chapter 7

"QED as probe of the structure of hadrons"

0) elastic $e^-\mu^- \to e^-\mu^-$ in LAB ($\equiv$ initial μ^- at rest)

1) Concept of form factor of charge distributions

2) $e^-p \to e^-p$ elastic scattering

3) $e^-p \to e^- \underset{\text{"anything"}}{X}$ deep inelastic scattering

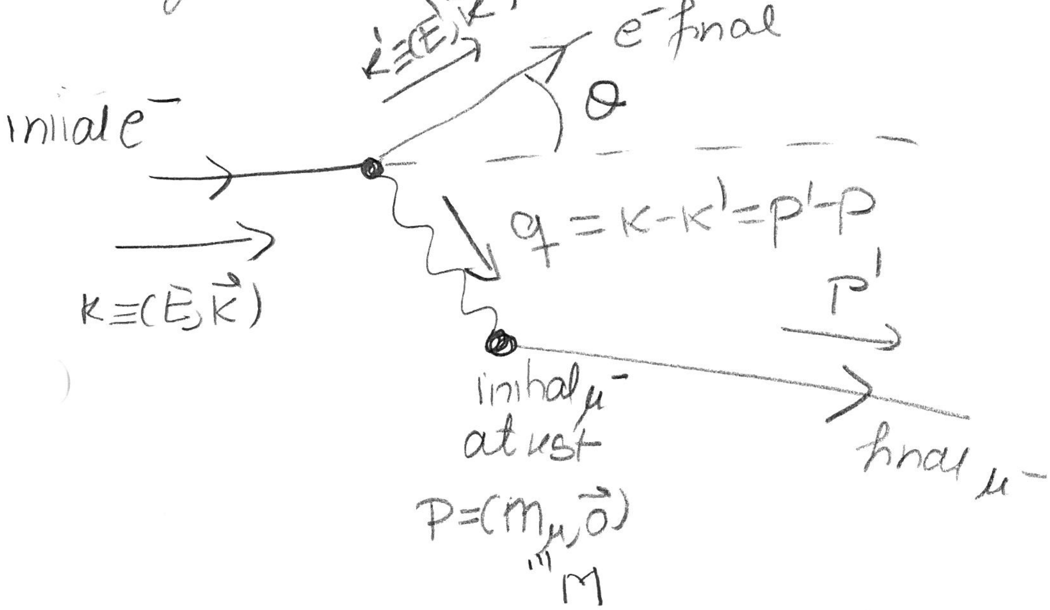
4) Bjorken scaling = partons

5) the quark-parton model of the hadrons

6) Parton model for hadron-hadron collisions

0) $e^-\mu^- \to e^-\mu^-$ in LAB

We are going to study the scattering of e^- and different targets in QED. We are going to graphically represent it with a "drawing" which is an admixture of a Feynman diagram and a physical representation



In chapter 6 we found that the amplitude

$$\mathcal{M} = -\frac{e^2}{q^2} j^\alpha_{elc}\, j_{\alpha,muon}$$

I am suppressing the helicity indixes

$$j^\alpha_e = \bar{u}_e(k')\gamma^\alpha u_e(k)$$

$$j_{\alpha,muon} = \bar{u}_m(p')\gamma_\alpha u_m(p)$$

and obtain (for $m_e = 0$)

$$\overline{|\mathcal{M}|^2} = \frac{8e^4}{q^4}\left[(k'p')(kp) - (kp')(k'p) - M_\mu^2 (kk')\right]$$

Using $k^2 = k'^2 = m_e^2 = 0$

$$q^2 = (k-k')^2 = -2kk' = -2EE'(1-\cos\theta) = -4EE'\sin^2\frac{\theta}{2}$$

$$kp = ME$$

$$kp' = k(k+p-k') = ME + \frac{q^2}{2}$$

$$k'p' = k'(k+p-k') = -\frac{q^2}{2} + ME'$$

$$p'^2 = M^2 = (p+q)^2 = M^2 + 2pq + q^2 \Rightarrow q^2 = -2pq = -2p(k-k') = -2M(E-E')$$

Altogether

$$|\overline{M}|^2 = \frac{8e^4}{q^4} M^2\left[2EE' - \frac{q^2}{2} + \frac{q^2}{4M^2}\right] = \frac{16e^4}{q^4} M^2 EE'\left[\cos^2\frac{\theta}{2} - \frac{q^2}{2M^2}\sin^2\frac{\theta}{2}\right]$$

so

$$d\sigma = \frac{1}{4ME} \frac{|M|^2}{(2\pi)^4} \frac{d^3k'}{2E'} \frac{d^3p'}{2p_0'} \delta^4(p+k-p'-k')$$

initial Flux in LAB frame

used to integrate d^3p_0'

$$\delta^4(p+k-p'-k') = \delta^3(\vec{k}-\vec{p}'-\vec{k}')\,\delta(M+E-E'-p_0') \qquad \vec{p}'=\vec{k}-\vec{k}'$$

$$\text{so } \delta(M+E-E'-p_0') = \delta(M+E-E'-\sqrt{M^2+|\vec{k}-\vec{k}'|^2})$$

$$\text{using } |\vec{k}-\vec{k}'|^2 = |\vec{k}|^2 + |\vec{k}'|^2 - 2\vec{k}\vec{k}' = E^2+E'^2-2EE'\cos\theta$$

$$= E^2+E'^2-2EE'-q^2$$

$$= (E-E')^2 - q^2$$

$$\delta(M^2+E'-E'-p_0') = \delta(M+\nu-\sqrt{M^2+\nu^2-q^2})$$

If we define $\nu \equiv E - E'$

$$\delta(M+E-E'-p'_0) = \delta\left(M+\nu-\sqrt{M^2+\nu^2-q^2}\right)$$

finding the zeros $(M+\nu)^2 = M^2+\nu^2-q^2 \Rightarrow 2M\nu = -q^2$

$$\delta(\quad) = \delta\left(\nu+\frac{q^2}{2M}\right)\frac{2p'_0}{2M} = \frac{1}{2MA}\,\delta\left(E'-\frac{E}{A}\right)$$

using $q^2 = -2EE'\sin^2\frac{\theta}{2}$

with

$$A = 1 + \frac{2E}{M}\sin^2\theta/2$$

$$d^3k' = |k'|^2 d|k'|\, d\Omega \overset{m_e=0}{=} E'^2 dE'\, d\Omega$$

so we get

$$\left.\frac{d\sigma(e\mu^-\to e\mu^-)}{d\Omega\, dE}\right|_{LAB} = \underbrace{\frac{(2\alpha E')^2}{q^4}}_{\equiv \frac{\alpha^2}{4E^2\sin^4\theta/2}}\left\{\cos^2\frac{\theta}{2} - \frac{q^2}{2M^2}\sin^2\frac{\theta}{2}\right\}\delta\left(\nu+\frac{q^2}{2M}\right)$$

link between E' and θ so only 1 independent kinematic variable

integrating E' with δ

$$\left.\frac{d\sigma(e\mu^-\to e\mu^-)}{d\Omega}\right|_{LAB} = \frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\,\frac{E'}{E}\left\{\cos^2\frac{\theta}{2} - \frac{q^2}{2M^2}\sin^2\theta/2\right\}$$

with $E' = \dfrac{E}{1+\frac{2E}{M}\sin^2\theta/2}$ and $\dfrac{q^2}{2M} = E'-E = \dfrac{2E^2\sin^2\theta/2}{M+2E\sin^2\theta/2}$

(1) Form factor of charge distribution

the scattering of e^- ~~over a~~ static charge distribution $Z\rho(\vec{r})$ is used in classical em to obtain information about its structure.

