrt{1 - \frac{v^2}{c^2}}}_{\ll 1}} \gg 660 m \text{ if } \frac{v}{c} \to 1$$

i.e. is $v = (1 - 10^{-7})\, c \Rightarrow \gamma = \frac{1}{\sqrt{1 - (1 - 10^{-7})^2}} \simeq 2240$

$\Rightarrow$ in Earth frame $\mu^-$ lives $\Delta t = 2240 \times 2.2 \times 10^{-6}\,s = 4.9\ ms$

$\Rightarrow$ can travel $L = v \Delta t = \gamma v \tau = 2240 \times 660 m = 1500\,Km$ before decay

- Length contraction : an object that at rest measures $L_{rest}$ (say $S$ $\Rightarrow L_{rest} = x_2 - x_1$) in a system $S'$ wrt it is moving at speed $\vec{v} = (-v\hat{x})$ is seen with length $L'$ ("seen" $\Rightarrow t'_1 = t'_2$)

sign is not important here

$$L' = x'_2 - x'_1 \text{ with } x_2 = \gamma(x'_2 + \beta t'_2) \quad x_1 = \gamma(x'_1 + \beta t'_1)$$

$$\Rightarrow L_{rest} = x_2 - x_1 = \gamma(x'_2 - x'_1) + \gamma\beta(\cancel{t'_2 - t'_1}) = \gamma L'$$

$$\Rightarrow L' = \frac{L_{rest}}{\gamma} << L_{rest} \Rightarrow \text{contracted}$$

Back to muon in $S'$ (muon at rest system) muon lives

$\tau = 2.2 \times 10^{-6}$ s

the atmosphere at rest ($S$) is 10 km thick but for the muon the atmosphere is moving toward him/her at $v = (1 - 10^{-7})c \Rightarrow \gamma = 2240 \Rightarrow L'_{atm} = 4.5$ m

$$L'_{atm} \simeq 4.5\,m < v \cdot \tau = 660\,m$$

$\Rightarrow$ the muon can see the whole atmosphere passing before decaying..

## Velocities do not add linearly

An object moves with velocity $\vec{u}' = u'\hat{x}$ in $S'$

$\Rightarrow$ in $\Delta t'$ it moves a distance $\Delta x' = u'\Delta t'$

$\Rightarrow$ in $S$ ($S'$ is moving wrt $S$ with $\vec{v} = v\hat{x}$)

it has moved a distance

$$\Delta x = \gamma(\Delta x' + v\Delta t')$$ in a time

$$\Delta t' = \gamma\left(\Delta t' + \frac{v}{c^2}\Delta x'\right)$$

$\Rightarrow$ its velocity in $S$: $u = \frac{\Delta x}{\Delta t} = \frac{\Delta x' + v\Delta t'}{\Delta t' + \frac{v}{c^2}\Delta x'}$

$$u = \frac{u' + v}{1 + \frac{v u'}{c^2}}$$

$\Rightarrow$ if moves at speed of light in $S'$ ($u' = c$)

$$u = \frac{c + v}{1 + v/c} = c$$ $\Rightarrow$ it moves at speed of light in $S$ as is should.

# 2 Four-vector notation

We have seen that for systems with $v \sim c$ time and spatial coordinates mix when changing reference frame.
To simplify notation we introduce 4-vector.
the contravariant position-time 4-vectors has coordin

$X^{\mu} \leftarrow$ upper Lorentz index
$\mu = 0, 1, 2, 3$

notation in class for 3 vectors

$$X \equiv \begin{pmatrix} x^0 = ct \overset{\text{NU}}{=} t \\ x^1 = x \\ x^2 = y \\ x^3 = z \end{pmatrix} \equiv \begin{pmatrix} x^0 \\ \vec{x} \end{pmatrix} \qquad \vec{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

