chapter 3

"Symmetries and conservation laws"

1) Symmetry groups and generators
2) Space time symmetries
   - translations and energy-momentum conservation
   - rotation and angular-momentum conservation
3) Discrete symmetries : charge conjugation and Parity
4) Bound states of strong interactions : hadrons
5) Flavour symmetries

1 Symmetry groups and generators

A group G is a set of elements R_i $G_i \equiv \{R_i\}$ $i = \ldots$
with an internal product "•" verifying

- closure if $R_i, R_j \in G \Rightarrow R_i \circ R_j \in G$
- Identity $I \in G$ / $I . R_i = R_i \circ I = R_i$
- Inverse of any $R_i \equiv (R_i)^{-1} \in G$ / $R_i \circ R_i^{-1} = R_i^{-1} \circ R_i = I$
- Associativity
$$R_i \circ (R_j \cdot R_k) = (R_i \cdot R_j) \circ R_k$$

If $R_i \circ R_j = R_j \cdot R_i \;\forall i, j \Rightarrow G$ is abelian
If $R_i R_j \neq R_j - R_i \;\forall i, j \Rightarrow G$ is not abelian

If all $R_i \in G$ are transformations that leave a system invariante $\equiv$ symmetry group of the system

In physics symmetries are important because they allow us to find conserve quantities of the system
In particle physics they are at the core of the characterization of the particles and of their interactions.

In particle physics we can make two type of transformations to a system:

- space-time transformation ≡ change of coordinate system i.e translations, rotations, parity,
- internal transformation ≡ transformation of particle states (or their wave function) into different ones
  i.e change all particles to antiparticle ≡ charge conjugation

Further classified in

- Global ≡ transformation is the same in all points in space-time
- Gauge ≡ transf is different in each point

Also classified in

- continuos ≡ transformation can be constructed by successive applications of infinitesimal steps
  i.e translations, rotations ⇒ Group is Lie group
- discrete ≡ finite group. i.e Parity, charge conjug. time reversal

Consider a space S of states $|\varphi\rangle$ which can be expressed in a basis $\{|\varphi_i\rangle\}$ $i = \dots k$

they transform under a transformation $R \in G$
So generically we can write the transformed states in terms of the transformed basis

$$|\varphi_i'\rangle = \sum_j [U_R]_{ij} |\varphi_j\rangle \quad [\,|\varphi'\rangle = U_R|\varphi\rangle\,]$$

$U_R \equiv$ operator representing R in the space S
If S has dimension $k \Rightarrow U_R = k \times k$ matrix

the probability of finding system described by $|\varphi\rangle$ in state $|\chi\rangle$ should not be changed by R

$$\Rightarrow \langle \chi' | \varphi' \rangle = \langle \chi | \varphi \rangle$$
$$\langle \chi | U_R^\dagger U_R | \varphi \rangle \Longrightarrow U_R^\dagger = U_R^{-1} \Rightarrow U_R \text{ unitary}$$

Moreover if R is a symmetry of the system the Hamiltonian must be unchanged

$$\langle \chi' | H | \varphi' \rangle = \langle \chi | H | \varphi \rangle =$$
$$\langle \chi | U_R^\dagger H U_R | \varphi \rangle \Rightarrow H = U_R^\dagger H U_R \Rightarrow U_R H = H U_R$$
$$\Rightarrow [U_R, H] = 0$$

(5)

$\Rightarrow \langle U_R \rangle_\varphi$ is a constant of motion

For continuous transformation a given $R \in G$ corresponds to some values of some continuous parameters α^i $i = 1 \ldots N$ (i.e a rotation in 3D is characterized by 3 angles $\Rightarrow$ group of rotations, SU(2) has dimension 3)
N ≡ dimension of G

In this case we can write $U_R \equiv e^{-i \sum_{i=1}^{N} \alpha_R^i g_i}$

$g_i \equiv$ generator of group in space S and it is represented by a matrix $K \times K$ ← dimensional space S

In this case the unitarity condition:

$U_R^+ = U_R^{-1} \Rightarrow e^{i \sum \alpha_R^i g_i^+} = e^{i \sum \alpha_R^i g_i}$

$\Rightarrow g_i = g_i^+ \equiv$ hermitian $\Rightarrow$ good quantum-mechanical observable

a. If they are asymmetries $[H, U_R] = 0 \Rightarrow$

- the eigenstates of G_i can be used as basis of the states with well defined eigenvalues ($\equiv$ quantum #'s)
- the corresponding quantum #'s of the system must be conserved in evolution of system

note: how many can be chosen simultaneously depends on if group is abelian or not)

(2) Spacetime symmetries

4 vector

$\equiv$ change of coordinate system $\quad x \to x' = R(x)$

and the wave function of system changes accordingly

$$\psi(x) \to \psi'(x') \equiv U_R\, \psi(x')$$

Physics cannot depend on how we define our system of coordinates $\Rightarrow \psi'(x') = \psi(x) \Rightarrow U_R \psi(x) = \psi(R^{-1}x)$

or eq $\psi'(x) = \psi(R^{-1}(x))$ ⇗

<u>Translations</u> $\equiv$ shift of origine in 4-D by constant 4 vector $\quad \epsilon \Rightarrow x'^{\mu} \equiv x^{\mu} - \epsilon^{\mu} \equiv R(x) \Rightarrow R^{-1}(x) = x^{\mu} + \epsilon^{\mu}$

$\Rightarrow$ For any system $\qquad \epsilon^{\mu}$ infinitesimal

$$\psi'(x^{\mu}) = \psi(x^{\mu} + \epsilon^{\mu}) \simeq \psi(x) + \epsilon^{\mu} \frac{\partial \psi}{\partial x^{\mu}}$$

Let us call P^{μ} the operator generating translations in direction μ ⇛

$$\psi'(x) = e^{-i\epsilon^{\mu} P_{\mu}}\, \psi(x) \simeq \psi(x) - i\epsilon^{\mu} P_{\mu}\, \psi(x)$$

$$\Rightarrow P_{\mu} = i\frac{\partial}{\partial x^{\mu}} \Rightarrow \begin{cases} P^0 = P_0 = i\dfrac{\partial}{\partial t} \\ P^j = -P_j = -i\dfrac{\partial}{\partial x^j} \end{cases}$$

We can take the eigenvalues of this operator to characterise the state

$$P = \begin{pmatrix} E = P^0 \\ P_x \\ P_y \\ P_z \end{pmatrix} \longleftarrow \text{numbers}$$

The corresponding eigenstates are the plane waves

$$\varphi(x) = e^{-ip^\mu x_\mu} = e^{-i(Et - \vec{p}\vec{x})}$$

We can chose the 4 eigenvalues to characterise the state because

$$[P^\mu, P^\nu] = \frac{\partial}{\partial x^\mu}\frac{\partial}{\partial x^\nu} - \frac{\partial}{\partial x^\nu}\frac{\partial}{\partial x^\mu} = 0$$

(derivatives commute)

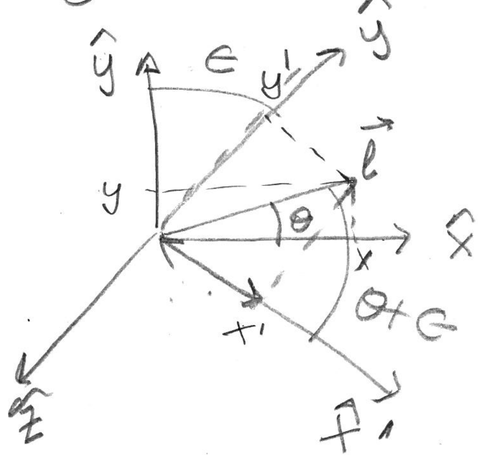
- Commutativity ⇒ a system composed of n particles each of them with well-defined 4-momenum P_i^μ has a total 4-momentum

$$P^\mu_{TOT} = \sum_{i=1}^{N} P_i^\mu$$

- Invariance under translation ⇒ P^μ_{TOT} is conserved in any physical process.