Static $\Rightarrow |\vec{K}| = |\vec{K}'| \simeq E$

$$\Rightarrow |\vec{q}|^2 = |\vec{K} - \vec{K}'|^2 = 4E^2 \sin^2\frac{\theta}{2}$$

So knowing E and measuring we know $|\vec{q}|$

To obtain information on $\rho(\vec{r})$ we compare

$$\frac{d\sigma}{d\Omega}(e\, Z_{\substack{\text{point}\\ \text{charge}}}) \quad \text{with} \quad \frac{d\sigma}{d\Omega}(e^- Z\rho)$$

and the ratio is the form factor

$$\frac{d\sigma}{d\Omega}(e^- Z\rho) \equiv F(\vec{q})\, \frac{d\sigma}{d\Omega}(e^- Z_{\substack{\text{point}\\ \text{charge}}})$$

$F(\vec{q})$ is the Fourier transform of ρ

$$F(\vec{q}) = \int d^3\vec{r}\; e^{i\vec{q}\vec{r}}\, \rho(\vec{r})$$

since $\int \rho(\vec{r})\, d^3\vec{r} = 1 \Rightarrow F(0) = 1$

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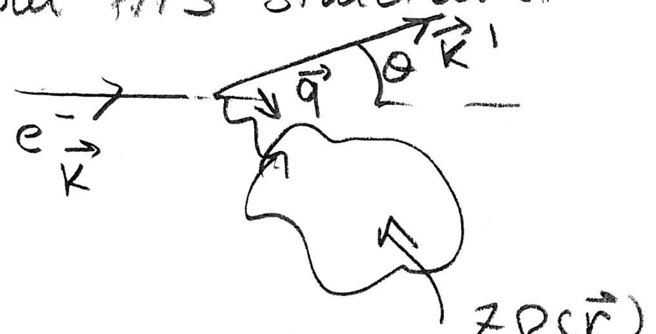

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For a spherically symmetric dist$^{\text{n}}$

$$F(q) = \int \rho(r)\, r^2 dr\, d\cos\theta\, d\varphi\, e^{+i|\vec{q}|r\cos\theta}$$

expanding in q

$$F(q) = 1 - i\int |\vec{q}|\, r^3 \rho(r)\, dr \underbrace{\int_{-1}^{1}\cos\theta\, d\cos\theta}_{0} \int d\varphi$$

$$\pm \frac{1}{2}\int |\vec{q}|^2\, r^4 \rho(r)\, dr \underbrace{\int_{-1}^{1}\cos^2\theta\, d\cos\theta}_{\frac{2}{3} = \frac{1}{3}\int_{-1}^{1} d\cos\theta} \int d\varphi$$

$r^2 d^3\vec{r}$

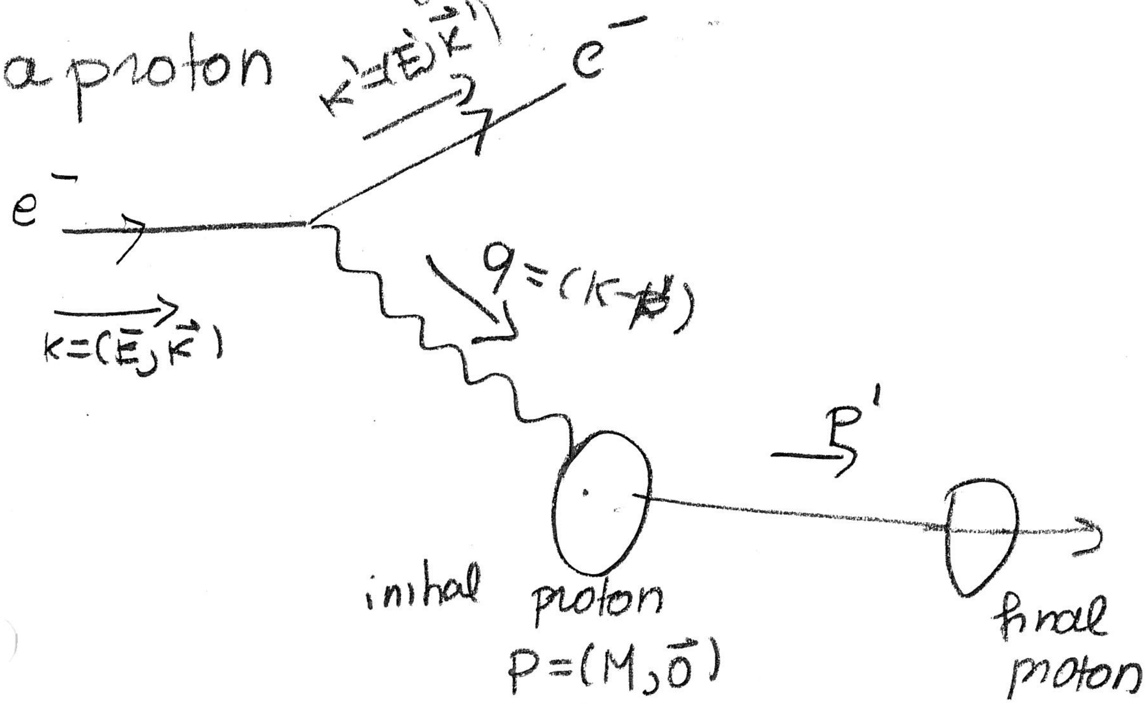
$$= 1 - \frac{|\vec{q}|^2}{6}\underbrace{\int r^2 d^3\vec{r}}_{\langle r^2\rangle}$$

So measuring the differential scattering CS at low scattering angles ($\Rightarrow$ low $|\vec{q}|$) we can infer the charge radius of the distribution.

✓

2 $e^- p^- \to e^- p^-$

We are now going to do elastic e^- scattering over a proton



As for $e^-\mu^- \to e^-\mu^-$ we can write

$$\mathcal{M} = \frac{-e^2}{q^2} j^\alpha_{elec} \, j_{\alpha\,prot}$$

with $j^\alpha_{elec} = \bar{u}_e(k')\gamma^\alpha u_e(k)$

but for $j_{\alpha\,prot} = \bar{u}_p(p')[\ ?\ ]^\alpha u_p(p)$

- $[\ ?\ ]^\alpha$ must be a 4 vector
- must be formed with $p^\alpha, p'^\alpha, q^\alpha$ or γ^α
- must be conserved $\Rightarrow \partial_\alpha j^\alpha_{prot} = 0 \Rightarrow q_\alpha j^\alpha_{prot} = 0$

the most general form compatible with these conditions

$$[\ ]^\alpha = F_1(q^2)\gamma^\alpha + \frac{\kappa}{2M_p} F_2(q^2)\sigma_{\alpha\beta} q^\beta$$

$F_1(q^2)$ and $F_2(q^2) \equiv$ electromagnetic form factors of the proton.

$\kappa \equiv g_{proton} - 2$ ($\equiv$ anomalous magnetic moment of proton.