So in this notation a Lorentz transf can be written in matrix form

$$\begin{pmatrix} x'^0 \\ x'^1 \\ x'^2 \\ x'^3 \end{pmatrix} = \underbrace{\begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{4 \times 4 \text{ matrix } \Lambda_{\text{boost}\,\hat{x}}} \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix}$$

which can be written

$$x'^{\mu} = \sum_{\nu=0}^{3} (\Lambda_{\text{boost}\,\hat{x}})^{\mu}{}_{\nu}\, x^{\nu} \equiv (\Lambda_{\text{boost}\,\hat{x}})^{\mu}{}_{\nu}\, x^{\nu}$$

Einstein's convention $\equiv$ repeated upper and lower indexes ($\equiv$ contracted) are summed from 0 to 3

We define the <u>covariant</u> position-time 4-vector

$$x_\mu = \sum_{\nu=0}^{3} g_{\mu\nu} x^\nu \equiv g_{\mu\nu} x^\nu$$

covariant ⇒ lower Lorentz index

metric tensor

In matrix form

$$g \equiv \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

but most time we omit the T

$$\Rightarrow (x_0, x_1, x_2, x_3) = (x^0, -x^1, -x^2, -x^3) = (x^0, -\vec{x}^T)$$

We define the 4-norm of the space-time 4-vector

$$(x)^2 \equiv x^{0\,2} - (x^1)^2 - (x^2)^2 - (x^3)^2 = c^2t^2 - |\vec{x}|^2 \equiv x^\mu x_\mu \equiv x_\mu x^\mu$$

$$\equiv x^\mu g_{\mu\nu} x^\nu$$

It has all Lorentz index contracted = take same values in any inertial systems.

It is the relativistic generalization of the norm of a 3-vector which is invariant under rotations

We

Lorents group is the group of transformation which leave the 4-norm invariant. It includes

- boost of velocity $\vec{\beta}$
- rotations in 3 dimension $\vec{x}' = R_{3\times3}\vec{x} \Rightarrow \Lambda_{rot} = \begin{pmatrix} 1 & 0 \\ 0 & R \end{pmatrix}$
- Parity : $\vec{x}' = -\vec{x}$ $\quad \Lambda_P = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$
- time reversal $t' = -t$ $\quad \Lambda_T = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$

We define a contravariant Lorent 4-vector a 4 dimension column object $\begin{pmatrix} a^0 \\ a^1 \\ a^2 \\ a^3 \end{pmatrix} \equiv \begin{pmatrix} a^0 \\ \vec{a} \end{pmatrix}$

which under any transformation of the Lorentz group transforms as

$$a'^\mu = \Lambda^\mu{}_\nu\, a^\nu$$

and the corresponding covariant 4-vector

$$a_\mu = g_{\mu\nu} a^\nu = (a_0, a_1, a_2, a_3) = (a^0, -a^1, -a^2, -a^3) \equiv (a^0, -\vec{a})$$

And the scalar product of 2 4-vector

$$a.b = a^\mu b_\mu = a_\mu b^\mu = a^0 b_0 + a^1 b_1 + a^2 b_2 + a^3 b_3$$
$$= a^0 b^0 - a^1 b^1 - a^2 b^2 - a^3 b^3 = a^0 b^0 - \vec{a}.\vec{b}$$

The scalar product of 2-four vectors is also Lorentz invariant (≡ same value in any inertial reference frame).

The norm of a 4-vector

$$a^2 = a^{0\,2} - |\vec{a}|^2$$

unlike the norm of a 3-vector

$$|\vec{a}|^2 = (a^1)^2 + (a^2)^2 + (a^3)^2 > 0$$

$a^2$ $a^2 > 0 \Rightarrow a^0 > |\vec{a}| \Rightarrow$ time-like 4-vector

$a^2 < 0 \Rightarrow a^0 < |\vec{a}| \Rightarrow$ space-like 4-vector

$a^2 = 0 \Rightarrow a^0 = |\vec{a}| \Rightarrow$ light-like 4-vector

# 3 Energy-momentum 4-vector

In special relativity motion of a particle of mass $m$ and velocity $\vec{\beta} \equiv \frac{d\vec{x}}{dx^0}$ is characterised by

do not forget

- its 3momentum $\vec{p} = m\gamma\vec{\beta} = \vec{\beta} E$
- its energy $E = m\gamma$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \Rightarrow \beta^2 = 1 - \frac{1}{\gamma^2}$$