Rotations

Infinitesimal rotation of system of coordinates of angle ϵ about $\hat{z}$



in S $\vec{\ell} = \begin{pmatrix} x = \ell\cos\theta \\ y = \ell\sin\theta \\ z = 0 \end{pmatrix}$

in S' ϵ small

$$x' = \ell\cos(\theta+\epsilon) \simeq \ell\cos\theta - \epsilon\ell\sin\theta = x - \epsilon y$$

$$y' = \ell\sin(\theta+\epsilon) \simeq y + \epsilon x$$

$$z' = z$$

$$x' = R_\epsilon(\vec{x}) = \begin{pmatrix} x - \epsilon y \\ y + \epsilon x \\ z \end{pmatrix}$$

Physics cannot depend on orientation of axis

any system must be invariant under rotation

$$\psi'(\vec{x}) = \psi(R_\epsilon^{-1}\vec{x}) = \psi\begin{pmatrix} x + y\epsilon \\ y - x\epsilon \\ z \end{pmatrix} \simeq \psi(x) + \epsilon\left(y\frac{\partial}{\partial x} - x\frac{\partial}{\partial y}\right)$$

Let us call J_z the generator of rotation about $\hat{z}$

by definition $\psi'(x) = e^{-i\epsilon J_z}\psi(x) \simeq \psi(x) - i\epsilon J_z \psi(x)$

$$\Rightarrow J_z = i\,y\frac{\partial}{\partial x} - i\,x\frac{\partial}{\partial y} = -y P_x + x P_y = (\vec{x} \wedge \vec{P})_z = L_z$$

$\Rightarrow$ generator of rotation about $\hat{z}$ axis is the angular momentum operator along $\hat{z}$ axis

We can repeat for $\hat{x}$ and $\hat{y}$

$J_x = (\vec{x} \wedge \vec{P})_x \qquad J_y = (\vec{x} \wedge \vec{P})_y$

but unlike the components of P^μ, the 3 compounds of $\vec{J}$ do not commute

they verify $[J_a, J_b] = i \sum_c \epsilon_{abc} J_c$ imaginary unit

$\epsilon_{ijk} \equiv$ total antisymmetric tensor

$\epsilon_{123} = \epsilon_{312} = \epsilon_{231} = 1 = -\epsilon_{213} = -\epsilon_{321} = -\epsilon_{132}$

this is the algebra of the group SU(2)

Because the 3 J_a operators do not commute we cannot chose the 3 eigenvalues $j_i = \langle \Psi | J_a | \Psi \rangle$ as quantum #'s.

$J_1^2 + J_2^2 + J_3^2$

But we can show that $[|\vec{J}|^2, J_a] = 0 \quad a = 1,2,3$

so we can use two quantum # to characterize the state: one associated with $|\vec{J}|^2$ (j) which can be integer or half-integer and one associated with one component of $\vec{J}$, i.e J_z which we call m

So we can write the basis of states of angular momentum "j" as

$|j, m\rangle$

with $|\vec{J}|^2 |j m\rangle = j(j+1) |j, m\rangle$

and $J_z |j, m\rangle = m |j m\rangle$ with $-j \leq m \leq j$

in units of 1 $\Rightarrow$ $2j+1$ values

And the corresponding quantum # for a composed system $|j_a, m_a\rangle |j_b, m_b\rangle \equiv |j_a, m_a; j_b, m_b\rangle$

$$J_z |j_a, m_a; j_b\, m_b\rangle = m_{TOT} |j_a m_a; j_b\, m_b\rangle$$

with $m_{TOT} = m_a + m_b$

$$|\vec{J}|^2 |j_a, m_a; j_b\, m_b\rangle = j_{TOT}(j_{TOT}+1) |j_a m_a; j_b m_b\rangle$$

j_{TOT} is not fully determined by j_a and j_b

j_{TOT} can take values between

$$|j_a - j_b| \leq j_{TOT} \leq j_a + j_b$$

So in general the composed state is a linear combination of states of well defined angular momentum