$F_1(q^2), F_2(q^2) \equiv$ parametrize our ignorance of the structure of the proton (also κ) - they must be determined experimentally by measuring the angular distribution of the scattered E^-

For $q^2 \to 0$ ($\lambda_\gamma \sim \frac{1}{\sqrt{q^2}}$ very large)

the photon "sees" the proton as a point source with anomalous magnetic moment κ $\Rightarrow$

$F_1(0) = F_2(0) = 1$

Integrating the phase space as for $e^-\mu^- \to e^-\mu^-$

$$\left.\frac{d\sigma}{dE' d\Omega}(e^-p \to e^-p)\right|_{LAB} = \frac{(2\alpha E')^2}{q^4}\left\{\left(F_1^2 - \frac{\kappa^2 q^2}{4M^2}F_2^2\right)\cos^2\frac{\theta}{2} - \frac{q^2}{2M^2}(F_1+\kappa F_2)^2\sin^2\frac{\theta}{2}\right\} \times \delta\left(\nu + \frac{q^2}{2M}\right)$$

link between E' and θ

integrating E' with δ

$$\left.\frac{d\sigma}{d\Omega}(e^-p \to e^-p)\right|_{LAB} = \frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\frac{E'}{E}\left\{\left(F_1^2 - \frac{\kappa^2 q^2}{4M^2}F_2^2\right)\cos^2\frac{\theta}{2} - \frac{q^2}{2M^2}(F_1+\kappa F_2)^2\sin^2\frac{\theta}{2}\right\}$$

still $E' = \dfrac{E}{1 + 2E/M \sin^2\frac{\theta}{2}}$ and $\dfrac{q^2}{2M} = E - E'$

In practice it is better to use linear combinations of F_1 and F_2 so no interference appear

$$G_E(q^2) = F_1(q^2) + \frac{\kappa q^2}{2M^2} F_2(q^2) \rightarrow \text{electric form fact}$$

$$G_M(q^2) = F_1(q^2) + \kappa F_2(q^2) \rightarrow \text{magnetic form facts}$$

$$\left.\frac{d\sigma_{ep}}{d\Omega}\right|_{LAB} = \frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\frac{E'}{E}\left\{\frac{G_E^2 + \tau G_M^2}{1+\tau}\cos^2\frac{\theta}{2} + 2\tau G_M^2 \sin^2\frac{\theta}{2}\right\}$$

with $\tau = -q^2/4M$

G_E and G_M are referred to as electric and magnetic form factors.

Data on $e^- p \to e^- p$ is used to fit these form factor. We get

$$G_E(q^2) \simeq \left(1 - \frac{q^2}{0.71\,\mathrm{GeV}^2}\right)^{-2} \xrightarrow{q^2 \to 0} 1 + 2\frac{q^2}{0.71\,\mathrm{GeV}^2}$$

So the mean charge radius of the proton

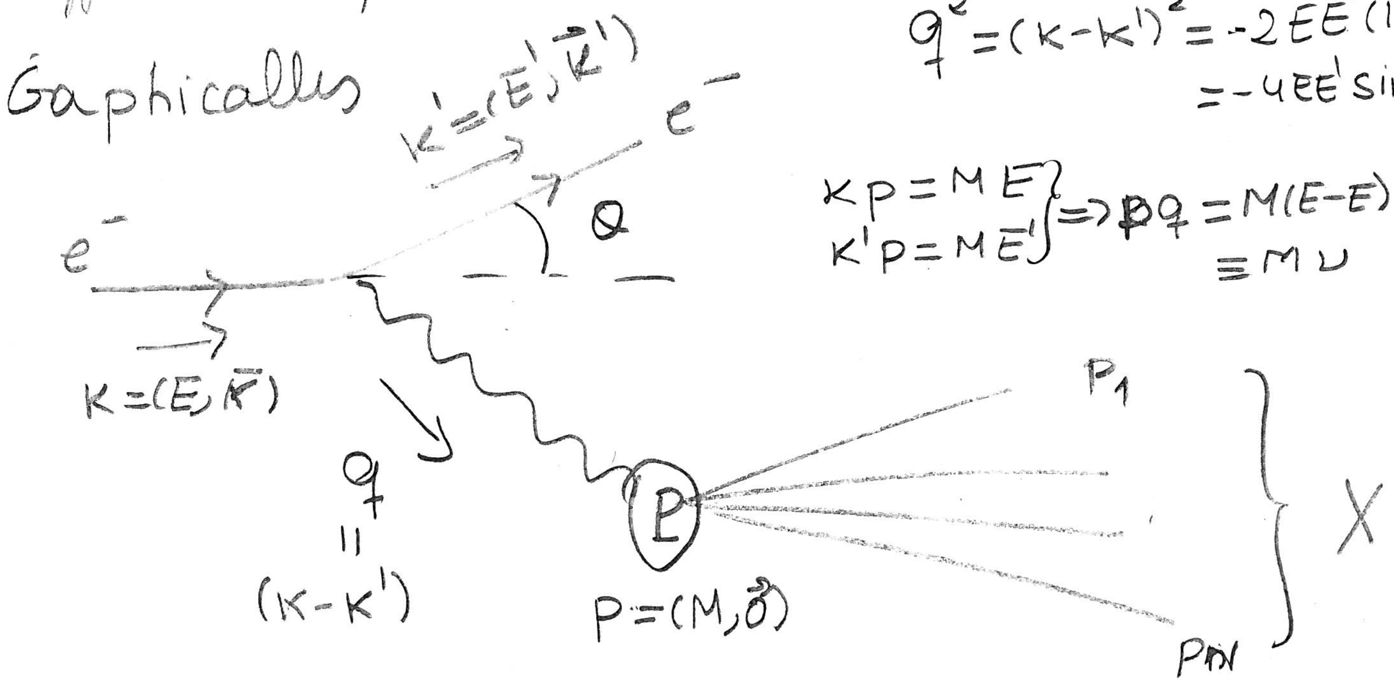
$$\Rightarrow \langle r^2 \rangle_{proton} \equiv 6 \left.\frac{dG_E}{dq^2}\right|_{q^2=0} = \frac{12}{0.71\,\mathrm{GeV}^2} = (0.81\,\mathrm{fm})^2$$

(3) Deep inelastic scattering $e^-p \to e^- X$

If in the e^-p collision e^- transfers a large $|q^2|$ to the proton it can "break it" and then in the final state we will have a collection of hadrons (protons, neutrons, pions, kaons...). Generically we reffer to this process as "deep inelastic scattering" (DIS)

$$q^2 = (k-k')^2 = -2EE'(1-\cos\theta) = -4EE'\sin^2\frac{\theta}{2}$$

Graphically



$$\left.\begin{matrix} kp = ME \\ k'p = ME' \end{matrix}\right\} \Rightarrow pq = M(E-E') \equiv M\nu$$

Now if we write

$$\mathcal{M} = -\frac{e^2}{q^2}\, j^{\alpha}_{elec}\, j_{\alpha, X} \qquad j^{\alpha}_{elec} = \bar{u}_e(k')\gamma^{\alpha} u_e(k)$$

but we do not know what to write for $j_{\alpha, X}$

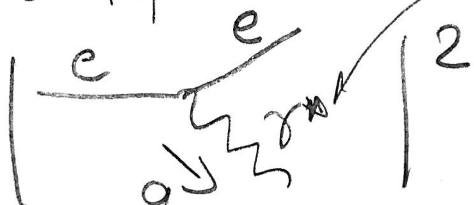
For whatever $J_\alpha X$ we can always write

$$|\overline{\mathcal{M}}|^2 = \frac{e^4}{q^4} L^{\alpha\beta}_{elec} K_{\alpha\beta}(X)$$