$\beta^2$ ($= |\vec{\beta}|$)

$$\Rightarrow |\vec{p}|^2 = m^2\gamma^2\beta^2 = m^2\gamma^2 - m^2 = E^2 - m^2 \Rightarrow E^2 = |\vec{p}|^2 + m^2$$

Since $\begin{pmatrix} x^0 \\ \vec{x} \end{pmatrix}$ is a 4-vector $\Rightarrow \begin{pmatrix} m\gamma = E \\ \gamma m\vec{\beta} = m\frac{d\vec{x}}{dx^0} = \vec{p} \end{pmatrix}$ is a 4vector

we call it the energy-momentum 4-vector $p$

$$\begin{pmatrix} p^0 = E = \sqrt{|\vec{p}|^2 + m^2} \\ \vec{p} \end{pmatrix}$$

Its 4-norm

$$p^2 = p^{0\,2} - |\vec{p}|^2 = |\vec{p}|^2 + m^2 - |\vec{p}|^2 = m^2 \quad ☺$$

$\Rightarrow$ mass is invariant under Lorentz transformations

$\Rightarrow$ same mass in any inertial system

So a particle with $m=0 \Rightarrow E = |\vec{p}| \equiv \beta E \Rightarrow \beta = 1$

$\Rightarrow$ moves at speed of light

Invariance under translations $\Rightarrow$ in any physical process the total Energy-momentum 4-vector is conserved

So any process:

N initial particles $\longrightarrow$ M final particles

each with 4-momentum

$$P_i^{init} \equiv \begin{pmatrix} E_i^{init} = \sqrt{|\vec{p}_i^{\,init}|^2 + m_i^2} \\ \vec{p}_i^{\,int} \end{pmatrix}$$

each with 4-momentum

$$P_j^{fin} = \begin{pmatrix} E_j^{fin} = \sqrt{|\vec{p}_j^{\,fin}|^2 + m_j^2} \\ \vec{p}_j^{\,fin} \end{pmatrix}$$

$i = 1 \ldots N$

$$P_{TOT}^{Init} = \begin{pmatrix} \sum_{i=1}^{N} E_i^{int} \\ \sum_{i=1} \vec{P}_i^{int} \end{pmatrix} = P_{TOT}^{Final} \equiv \begin{pmatrix} \sum_{j=1}^{M} E_j^{fin} \\ \sum_{j=1}^{n} \vec{P}_j^{fin} \end{pmatrix}$$

So knowing the energy-momentum of all initial particles $\Rightarrow$ relations among energy and 3-momenta (ie the directions) of final particles $\equiv$ Kinematic constraints

# ④ Examples

1) Two body decay : $A \longrightarrow B\ C$

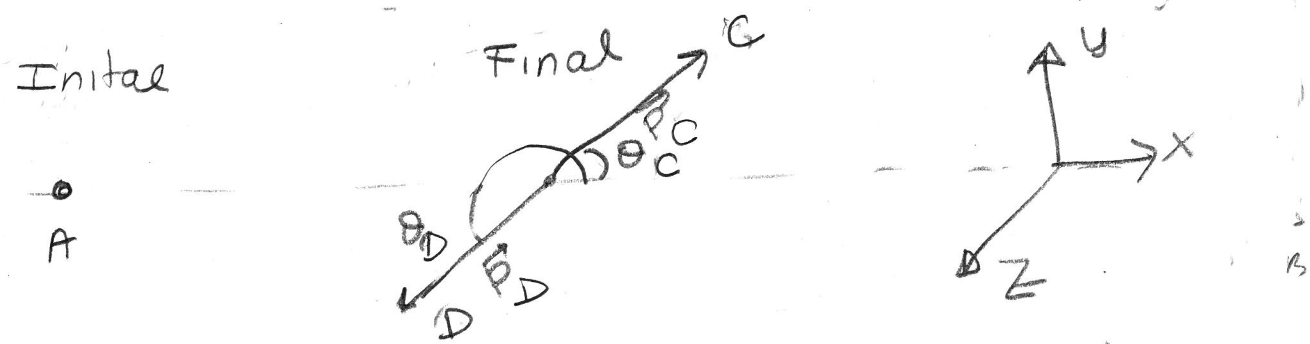
Lets start with A at rest (R system)