$$|j_a\, m_a; j_b\, m_b\rangle = \sum_{j=|j_a-j_b|}^{j_a+j_b} \underbrace{C^{j\, j_a\, j_b}_{m\, m_a\, m_b}}_{\text{coefficients of Clebbsch-Gordon}} |j, m = m_a + m_b\rangle$$

$$\Rightarrow \left| C^{j\, j_a j_b}_{m\, m_a m_b} \right|^2 = \left| \langle jm \mid j_a m_a ; j_b m_b \rangle \right|^2$$

is the probability of finding a state $|jm\rangle$ when measuring the angular momentum of a system composed of $|j_a m_a\rangle$ and $|j_b, m_b\rangle$

$$|jm \text{ (from } \vec{J}_a + \vec{J}_b)\rangle = \sum_{m_a} C^{J\, J_a J_b}_{m, m_a, m-m_a} \, |j_a m_a\rangle |j_b \; m_b = m - m_a\rangle$$

The angular momentum of quantum states $|\varphi\rangle$

two contributions

- orbital angular momentum which depends on its state of motion $\vec{L} = \vec{X} \wedge \vec{P}$
- intrinsic angular momentum $\equiv$ spin $\vec{S}$

the total angular momentum of a state $|\varphi\rangle$ is the sum of both $\vec{J} = \vec{L} + \vec{S}$

with $\quad |\vec{L}|^2 |\varphi\rangle = \ell(\ell+1) |\varphi\rangle$

$\ell \equiv$ orbital angular momentum quantum # which integer

and $\quad |\vec{S}|^2 |\varphi\rangle = s(s+1) |\varphi\rangle$

$s \equiv$ spin quantum # (or simply spin) which can be integer or half-integer

$\Rightarrow \quad |\vec{J}|^2 |\varphi\rangle = j(j+1) |\varphi\rangle \quad$ with $|s-\ell| \leq j \leq s+\ell$

Invariance under rotations $\Rightarrow$ we can characterize a elementary particle irrespective of its motion by its spin quantum # (s) and one component $\langle S_z \rangle = m$

so we can write the spin part of its vector state as $|s, m\rangle$ with
$$|\vec{S}|^2 |s,m\rangle = s(s+1)|s,m\rangle$$
$$S_z |s,m\rangle = m|s,m\rangle$$

For a massive particle "m" can take all possible values between $-s$ and s in units of 1 $\Rightarrow$ $2s+1$ polarization states (also called spin states)

For massless particle "m" can only take the extreme value $m = \pm s \Rightarrow$ 2 polarization states.

- Any fermion of standard model: $s = \frac{1}{2}$ and $\Rightarrow$ 2 spin states $\begin{cases} |\frac{1}{2}\,\frac{1}{2}\rangle \equiv f^{\uparrow} \text{ spin up} \\ |\frac{1}{2}, -\frac{1}{2}\rangle \equiv f^{\downarrow} \text{ spin down} \end{cases}$

- photon, gluon $s = 1$ mass $= 0 \Rightarrow$ 2 polarization states $\begin{cases} |1,1\rangle \equiv \text{positively polarized} \\ |1,-1\rangle \equiv \text{negatively polarized} \end{cases}$

- W, Z $s = 1$, mass $\neq 0 \Rightarrow$ 3 polarization states $\begin{cases} |1,1\rangle \\ |1,0\rangle \equiv \text{longitudinally polarized} \\ |1,-1\rangle \end{cases}$

- Higgs boson: $S=0 \Rightarrow 1$ state $|0,0\rangle$

naming $\left\{ \begin{array}{lll} \text{particle of } S=0 & \text{scalar} & \text{(boson)} \\ \text{" " } S=1/2 & \text{spin } 1/2 & \text{fermion} \\ \text{" " } S=1 & \text{vector} & \text{(boson)} \\ \text{" " } S=3/2 & \text{spin } 3/2 & \text{fermion} \\ \text{" } S=2 & \text{tensor} & \text{(boson)} \end{array} \right.$

Invariance under rotations $\Rightarrow$ in any physical process the total angular momentum must be conserved $\Rightarrow$

In any process $|\text{Initial}\rangle \rightarrow |\text{Final}\rangle$

$$j_{\text{initial}} = j_{\text{final}}$$
$$m_{\text{initial}} = m_{\text{final}}$$

In general $|\text{initial}\rangle$ and $|\text{final}\rangle$ are made of particles and to get j_{int} and j_{fin} one needs to compose their spin and orbital angular momentum.