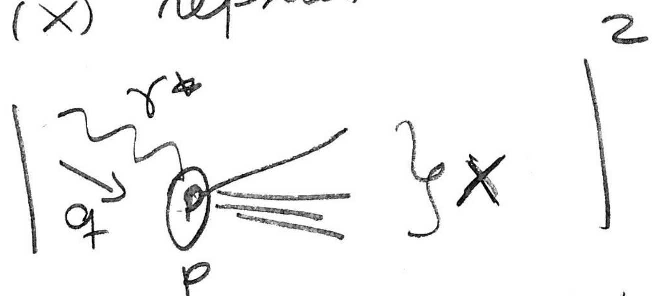
where $L^{\alpha\beta}_{elec} = \frac{1}{2}\sum_{ss'} [\bar{u}^{s'}(k')\gamma^\alpha u^s(k)][\bar{u}^{s'}(k')\gamma^\beta u^s(k)]^*$

$$= 2\left[k^\alpha k'^\beta + k^\beta k'^\alpha - 2g^{\alpha\beta}(k k')\right]$$

which represents $\qquad * \equiv$ virtual photon



and $K_{\alpha\beta}(X)$ represent



The cross section in the LAB system summing over all possible X

$$d\sigma^{DIS}\Big|_{LAB} = \frac{1}{4ME}\sum_X |\overline{\mathcal{M}}|^2 \frac{d^3k'}{(2\pi)^3 2E'} \prod_{i\in X} \frac{d^3p_i}{(2\pi)^3 2E_i} (2\pi)^4\delta^4(p+k-k'-\textstyle\sum p_i)$$

$$= \frac{1}{4ME}\frac{e^4}{q^4} L^{\alpha\beta}_{ele} \frac{d^3k'}{(2\pi)^3 2E'} \sum_X K_{\alpha\beta}(X) \prod_i \frac{d^3p_i}{(2\pi)^3 2E_i}(2\pi)^4\delta^4($$

$d^3k' = E'^2 dE' d\Omega$

$$\Rightarrow \frac{d\sigma}{dE' d\Omega}\bigg|_{LAB} = \frac{e^4}{16\pi^2 q^4} \frac{E'}{E} L_{ele}^{\alpha\beta} \underbrace{\left[\frac{1}{4\pi M} \sum_X \int \prod_i \frac{d^3 p_i}{(2\pi)^3 2E_i} K_{\alpha\beta}(X) (2\pi)^4 \delta^4 (\right]}_{\equiv W^X_{\alpha\beta}}$$

So now we have to figure out the most general form of $W_{\alpha\beta}$

First notice that we have lost the constraint between E' and θ because for each X the δ will give a different relation between E' and θ and we are summing over all possible X

For each X we can define ~~the~~ invariant mass of the ~~X~~ , from $\delta(p + \underbrace{k - k'}_{q} - \sum p_i)$

$$M_X^2 \equiv \sum_i p_i^2 = (p+q)^2 = M^2 + 2pq + q^2 = M^2 + 2M\nu + q^2 \Rightarrow q^2 = M_X^2 - M^2 - 2M\nu$$

not fixed

So for each M_X there is a link between q^2 and ν $\Rightarrow$ link between E' and θ. But as M_X is not fixed we do have 2 independent variables (θ, E') or we can also use (ν, q^2)

What is the most general form of $W^{\alpha\beta}(x)$?

$W^{\alpha\beta}$ is a Lorentz tensor with no spin structure and it can be made of p or q and it can depend on the 2 independent variables

So the most general form is

$$W^{\alpha\beta} \equiv -W_1(q^2,\nu)\, g^{\alpha\beta} + \frac{W_2(q^2,\nu)}{M^2}\, p^\alpha p^\beta$$

$$+ \frac{W_4(q^2,\nu)}{M^2}\, q^\alpha q^\beta + \frac{W_5(q^2,\nu)}{M^2}\,(p^\alpha q^\beta + q^\alpha p^\beta)$$

(symmetric under $\alpha \Leftrightarrow \beta$ because $L^{\alpha\beta}_{elec}$ is symmetric)

$W_{1,2,4,5}(q^2,\nu)$ parametrize our ignorance and they need to be extracted from data. Not all of them are independent because conservation of em current

$$\Rightarrow W^{\alpha\beta} \equiv J^\alpha_X\, J_{X,\beta} \quad \text{with } \partial^\alpha J_{\alpha,X} = 0$$

$$\Rightarrow q_\alpha W^{\alpha\beta} = q_\beta W^{\alpha\beta} = 0$$

$$0 = -q^\alpha W_1 + \frac{W_2}{M^2}(pq)\, p^\alpha + \frac{W_4}{M}\, q^2 q^\alpha + \frac{W_5}{M^2}(p^\alpha q^2 + q^\alpha (p\cdot$$

$$= q^\alpha\Big[-W_1 + \frac{W_4}{M^2}\, q^2 + \frac{W_5}{M^2}(pq)\Big]$$

$$+ p^\alpha\Big[\frac{W_2}{M^2}(pq) + \frac{W_5}{M^2}\, q^2\Big]$$

this must vanish for any q and p so each coeffcient must vanish

$$\Rightarrow W_5 = -W_2 \frac{(pq)}{q^2} \quad , W_4 = \frac{M^2}{q^2} W_1 + W_2 \frac{(pq)^2}{q^4}$$

$$\text{So } W^{\alpha\beta}(q^2,\nu) = W_1(q^2,\nu)\left[-g^{\alpha\beta} + \frac{q^\alpha q^\beta}{M^2}\right)$$

$$+ \frac{W_2(q^2,\nu)}{M^2}\left[p^\alpha - \frac{(pq)}{q^2} q^\alpha\right]\left[p^\beta - \frac{(pq)}{q^2} q^\beta\right]$$

$$\text{So } L^{\alpha\beta} W_{\alpha\beta} = 4 W_1(q^2\nu)(kk') + 2\frac{W_2(q^2\nu)}{M^2}\left[2(pk)(pk') - M^2 kk'\right]$$

$$\overset{\text{LAB}}{=} 4EE'\left\{2W_1(q^2,\nu)\sin\frac{\theta}{2} + W_2(q^2\nu)\cos^2\frac{\theta}{2}\right\}$$

$$\Rightarrow \left.\frac{d\sigma^{DIS}}{dE'd\Omega}\right|_{LAB} = \frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\left\{W_2(q^2\nu)\cos^2\frac{\theta}{2} + 2W_1(q^2\nu)\sin^2\frac{\theta}{2}\right\}$$

mass scale is explicitly present; it is set by the empirical value 0.71 GeV in the dipole formula for $G(Q^2)$ which reflects the inverse size of the proton, see (8.20). As Q^2 increases above $(0.71\ \mathrm{GeV})^2$, the form factor depresses the chance of elastic scattering; the proton is more likely to break up. The point structure functions, on the other hand, depend only on a dimensionless variable $Q^2/2m\nu$, and no scale of mass is present. The mass m merely serves as a scale for the momenta Q^2, ν.