Initial $\qquad\qquad$ Final

$A \longrightarrow C + D$

4-momenta

$$P_A^R = \begin{pmatrix} E_A^R = M_A \\ \vec{P}_A^R = 0 \end{pmatrix} \qquad P_C^R = \begin{pmatrix} E_C^R = \sqrt{m_C^2 + |\vec{p}_C|^2} \\ \vec{p}_C^{\,R} \end{pmatrix} ; \; P_D^R = \begin{pmatrix} E_D = \sqrt{m_D^2 + |\vec{p}_D|^2} \\ \vec{p}_D^{\,R} \end{pmatrix}$$




4-momentum conservation $\Rightarrow P^{INI} = P_A = F^{INAL} = P_C + P_c$
in any system

$\Rightarrow$ 4 equations

(0) $\quad E_A^R = M_A = E_C^R + E_D^R = \sqrt{M_C^2 + |\vec{p}_C|^2} + \sqrt{M_D^2 + |\vec{p}_D|^2}$

(1-3) $\quad \vec{0} = \vec{P}_C^R + \vec{P}_D^R \Rightarrow \vec{P}_C^R = -\vec{P}_D^R \Rightarrow$ C, D produced back to back

$\Rightarrow \theta_D = \theta_C + \pi$

$\equiv \theta_R$ ← A Rest frame

$\Rightarrow |\vec{P}_C^R| = |\vec{P}_D^R| \equiv P_f$

with Posible to chose C, D with $\theta_R > 1$.

$$M_A = \sqrt{P_f^2 + M_C^2} + \sqrt{P_f^2 + M_D^2} \Rightarrow \text{solvable for } P_f$$

solution

$$P_f = \frac{1}{2M}\sqrt{(M_A^2 - (M_D+M_C)^2)(M_A^2 - (M_D-M_C)^2)}$$

notice that if $M_B + M_C < M_A \Rightarrow P_f$ imaginary

$\Rightarrow$ decay is not possible ($\equiv$ not kinematically allowed

$$E_C^R = \sqrt{P_f^2 + M_C^2} = \frac{M_A^2 + M_C^2 - M_D^2}{2M_A}$$

$$E_D^R = \sqrt{P_f^2 + M_D^2} = \frac{M_A^2 + M_D^2 - M_C^2}{2M_A}$$

Totally fixed by masses independent of emission angle $\theta_R$ which undetermined

Lets look at the same process in a frame in which A is moving at $\vec{\beta} = \beta\hat{x}$

Let us call that "F" system (Decay in flight)

We can obtain $P_{C,D}^F$ from $P_{C,D}^R$ using that they are 4 vectors so under the Lorentz transformation changing from A to F they are

$$\begin{pmatrix} E_C^F \\ P_{C,x}^F \\ P_{C,y}^F \\ P_{C,z}^F \end{pmatrix} = \begin{pmatrix} \gamma & \gamma\beta & 0 & 0 \\ \gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} E_C^R \\ P_{C,x}^R = P_f\cos\theta_R \\ P_{C,y}^R = P_f\sin\theta_R \\ P_{C,y}^R = 0 \end{pmatrix}$$

$$\Rightarrow E_C^F = \gamma\left(E_C^R + \beta P_f \cos\theta_R\right)$$

$$P_{C,x}^F = \gamma\left(P_f\cos\theta_R + \beta E_C^R\right)$$

$$P_{C,y}^F = P_{Cy}^R = P_f \sin\theta_R$$

$$P_{Cz}^F = P_{Cz}^R = 0$$

For D the expressions are the same with change $\sin\theta_R \to -\sin\theta_R$

$\cos\theta_R \to -\cos\theta_R$

So the emission angle of C (D) in the F frame

$$\tan\theta_C^F = \frac{P_{C,y}^F}{P_{C,x}^F} = \frac{P_f\sin\theta_R}{\gamma(P_f\cos\theta_R + \beta E_C^R)}$$

$$\tan\theta_D^F = \frac{P_{D,y}^F}{P_{D,x}^F} = \frac{-P_f\sin\theta_R}{\gamma(-P_f\cos\theta_R + \beta E_D^R)}$$