But for example $Z \not\rightarrow \gamma\, e^-$

because $j_{\text{int}} = 1$

$S_\gamma = 1$
$S_e = 1/2$ $\Rightarrow 1-\frac{1}{2} \leq S_{\text{final}} \leq 1+\frac{1}{2} \Rightarrow S_{\text{final}} = 1/2 \text{ or } 3/2$

and adding the orbital angular momentum ℓ

$$|s_{final}-\ell| \leq j_{final} \leq s_{final}+\ell \qquad \ell \equiv \text{integer}$$

so j_{final} is always half-integer $\neq j_{init}$

③ Discrete symmetries :

Discrete transformation ≡ cannot be constructed by successive applications of infinitesimal steps

- Parity $\mathcal{P}$: $\vec{x} \longrightarrow -\vec{x}$ it is a space-time transf.
  Its effect on the state is represented by unitary operator U_P
  $$\mathcal{P} : |\psi\rangle \rightarrow |\psi'\rangle \equiv U_P|\psi\rangle$$
- charge conjugation : $\mathcal{C}$ = particle antiparticle state
  Its effect on the state is represented by unitary op. U_C
  $$\mathcal{C} : |\psi\rangle \rightarrow |\psi^c\rangle \equiv U_C|\psi\rangle$$

[Time reversal ; $x^0 \rightarrow -x^0$
Its effect on state is represented by anti-unitary op U_T
$$\mathcal{T} : \psi\rangle \rightarrow |\psi^T\rangle \equiv U_T|\psi\rangle$$]

For $\mathcal{C}$ and $\mathcal{P}$: the group only has two elements since in both cases $U^2 = I \Rightarrow G = \{U, I\}$

Also in both cases $U = U^+$ and $U^2 = I \Rightarrow U = U^+$

$\Rightarrow$ the U operator is a hermitian $\Rightarrow$ good QM observable

We denote C_ψ and P_ψ the quantum # of for U_C and U_P for some state $|\psi\rangle$. For $A = P$ or C

$$|\psi\rangle = U_A^2|\psi\rangle = C_A^2|\psi\rangle \Rightarrow C_A = \pm 1$$

Parity and charge conjugation are symmetries of strong and em interactions. Consequences

$\Rightarrow$ hadrons (bound states of strong interactions) have well defined C and P quantum #

$\Rightarrow$ neutral hadrons have well defined C quantum #

$\Rightarrow$ in strong and em processes C_{TOTAL} and P_{TOTAL} conserved

Notation: we denote states with well defined P using a geometrical notation

- vector $\vec{v}$ as an object that $\mathcal{P}: \vec{v} \to -\vec{v} \Rightarrow P_{vect} = -1$
- scalar $S = \vec{v}_1 \cdot \vec{v}_2 \Rightarrow$ $\mathcal{P}: \vec{S} \to \vec{S} \Rightarrow P_{scal} = 1$
- pseudo vector $\vec{a} = \vec{v}_1 \wedge \vec{v}_2$ $\quad \mathcal{P}: \vec{a} \to \vec{a} \Rightarrow P_{\substack{pseudo\\vector}} = 1$
- pseudoscalar $p = \vec{a} \cdot \vec{v}$ $\quad \mathcal{P}: p \to -p \Rightarrow P_{\substack{pseudo\\scalar}} = -1$

For quark, leptons and gauge bosons and Higgs the assignment of sign of P (and C for γ and g) are conventions

$P_{quark} = P_{lepton} = 1 \Rightarrow P_{\bar{q}} = P_{\bar{e}} = -1$

$P_\gamma = P_g = -1 =$ $\qquad P_H = 1$

$C_\gamma = -1$

4) Bound states of strong interactions

Hadrons ≡ bound states of strong interactions

- mesons : quark-antiquark $(q\bar{q}')$
- baryons : 3 quarks $(qq'q'')$

Since strong interactions conserve angular momentum, parity and charge conjugation the hadrons must have well define spin, P and C.

