Chapter 9

"Weak interactions"

1) Weak decays and parity violation: V-A charge current
2) W-boson
3) Tests: π-decay, μ-decay, β-decay
4) Fermionic mixing matrix
5) CP violation
6) Neutral currents: Z boson

1) Weak decays and Parity Violation

the lifetime of processes like

$$\Delta^{++} \to p\pi^{+} \qquad \tau_{\Delta} \sim 10^{-23}\,s$$
$$\pi^{0} \to \gamma\gamma \qquad \tau_{\pi^0} \sim 10^{-16}\,s$$

can be understood as mediated by strong and em interactions. Since at lowest order

$$\tau \sim \frac{1}{\Gamma} \sim \frac{1}{g^{\text{some power}}} \Rightarrow \tau_{\text{strong}} \ll \tau_{\text{em}}$$

But there are decay like

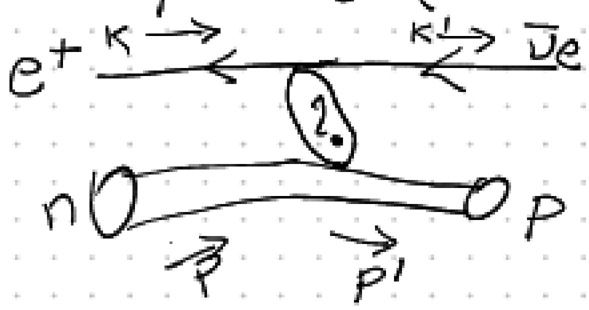
$$\Sigma^{+} \to p\,\pi^{0} \qquad \tau_{\Sigma} \sim 10^{-9}\,s$$
$$\pi^{\pm} \to \mu^{\pm} \overset{(-)}{\nu}_{\mu} \qquad \tau_{\pi^\pm} \sim 10^{-8}\,s$$

β decay $\quad n \to p e^{-} \bar{\nu}_e \qquad \tau_n \sim 800\,s$

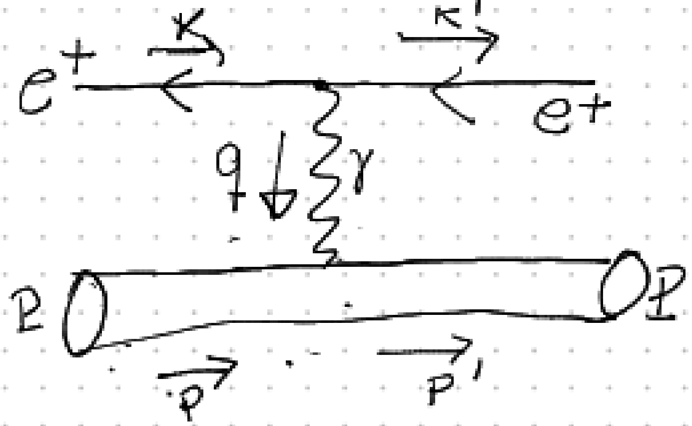
But $m_{\Sigma} \sim m_{\Delta}$ so why $\tau_{\Sigma} \gg \tau_{\Delta}$?

To explain why these decays were slower a new and weaker interaction was proposed (for $n \to p e^- \nu$ since $m_n \sim m_p$ there is an additional kinematic suppression)

If we want to build an amplitude for $n \to p \bar{e} \bar{\nu}_e$ (let us take $n e^{+} \to p \bar{\nu}_e$)



We can take $e^+p \to e^+p$ in QED as guidance ③



$$\mathcal{M}^{QED}_{e^+p\to e^+p} = -\frac{e^2}{q^2} j^{\alpha}_{e e P} j'_{\alpha, P}$$

$$j^{\alpha}_{e} = \bar{v}_e \gamma^{\alpha} v'_e$$

$$j_{\alpha,P} = \bar{u}'_p \gamma_{\alpha} [F_1(q^2) + \ldots] u_p$$

Proton form factor

$F_1(0) = 1$

with this analogy in mind Fermi proposed

$$\mathcal{M}_{n\to p e \bar{\nu}_e} = -G[\bar{u}_e \gamma^{\alpha} v_{\nu}][\bar{u}_p \gamma_{\alpha} u_n]$$

with G which must have dimension E^{-2}

Experimentally $G \sim 10^{-5}\ \text{GeV}^{-2} \Rightarrow$ range $< 10^{-4}$ fm

Also unlike in QED, in β-decay the vertex changes electric charge.