Take $\cos\theta_R > 0$ (always possible to choose C, D that way)

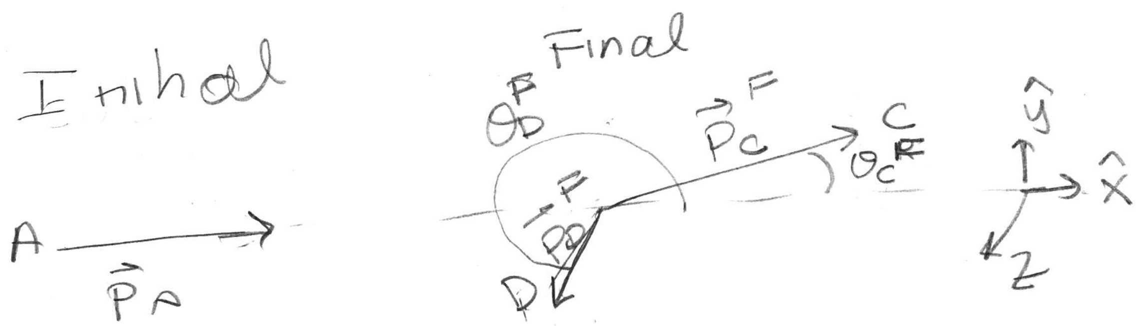
- if $\beta \to 1 \Rightarrow \gamma$ very large

- $\tan\theta_C^F \simeq \frac{1}{\gamma}\frac{\sin\theta_R}{\left(\cos\theta_R + \beta\frac{E_C^R}{P_g}\right)} << \tan\theta_R$

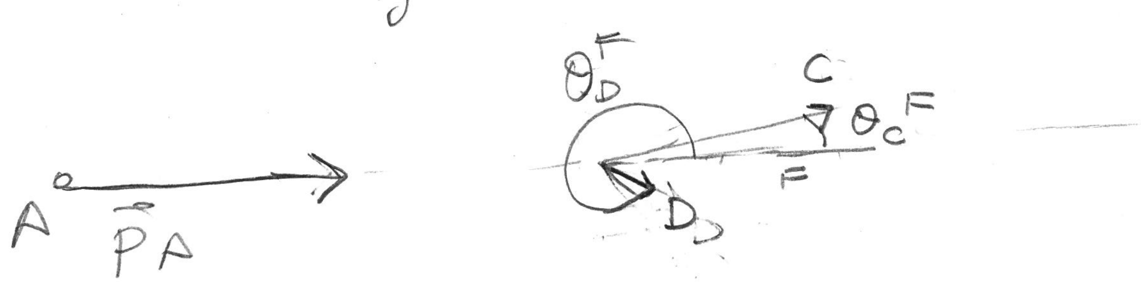
$\Rightarrow$ C emitted closer to A flight direction

- $\tan\theta_D^F = \frac{1}{\gamma}\frac{-\sin\theta_R}{-\cos\theta_R + \beta\frac{E_D^R}{P_g}} =$