Take a meson $M = (q\bar{q}')$ its angular momentum at rest ≡ its spin

$$\vec{S}_M = \underbrace{\vec{S}_q + \vec{S}_{\bar{q}'}}_{\vec{S}_{q\bar{q}'}} + \vec{L}_{q\bar{q}'}$$

relative orbital angular momentum of pair $q\bar{q}'$

$$\langle M | \vec{S}_M^{\,2} | M \rangle = s_M(s_M + 1)$$

what values can s_M take?

We know that

$$\langle M | \vec{S}_{q\bar{q}'}^{\,2} | M \rangle = s_{q\bar{q}'}(s_{q\bar{q}'} + 1)$$

with $|s_q - s_{\bar{q}'}| \leq s_{q\bar{q}'} \leq |s_q + s_{\bar{q}'}|$ in steps of 1

$0 = \frac{1}{2} - \frac{1}{2}$ $\qquad$ $\frac{1}{2} + \frac{1}{2} = 1$

$$\Rightarrow s_{q\bar{q}'} = 0, 1$$

We know that

$$\langle M | \vec{L}^2_{q\bar{q}'} | M \rangle = l_{q\bar{q}'}(l_{q\bar{q}'}+1) \quad l_{q\bar{q}} = \text{integer}$$

$$\Rightarrow |S_{q\bar{q}'} - l_{q\bar{q}'}| \leq S_M \leq |S_{q\bar{q}'} + l_{q\bar{q}'}| \quad \text{in steps of } 1$$

- mesons with $l_{q\bar{q}'} = 0$ ($\equiv q\bar{q}'$ in s wave)
  usually lightest states with same $q\bar{q}'$ composition

  we can have

  Examples

  - $S_M = S_{q\bar{q}} = 0 \Rightarrow$ "scalar" meson $\pi^\pm, K^\pm, \pi^0, K^0$
  - $S_M = S_{q\bar{q}'} = 1 \Rightarrow$ "vector" meson ρ, ω

their spin wave function in terms of q and $\bar{q}'$ ones

$$|S_M, m_M\rangle = \sum_{m=-1/2}^{1/2} C^{S_M, \frac{1}{2}\frac{1}{2}}_{m_M, m, m_M - m} \underbrace{|\tfrac{1}{2}, m\rangle}_{S_q} \underbrace{|\tfrac{1}{2}, m_M - m\rangle}_{S_{\bar{q}'}}$$

For scalar meson $S_M = 0$, $m_M = 0 \Rightarrow$

$$|0,0\rangle = \underbrace{C^{0\frac{1}{2}\frac{1}{2}}_{0\frac{1}{2}-\frac{1}{2}}}_{\frac{1}{\sqrt{2}}} \underbrace{|\tfrac{1}{2}\tfrac{1}{2}\rangle}_{q\uparrow} \underbrace{|\tfrac{1}{2}\tfrac{-1}{2}\rangle}_{\bar{q}\downarrow} + \underbrace{C^{0\frac{1}{2}\frac{1}{2}}_{0-\frac{1}{2}\frac{1}{2}}}_{-1/\sqrt{2}} \underbrace{|\tfrac{1}{2}-\tfrac{1}{2}\rangle}_{q\downarrow} \underbrace{|\tfrac{1}{2}\tfrac{1}{2}\rangle}_{\bar{q}\uparrow}$$

$$= \frac{1}{\sqrt{2}} \left(q\uparrow \bar{q}'\downarrow - q\downarrow \bar{q}'\uparrow \right)$$