The discussion can be summarized as follows: if large Q^2 virtual photons resolve "point" constituents inside the proton, then

$$MW_1(\nu, Q^2) \xrightarrow[\text{large } Q^2]{} F_1(\omega),$$

$$\nu W_2(\nu, Q^2) \xrightarrow[\text{large } Q^2]{} F_2(\omega), \tag{9.5}$$

where

$$\omega = \frac{2q \cdot p}{Q^2} = \frac{2M\nu}{Q^2}. \tag{9.6}$$

Note that in (9.5) we have changed the scale from what it was in (9.3). We have introduced the proton mass instead of the quark mass to define the dimensionless variable ω. The presence of free quarks is signaled by the fact that the inelastic structure functions are independent of Q^2 at a given value of ω [see (9.5)]. This is equivalent to the onset of $\sin^{-4}(\theta/2)$ behavior for large momentum transfers in the Rutherford experiment, which reveals the "point" charge of the nucleus in the atom. A sample of data is shown in Fig. 9.2. νW_2 at $\omega = 4$ is independent of Q^2; the photon is indeed interacting with point-like particles. No form factors, leading to additional Q^2 dependence as in (9.4), are present. Are these particles (called partons by Bjorken) the same as the quarks discovered in the spectroscopy of hadrons (Chapter 2)?

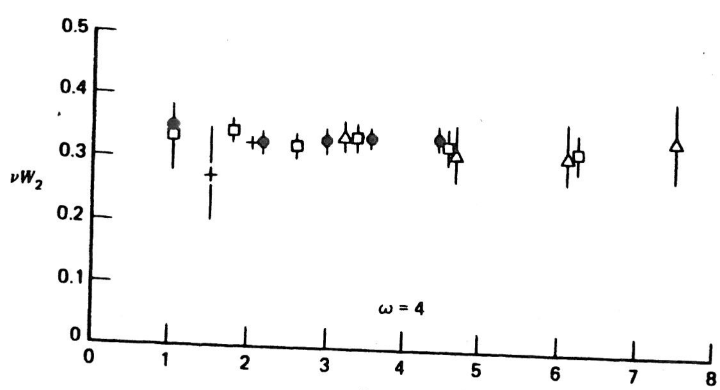


Fig. 9.2 The structure function νW_2 determined by electron–proton scattering as a function of Q^2 for $\omega = 4$. Data are from the Stanford Linear Accelerator.

4) Bjorken Scaling

change of variable

Let's collect the expression we got ($Q^2 = -q^2 > 0$)

$$\left.\frac{d\sigma^{DIS}}{dE' d\Omega}\right|_{LAB} = \frac{\alpha^2}{4E^2 \sin^4\frac{\theta}{2}} \left[W_2(Q^2,\nu)\cos^2\frac{\theta}{2} + 2W_1(Q^2,\nu)\sin^2\frac{\theta}{2} \right]$$

with $Q^2 = -q^2 = 4EE'\sin^2\frac{\theta}{2}$ and $\nu = E - E'$ being 2 indep variable

$$\left(F_1^2 - \frac{\kappa^2 q^2}{4M^2} F_2^2 \right) \qquad -\frac{q^2}{2M^2}(F_1 + \kappa F_2)^2$$

$$\left.\frac{d\sigma^{ep\to ep}}{dE' d\Omega}\right|_{LAB} = \frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}} \left[G_2(Q^2)\cos^2\frac{\theta}{2} + 2G_1(Q^2)\sin^2\frac{\theta}{2} \right] \delta\left(\nu - \frac{Q^2}{2M}\right)$$

so we can say that

$$W_{1,2}^{ep\to ep} = G_{1,2}(Q^2)\,\delta\left(\nu - \frac{Q^2}{2M}\right)$$

which depends on Q^2 and ν independently.

$$\left.\frac{d\sigma^{e\mu^-\to e\mu^-}}{dE' d\Omega}\right|_{LAB} = \frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}} \left[\cos^2\frac{\theta}{2} + \frac{Q^2}{2M_\mu^2}\sin^2\frac{\theta}{2} \right] \delta\left(\nu - \frac{Q^2}{2M_\mu}\right)$$

so

$$W_2^{e\mu\to e\mu}(Q^2,\nu) = \delta\left(\nu - \frac{Q^2}{2M_\mu}\right) = \frac{1}{\nu}\,\delta\left(1 - \frac{Q^2}{2M_\mu \nu}\right)$$

$$2W_1^{e\mu\to e\mu} = \frac{Q^2}{2M_\mu^2}\,\delta\left(\nu - \frac{Q^2}{2M_\mu}\right) = \frac{Q^2}{2M_\mu^2 \nu}\,\delta\left(1 - \frac{Q^2}{2M_\mu \nu}\right)$$

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So when e^- scatters on μ^- (point like fermion particles)

$$M\, W_1^{e^- point}(Q^2,\nu) = \frac{1}{2}\frac{Q^2}{2m\nu}\,\delta\!\left(1-\frac{Q^2}{2m\nu}\right) \equiv F_1(\omega)$$

$$\nu\, W_2^{e^- point}(Q^2,\nu) = \delta\!\left(1-\frac{Q^2}{2M\nu}\right) \equiv F_2(\omega)$$

$$\frac{Q^2}{2m\nu} \equiv \frac{1}{\omega}$$

with $F_1(\omega) = \dfrac{1}{2\omega}\, F_2(\omega)$

So for e^- scattering on point like particle the coefficients of $\sin^2\frac{\theta}{2}$ and $\cos^2\frac{\theta}{2}$ of the e^- angular energy distribution can be combined in two functions of a single variable $\omega = \dfrac{2M\nu}{Q^2} = \dfrac{2M(E-E')}{2EE'\sin^2\theta/2}$

Now if we look at data for $e^-p \to e^-X$ DIS at large Q^2 we find

in figure

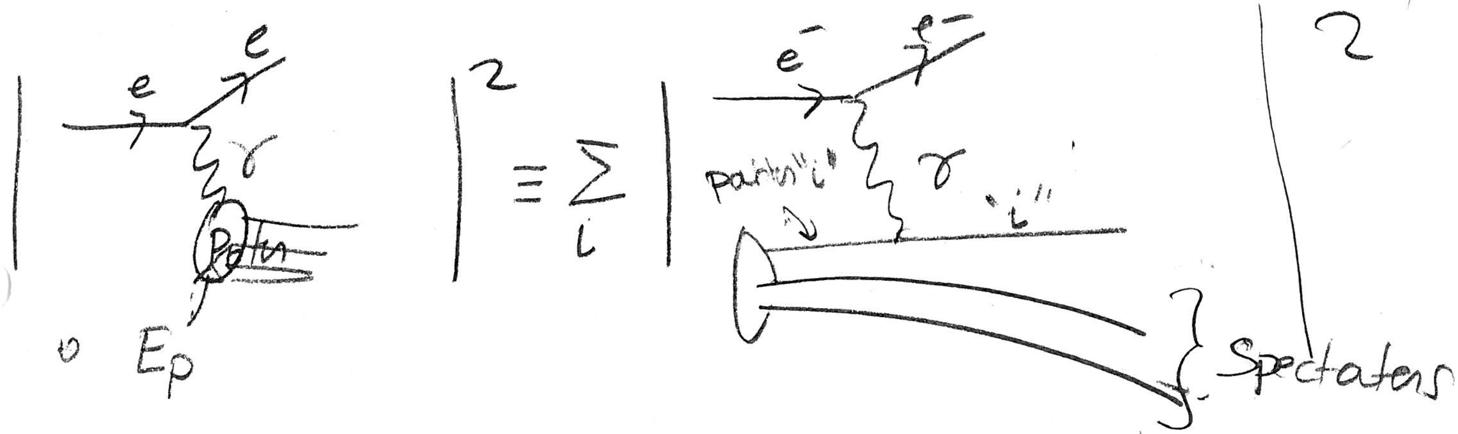
$$\left.\begin{aligned} M\, W_1^{DIS}(Q^2,\nu) &\xrightarrow{Q^2 \text{ large}} F_1(\omega) \\ \nu\, W_2^{DIS}(Q^2,\nu) &\longrightarrow F_2^{ep}(\omega) \end{aligned}\right\} \equiv \text{Bjorken scaling}$$

So when Q^2 large (γ^* short wavelength) the DIS behaves like e^- scattering on a point-like state

what are these point like particles? What is meaning of ω?