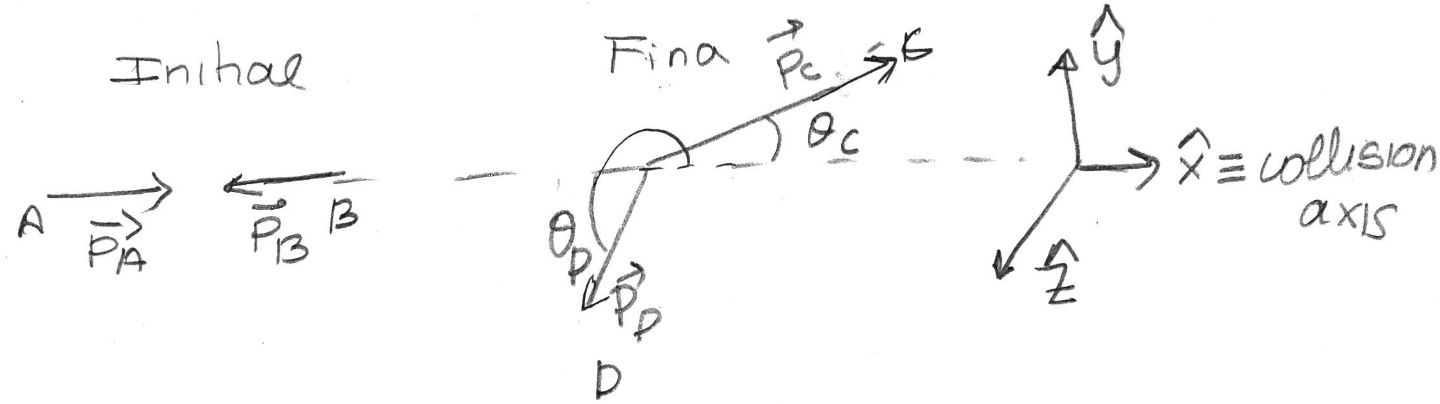
- I - If $\cos\theta_R > \beta\frac{E_D^R}{P_g} \Rightarrow P_{D,x}^F < 0 \Rightarrow$ D still emitted backwards WRT A, and C

$\Rightarrow$ Initial 

- If $\cos\theta_R < \beta\frac{E_D^R}{P_g} \Rightarrow P_{D,x}^F > 0 \Rightarrow$ D also emitted forward



Example 2: Scattering $A+B\to C+D$




In any reference frame $P_{TOT}^{INIT} = P_{TOT}^{FINAL}$

$$\overset{\text{4-momentum}}{\longrightarrow} P_A + P_B = P_C + P_D$$

$$\begin{pmatrix} E_A=\sqrt{|\vec{p}_A|^2+M_A^2} \\ \vec{p}_A \end{pmatrix} + \begin{pmatrix} E_B=\sqrt{|\vec{p}_B|^2+M_B^2} \\ \vec{p}_B \end{pmatrix} = \begin{pmatrix} E_C=\sqrt{|\vec{p}_C|^2+M_C^2} \\ \vec{p}_C \end{pmatrix} + \begin{pmatrix} E_D=\sqrt{|\vec{p}_D|^2+M_D^2} \\ \vec{p}_D \end{pmatrix}$$

$$\Rightarrow \begin{matrix} E_A+E_B=E_C+E_D \\ \vec{p}_A+\vec{p}_B=\vec{p}_C+\vec{p}_C \end{matrix} \Rightarrow 4 \text{ constraints}$$

We define the center of mass system (COM) as that where $\vec{p}_A+\vec{p}_B=0 \Rightarrow |\vec{p}_A|=|\vec{p}_B| \equiv p_{in}$

$$\Rightarrow \vec{p}_C+\vec{p}_D=0 \Rightarrow \text{C, D come out back to back} = \begin{cases} |\vec{p}_C|=|\vec{p}_D|\equiv p_f \\ \theta_D=\theta_C+\pi \end{cases}$$

It is the LAB system for "symmetric" colliders (e.g. "LEP" $e^+e^-$ collider)

Let us define the Mandelstam variable S

$$S \overset{\text{4 vector}}{\equiv} (P_A+P_B)^2 = (E_A+E_D)^2 - |\vec{p}_A+\vec{p}_B|^2$$

$$= (P_C+P_D)^2 = (E_C+E_D) - |\vec{p}_C+\vec{p}_D|^2$$

$S \equiv$ norm of a 4 vector $\Rightarrow$ same value in any inertial ref.