which is asymmetric under exchange $q \leftrightarrow \bar{q}'$

For vector mesons $S_M = 1$ and $m_M = -1, 0, 1\rangle$

$$|S_M=1, m_M=1\rangle = \underset{=1}{C^{1\frac{1}{2}\frac{1}{2}}_{1\frac{1}{2}\frac{1}{2}}} \underbrace{|\tfrac{1}{2}\,\tfrac{1}{2}\rangle}_{q\uparrow} \underbrace{|\tfrac{1}{2},\tfrac{1}{2}\rangle}_{\bar{q}'\uparrow} = q\uparrow \bar{q}'\uparrow$$

$$|S_M=1, m_M=-1\rangle = C^{1\,1/2\,1/2}_{-1\,-1/2\,-1/2} |\tfrac{1}{2}\,-1/2\rangle |\tfrac{1}{2}\,-1/2\rangle = q\downarrow \bar{q}'\downarrow$$

$$|S_M=1\ m_M=0\rangle = C^{1\,1/2\,1/2}_{0\,1/2\,-1/2}\ q\uparrow\bar{q}'\downarrow + C^{1\frac{1}{2}\frac{1}{2}}_{0\,-1/2\,1/2}\ q\downarrow\bar{q}'\uparrow$$

$$= \frac{1}{\sqrt{2}}\left(q\uparrow\bar{q}'\downarrow + q\downarrow\bar{q}'\uparrow\right)$$

all symmetric under $q \leftrightarrow \bar{q}'$

from property of spatial part of wave fn

For the mesons parity $P_M = \underset{=1}{P_q}\,\underset{=-1}{P_{\bar{q}'}}\,(-)^{\ell_{q\bar{q}'}}$ $\qquad Y_{\ell m}(\theta,\phi) = (-)^{\ell}\, Y(\pi-\theta, \phi+\pi)$

$= (-1)^{1+\ell_{q\bar{q}'}}$

so for $\ell_{q\bar{q}'} = \cancel{0}$ $\Rightarrow P = 1$ $\rangle$

$\Rightarrow$ mesons with $S_M = S_{q\bar{q}} = 0$ $[\Rightarrow \ell_{q\bar{q}} = \cancel{1}] \Rightarrow$ pseudoscalar mesons

mesons '' $S_M = S_{q\bar{q}} = 1$ $[\Rightarrow \ell_{q\bar{q}'} = 0] \Rightarrow$ vector mesons

For neutral mesons $(q\bar{q})$

$$C_{m^0} = (-1)\,(-1)^{S_{q\bar{q}}+1}\,(-1)^{\ell_{q\bar{q}}} = (-1)^{S_{q\bar{q}}+\ell_{q\bar{q}}}$$

from exchange fermions

from spin wave functs

from $q \leftrightarrow \bar{q}$ in $Y_{\ell m}(\theta,\phi)$

So for example π^0 $C_{\pi^0} = (-1)(-1)^1(-1)^0 = +1$

ρ^0 $C_{\rho^0} = (-1)(-1)^2(-1)^0 = -1$

- For M with $l_{q\bar{q}'} = 1$ ($q\bar{q}'$ in p-wave)

- for $S_{qq'} = 0 \Rightarrow S_M = 1$ and $P_M = (-1)^{1+1} = 1$ $\Bigg\} \Rightarrow$ pseudo vector meson

neutral ones have $C_{M^0} = -1$

- for $S_{qq'} = 1 \Rightarrow |1-1| \leq S_M \leq 1+1 \Rightarrow S_M = 0, 1, 2$ all with $P_M = 1$.
  - $S_M = 0$, $P_M = 1 \Rightarrow$ scalar meson and
  - $S_M = 1$, $P_M = 1 \Rightarrow$ pseudo vector meson
  - $S_M = 2$, $P_M = 1 \Rightarrow$ tensor meson

in all the neutral ones have $C_{M^0} = 1$

For Baryons $B \equiv qq'q''$

$$\vec{S}_B = \underbrace{\vec{S}_q + \vec{S}_{q'}}_{S_{q\bar{q}'}} + \vec{S}_{q''} + \vec{L}_{qq'q''}$$

$$\langle B | |\vec{S}_B|^2 | B \rangle = S_B(S_B + 1)$$

For baryons with $\ell_{qq'q''}=0$ we obtain S_B by composing $S_q, S_{q'}, S_{q''}$