(5) The parton (≡quark) model of hadrons

In the quark model of the proton the observed Bjorken scaling in $e^-p \to e^-X$ DIS is understood as due to the interaction of the exchanged γ with one of the quarks (≡point-like state) which is "inside" the proton. Schematically



So γ interaction with only one of the partons in the proton and that parton carries a fraction "x" of the energy of the proton E_p. The other partons do not participate in the interaction (≡ spectators)

Let us define the parton distribution function of the parton "i" in the proton $f_p^i(x)$ as the probability of the parton "i" to carry a fraction x of the proton energy and momentum. We will assume "i" to follow the proton direction

	Proton	Parton "i"
energy	E_p	$E_i = x\, E_p$
3-momentum	$\vec{p}$	$\vec{p}_i = x\, \vec{p}$
mass	$M \equiv \sqrt{E^2 - \lvert\vec{p}\rvert^2}$	$m_i = \sqrt{E_i^2 - \lvert\vec{p}_i\rvert^2} = xM$

Conservation of momentum

$$\Rightarrow \sum_i \int_0^1 dx\, \vec{p}_i\, f_i^P(x) = \vec{P} \Rightarrow \sum_i \int_0^1 dx\, x\, f_i^P(x) = 1$$

the cs for the interaction $e\text{"}i\text{"} \to e\text{"}i\text{"}$ is like $e\mu^- \to e\mu^-$ just introducing $e_i \equiv$ charge of parton "i" (assuming "i" is a fermion)

so

$$M\, W_1^{ei\to ei}(Q^2, \nu) = \frac{M}{2} \underbrace{\frac{Q^2}{2m_i^2 \nu}}_{m_i^2 \,=\, x^2 M^2} e_i^2\, \delta\Big(1 - \frac{Q^2}{2\nu \underbrace{m_i}_{=\,Mx}}\Big)$$

$$= \frac{e_i^2}{2\omega x^2}\, \delta\Big(1 - \frac{1}{x\omega}\Big) \equiv F_1^i(\omega)$$

$$\Longrightarrow x = 1/\omega$$

$$\nu W_2^{ei\to ei}(Q^2, \nu) = e_i^2\, \delta\Big(1 - \frac{1}{x\omega}\Big) \equiv F_2^i(\omega) = \frac{2}{\omega} F_1^i(\omega)$$

the δ's $\Rightarrow$ the variable $\omega = \frac{2M\nu}{Q^2}$ which we found to be ~~relevant~~ theorely in DIS data is the inverse of the "x" $\equiv$ the momentum fraction carried by the parton participating in the collision

$e_i^2\delta(1-\frac{1}{x\omega})$

In this model

$$F_2^{ep,DIS}(\omega) = \int \sum_i f_i(x)\, F_2^i(\omega)\, dx$$

$$= \sum_i e_i^2 f_i(x=\tfrac{1}{\omega})\frac{1}{\omega}$$

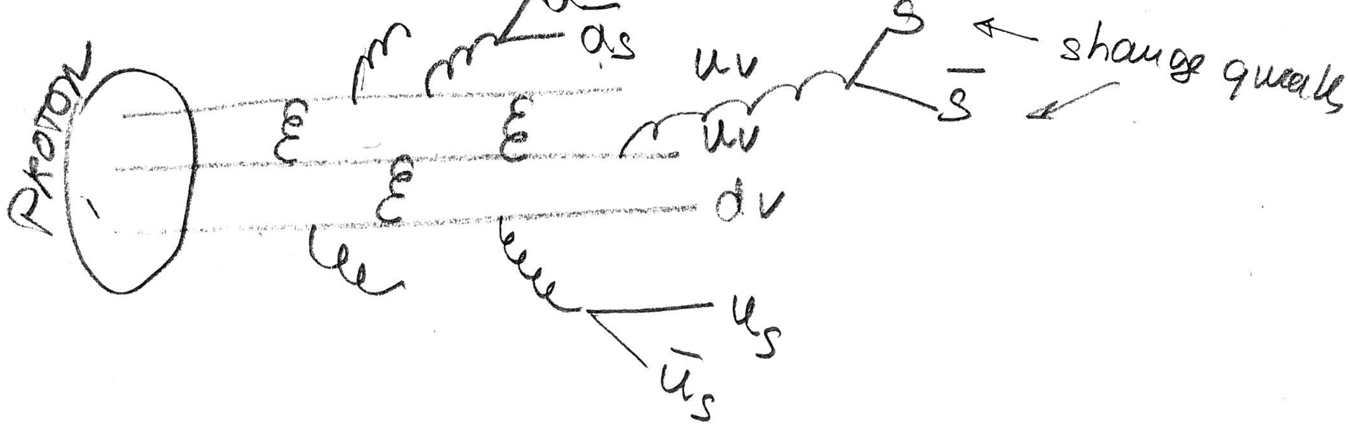
$$F_1^{ep,DIS}(\omega) = \frac{\omega}{2} F_2^{ep,DIS}(\omega)$$ Callen-Gross Relation

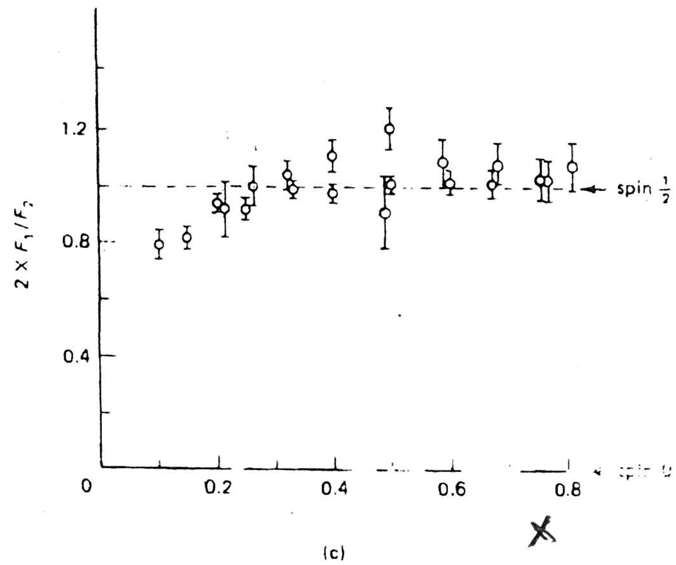
or equivalently

$$F_2^{epDIS}(x) = \sum_i e_i^2\, x\, f_i^p(x) \equiv F_2^{ep}(x)$$

$$F_1^{epDIS}(x) = \frac{1}{2x} F_2^{epDIS} \equiv F_1^{ep}(x)$$

In the simplist model the proton is understood as made of 3 valence quarks $u_v d_v d_v$ and a sea of quark-antiquark ($q_s \bar{q}_s$) and gluons (g) which are there because of strong interactions





(c)

Figure 8.7 Scaling functions and the Callan–Gross relation. In (a) and (b) I plot the experimental measurements of $F_1(x)$ and $F_2(x)$. In (c) the ratio $2xF_1/F_2$ is plotted against x, as a test of the Callan–Gross relation, which evidently holds well for $x \gtrsim 0.2$. [Data from A. Bodek et al., *Phys. Rev.* **D20,** 1471 (1979).]