In COM

$S = (E_A + E_B)^2 \equiv$ energy available in the collision
(usually a known property of the experiments)

$$= \left[\sqrt{P_i^2 + M_A^2} + \sqrt{P_i^2 + M_B^2}\right]^2$$

$$\Rightarrow \quad P_i = \frac{1}{2\sqrt{S}} \sqrt{[S-(M_A+M_B)^2][S-(M_A-M_B)^2]} \xrightarrow{S \gg M_A^2,\, M_B^2} \frac{\sqrt{S}}{2}$$

$$E_A = \frac{1}{2\sqrt{S}} (S + M_A^2 - M_B^2) \longrightarrow \frac{\sqrt{S}}{2}$$

$$E_B = \frac{1}{2\sqrt{S}} (S - M_B^2 - M_A^2) \longrightarrow \frac{\sqrt{S}}{2}$$

In most experiments $\sqrt{S} \gg$ mass of colliding particles

Knowing $S$ we can solve for $P_f$ $\qquad \sqrt{S} \gg M_C, M_D$

$$S = (E_C + E_D)^2 = \left(\sqrt{P_f^2 + M_C^2} + \sqrt{P_f^2 + M_D^2}\right)$$

$$\Rightarrow P_f = \frac{1}{2\sqrt{S}} \sqrt{[S-(M_C+M_D)^2][S-(M_C-M_D)^2]} \longrightarrow \sqrt{S}/2$$

$$\Rightarrow E_C = \frac{1}{2\sqrt{S}} (S + M_C^2 - M_D^2) \longrightarrow \sqrt{S}/2$$

$$E_D = \frac{1}{2\sqrt{S}} (S + M_D^2 - M_C^2) \longrightarrow \sqrt{S}/2$$

Notice that if $\sqrt{s} < (M_C + M_D) \Rightarrow P_f$ imaginary

$\Rightarrow$ collision $A+B \to C+D$ not kinematically allowed

Notice expressions of $E_C$, $E_D$, $P_f$ in COM similar to

$A \to C+D$ in A rest frame with $M_A \to \sqrt{s}$

In general if we want to produce some state!

with total mass $M_{FIN} = \sum_i M_{F_i}$ ← set of final particles

in a collision $A+B \to F_1 \dots F_N$

one needs

$\sqrt{s} >> M_{FIN}$ in any reference frame

In a collider experiment with beam-beam symmetric collisions

(so LAB frame is COM frame) this condition implies

that the beam energies (take $\sqrt{s} >> M_{\text{beam particles}}$)

$$E_A^{beam} = E_B^{beam} = \frac{\sqrt{s}}{2} >> \frac{M_{FIN}}{2}$$

In a FIX target experiment one of the initial

particles is at rest (take B)

In

$$A \xrightarrow{\vec{p}_A} \underset{B}{\bullet} \qquad E_B^{FT} = M_B \qquad \vec{p}_B^{\,FT} = 0$$

$$S_{FT} = (E_A^{FT} + E_B^{FT})^2 - |\vec{p}_A^{FT} + \vec{p}_B^{FT}|^2 = E_A^2 + 2M_B E_A + M_B^2 - |\vec{p}_A|^2$$

$$= M_A^2 + M_B^2 + 2M_B E_A^{FT} \simeq 2M_B E_A^{FT}$$

$$E_{beam} = E_A^{FT} \gg M_A, M_B$$

So the condition to produce an state with $M_{FIN}$

$$\sqrt{s} \gg M_{FIN} \Rightarrow E^{beam} = \frac{S_{FT}}{2M_B} \gg \frac{M_{FIN}^2}{2M_B} \gg \frac{M_{FN}}{2}$$

So to produce new heavy particles we are better off with a collider experiment because we need less acceleration of the beams.

F.e $M_H \simeq 126$ GeV

- in $e^+e^-$ collider with $E_{beam} = E_{e^+} = E_{e^-} > 63$ GeV
- in a $e^-p$ FT (p at rest) $E_{e^+}^{beam} > \frac{(125 \text{GeV})^2}{2 \times 1 \text{GeV}} =$ (1 GeV = $m_{proton}$) $\gtrsim 8000$ GeV