We have seen tha $S_{qq'}=0, 1$

composing with $S_{q''}=1/2$ we can make

$|S_{qq'}-\frac{1}{2}| \leq S_B \leq |S_{qq'}+\frac{1}{2}|$ $\to$ for $S_{qq'}=0$ $S_B=1/2$

for $S_{qq'}=1/2$ $S_B=1/2, 3/2$

$\Rightarrow$ Baryons with $S_B=1/2$ in $\ell_{qqq'}=0$ are admixtures of $S_{qq'}=0$ and $S_{qq'}=1$ states

<u>Exampl</u> p, n, Σ, Λ

Baryons with $S_B=3/2$ in $\ell_{qqq'}=0$ are pure $S_{qq'}=1$ states $\Big\}$ Δ, Σ^*, Ω

. Parity of baryons $P_B=(+1)^3(-1)^{\ell_{qq'q''}}$ (with $P_q^3 \downarrow$)

So for all baryon in $\ell_{qq'q''}$ $P=+1$

Baryons in higher $\ell_{qq'q''}>0$ are generically heavier

Angular momentum, parity and C must be conserved in strong and em interactions.
this makes some processes forbidden even if Kinematically allowed . Fex

$\rho^0 \to \underset{\overset{\|}{\text{final}}}{\pi^+\pi^-}$ with

$\rho^0 \equiv$ vector meson $S_{\rho^0} = 1$, $P_\rho = -1$ $C_{\rho^0} = -1$

$\pi^{\pm,0}$ pseudoscalar " $S_\pi = 0$, $P_\pi = -1$ $C_{\pi^0} = +1$

$J_{inihal} = J_\rho = 1$

$$\vec{J}_{final} = \underbrace{\vec{S}_{\pi^+} + \vec{S}_{\pi^-}}_{\vec{S}_{\pi^+\pi^-}} + \vec{L}_{\pi^+\pi^-}$$

Since $S_\pi = 0 \Rightarrow \langle \text{final} | \vec{S}_{\pi^+\pi^-} | \text{final} \rangle \equiv S_{\pi^+\pi^-}(1+S_{\pi^+\pi^-})$

with $\underset{0}{S_{\pi^+} - S_{\pi^-}} \le S_{\pi^+\pi^-} \le \underset{0}{S_{\pi^+} + S_{\pi^-}} \Rightarrow S_{\pi^+\pi^-} = 0$

$\Rightarrow J_{final} = \ell_{\pi^+\pi}$

so conservation angular momentum $\Rightarrow \ell_{\pi^+\pi^-} = 1$

$P_{inihal} = P_{\rho^0} = -1$

$P_{final} = P_{\pi^+} \cdot P_{\pi^-} \cdot (-)^{\ell_{\pi\pi}} = -1 = P_{init}$ OK

$C_{\rho^0} = -1$

$\mathcal{C}\, |\pi^+\pi^-\rangle = |\pi^-\pi^+\rangle$ ≡ same as parity effect

$\mathcal{C}_{(\pi^+\pi^-)} = -1 = C_{\rho^0} \Rightarrow$ OK

How about $\rho^0 \to \pi^0 \pi^0$?

In this case angular momentum conservation

$\Rightarrow \ell_{\pi^0\pi^0} = 1 \Rightarrow$ $\pi^0\pi^0$ wave function

is antisymmetric under exchange $\pi^0 \Leftrightarrow \pi^0$

But π^0's are bosons (integer spin) $\Rightarrow$
their wave funct must be symmetric under exchange
$\pi^0 \Leftrightarrow \pi^0$

$\Rightarrow$ this decay is not possible