Stop

Strong interaction ⇒ probability of emission of gluons or a $q_s\bar{q}_s$ pair with momentum fraction

x goes as $\sim \frac{1}{x}$

Notation $f_q^P(x) \equiv q^P(x)$

In this model
$$\frac{1}{x} F_2^{ep}(x) = \overset{e_u}{\left(\frac{2}{3}\right)^2} [u_v^P(x) + u_s^P(x) + \bar{u}_s^P(x)]$$
$$e_d \longrightarrow + \left(\frac{1}{3}\right)^2 [d_v^P(x) + d_s^P(x) + \bar{d}_s^P(x)]$$
$$e_s \longrightarrow + \left(\frac{1}{3}\right)^2 [s^P(x) + \bar{s}^P(x)]$$

as sea quarks are always produced in pair one expects $q_s^P = \bar{q}_s^P$

Further more neglecting mass effects
$$u_s^P = d_s^P = s^P \equiv S^P(x) \sim \frac{1}{x}$$

So
$$\frac{1}{x} F_2^{ep}(x) = \left(\frac{2}{3}\right)^2 (u_v^P + 2S^P) + \left(\frac{1}{3}\right)^2 (d_v^P + 2S^P) + \left(\frac{1}{3}\right)^2 (2S^P) = \frac{4}{9} u_v^P + \frac{1}{9} d_v^P + \frac{4}{3} S^P$$

For $en \to eX$ $n = uud$ and for strong interactions $u \leftrightarrow d$ makes no difference so we can assume

$$u_v^n = d_v^p \equiv d_v$$
$$d_v^n = u_v^p \equiv u_v$$
$$S^p = S^n \equiv S$$

In this simplified model

$$\frac{1}{x} F_2^{ep}(x) = \frac{1}{9}\left[4u_v(x) + d_v(x)\right] + \frac{4}{3} S(x)$$
$$\frac{1}{x} F_2^{en}(x) = \frac{1}{6}\left[4d_v(x) + u_v(x)\right] \neq \frac{4}{3} S(x)$$

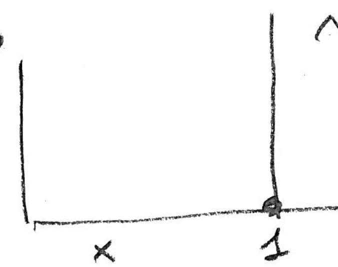
Further more they must verify

$$\int u_v(x)\,dx = 2 \qquad \int d_v(x)\,dx = 1 \Rightarrow u_v > d_v$$

How do we expect $F_2^{ep}(x)$ depend on x

If proton was made

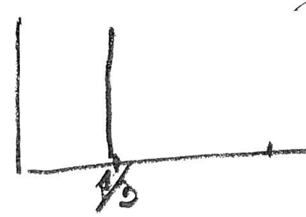
F_2^{ep}



$\sim x\,\delta(1-x)$

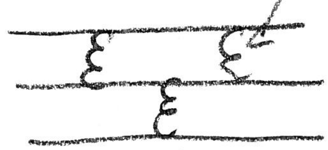
• only 1 quark $\Rightarrow x = 1$

$\sim x\,\delta(\frac{1}{3}-x)$

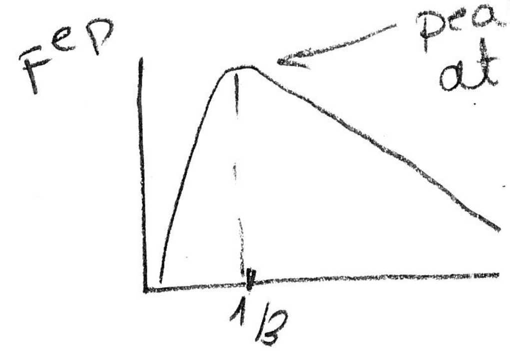


3 valence quarks no interaction $\Rightarrow$ each carries $x = 1/3$

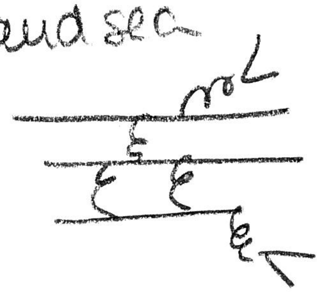
- 3 valence quarks interacting

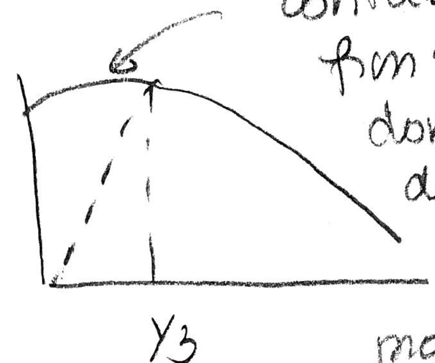


$\Rightarrow x \neq \frac{1}{3}$ possible



- 3 valence quark interacting and sea





In this simplified model

$$\frac{F_2^{en}(x)}{F_2^{ep}(x)} \xrightarrow{x \to 0} 1$$

$$\frac{F_2^{en}(x)}{F_2^{ep}(x)} \xrightarrow[u_v \gg d_v]{x \to 1} \frac{1}{4}$$

also

$$F_2^{ep}(x) - F_2^{en}(x) = \frac{1}{3}\left(u_v(x) - d_v(x)\right)$$ which must peak at $x = 1/3$ (Figures)

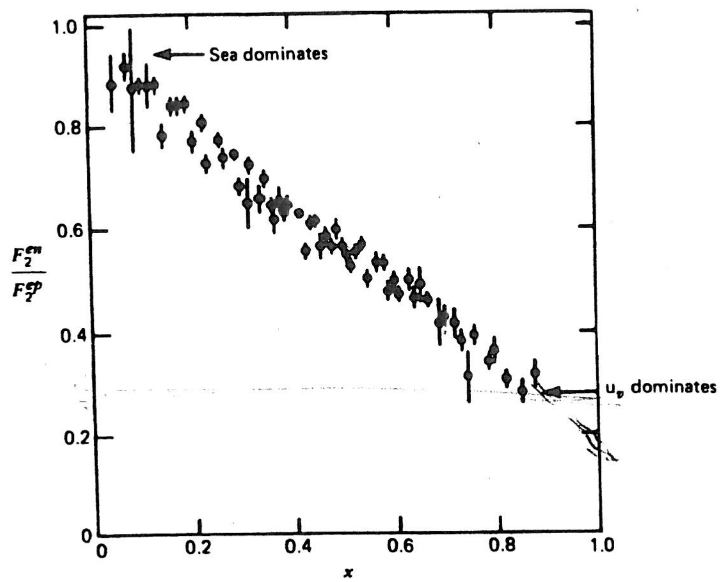


Fig. 9.6 The ratio F_2^{en}/F_2^{ep} as a function of x, measured in deep inelastic scattering. Data are from the Stanford Linear Accelerator.

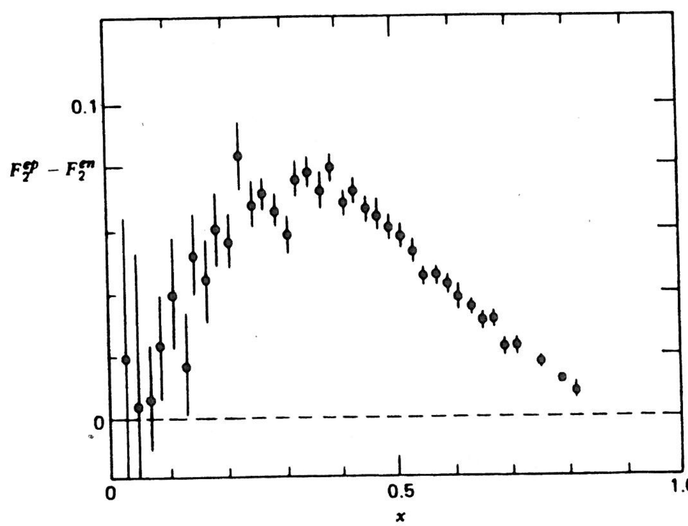


Fig. 9.8 The difference $F_2^{ep} - F_2^{en}$ as a function of x, as measured in deep inelastic scattering. Data are from the Stanford Linear Accelerator.

An alternative phenomenological approach is ...

From the experimental results of $F_2^{ep}(x)$ and $F_2^{en}(x)$

We find

$$\int_0^1 dx\, F_2^{ep}(x) \simeq 0.18 \equiv I_1$$

$$\int_0^1 dx\, F_2^{en}(x) \simeq 0.12 \equiv I_2$$

neglecting the contribution of strange quarks

$$I_1 = \frac{4}{9}\int dx\,(\bar{u}+u)x + \frac{1}{9}\int dx\, x(d+\bar{d}) \equiv \frac{4}{9}\langle x\rangle_u + \frac{1}{9}\langle x\rangle_d$$

$$I_2 = \frac{4}{9}\langle xd\rangle + \frac{1}{9}\langle xu\rangle$$

solving $\Rightarrow$ $\langle xu\rangle = 0.36 = 2\langle xd\rangle$

So $\langle xu\rangle + \langle xd\rangle \simeq 0.54 \Rightarrow$ only about 54% of the momentum of the proton is carried out by light quarks. the 46% missing is carried by the gluons. So we must also consider a gluon distribution function $g^p(x) = g^n(x) \equiv g(x)$

But as gluons have no electric charge they do not contribute to the $F_{1,2}$ of DIS

6) Parton model for hadron-hadron collisions

the quark and gluon distribution functions extracted from DIS and other data are "universal" $\equiv$ they characterize the parton in any proton (or neutron).

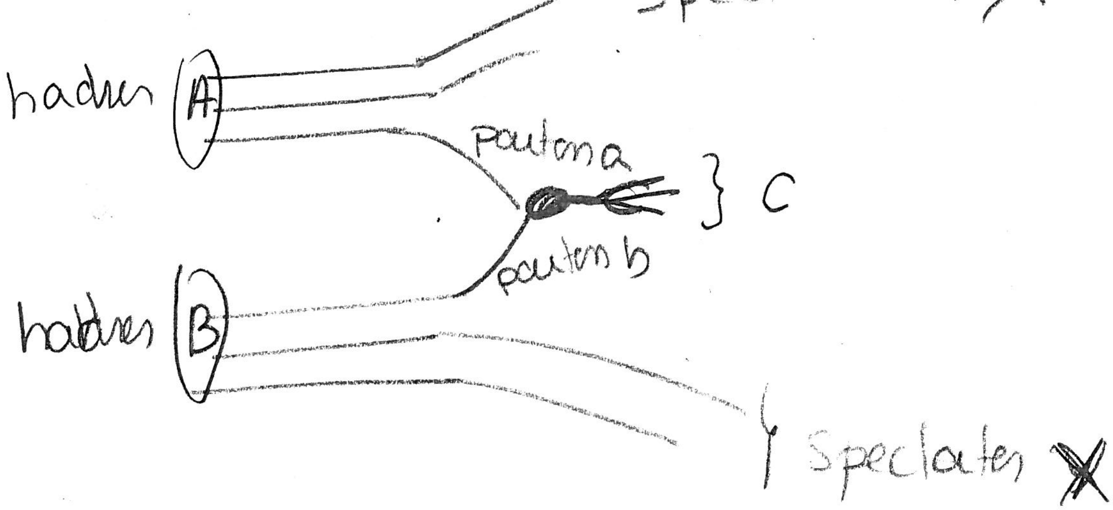
For antiparticles

$$u_V^p = \bar{u}_V^{\bar{p}} \qquad d_V^p = \bar{d}_V^{\bar{p}} \qquad s^p = s^{\bar{p}}$$

We use them to predict cross section in pp or $p\bar{p}$ collisions

Schematically
$$\underset{\text{hadron}}{A} + \underset{\text{hadr}}{B} \longrightarrow \overset{\substack{\text{set of high energy}\\ \text{particles}}}{C} + X$$

proceeds via
$$\text{parton } a + \text{parton } b \rightarrow C$$
$$\text{spectator} \Rightarrow X$$



So we can obtain

color factors

$$\sigma(A+B\to C+X)(s)=\sum_{a,b} C_{ab}\int dx_a\, dx_b\, [f_{a/A}(x_a)\, f_{b/B}(x_b) + A\leftrightarrow B \ (\text{if } a\neq b)]\ \hat{\sigma}(a+b\to C)(\hat{s}=)$$

Neglecting A, B a, b mass

$$p_a \doteq x_a P_A \qquad \Rightarrow \hat{s}\equiv(p_a+p_b)^2 \doteq 2x_a x_b P_A P_B$$
$$p_b = x_b P_B \qquad = x_a x_b (P_A+P_B)^2 = x_a x_b s$$

So to produce a system C with mass M_C

$$\Rightarrow \hat{s} > M_C^2 \Rightarrow s > \frac{M_C^2}{x_a x_b}$$

Since $x_a, x_b < 1$ and in fact well below most cases $s \gg M_C^2$

Fe at LHC with $\sqrt{s} = 13\,\text{TeV}$ we can hardly see particles with $M \gtrsim 2\,\text{TeV}$

this is more explicit, if we make a change of variable for x_a, x_b to x_a $\tau = x_a x_b$

so $\int_0^1 dx_a \int_0^1 dx_b \Rightarrow \int_{\tau_{min}}^{1} d\tau \int_{\tau}^{1} \frac{dx_a}{x_a}$

Finally the colour factors account for the fact that we have 3 possible colour of quarks (and 8 for gluons), but when we define the distribution functions $q(x)$, $g(x)$ we are summing over all colours. So the quark dist function per colour is $\frac{1}{3} q(x) \Rightarrow C_{qq} = C_{q\bar{q}} = C_{\bar{q}\bar{q}} = \frac{1}{9}$.

On the other hand when summing $\sum_{ab}$ we must sum over all colours. So if $a + b \to c$ is a strong interaction process $\Rightarrow$ (colour$_a$ = colour$_b$)

$$\Rightarrow \sum_{\substack{\text{colour } a \\ \text{colour } b}} C_{ab} = 3 \times \frac{1}{9} = \frac{1}{3}$$