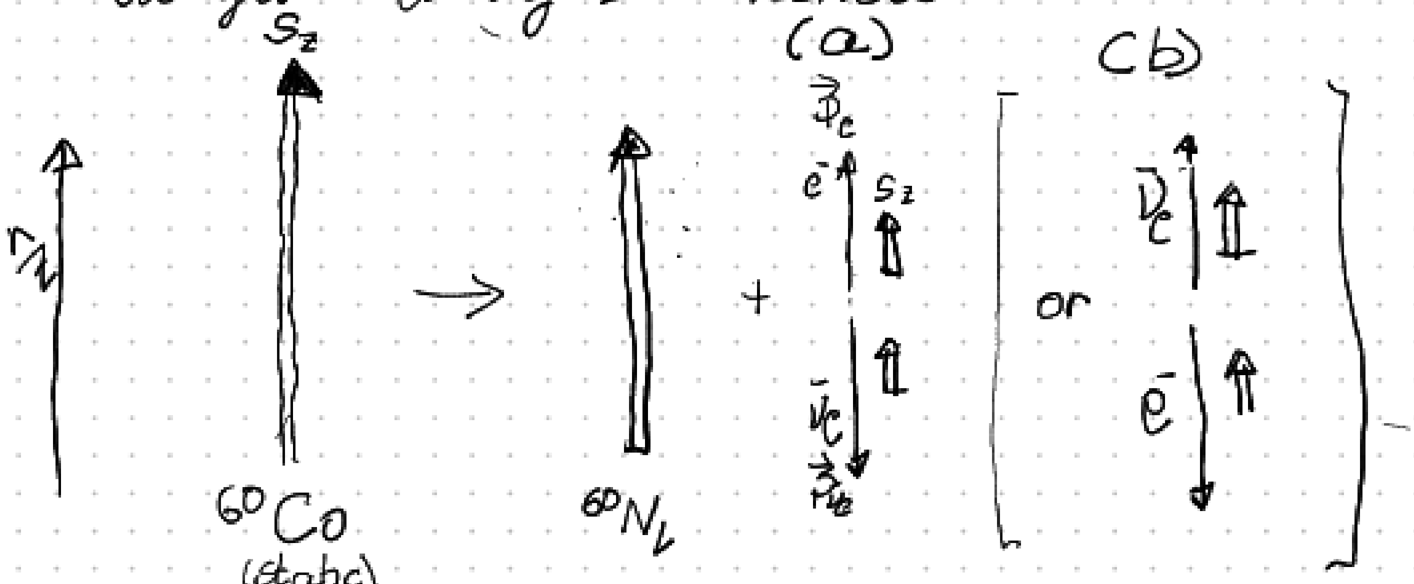
We call this weak charge interaction

In homeworks we showed that interaction $\bar{\psi}\gamma^{\mu}\psi$ conserves Parity and charge conjugation

To verify if Parity was conserved in β-decay Lee and Yang proposed and experiment and Wu made it the proposal was to measure the direction of emission of e^- in β-decay of Cobalt

$$\begin{array}{lcccccc} & {}^{60}Co & \longrightarrow & {}^{60}Ni & + e^- & + \bar{\nu}_e \\ \text{spin} \downarrow & & & & & \\ S & 5 & & 4 & \frac{1}{2} & \frac{1}{2} \end{array}$$

with respect to an external magnetic field $\vec{B}$ put there to ensure that $(S_z)_{Co}$ and $(S_z)_{Ni}$ were aligned along $\hat{z}$ direction



$$J_z = 5 \;=\; 4 \;+\; \tfrac{1}{2}+\tfrac{1}{2} \;\text{ or }\; \tfrac{1}{2}+\tfrac{1}{2}$$

there are two possible configurations allowed by angular momentum conservation

a) e^- is emitted in direction of spin of the nucleus
helicity of $\bar{\nu}_e$ is -1 $\Rightarrow v_\nu \simeq v_R$
helicity of e^- is $+1$ $\Rightarrow u_e \simeq u_R$
chirally right-handed

b) e^- is emitted in direction opposite of spin of nucleus
helicity of $\bar{\nu}_e$ is $+1$ $\Rightarrow v_\nu \simeq v_L$
helicity of e^- is -1 $\Rightarrow u_e \simeq u_L$
chirally left-handed

Under Parity

$$\vec{S} \xrightarrow{\mathcal{P}} \vec{S} \quad \text{(spin is a pseudovector)}$$

$$\vec{p}_e \rightarrow -\vec{p}_e$$

$$\vec{p}_{\bar{\nu}_e} \rightarrow -\vec{p}_{\bar{\nu}_e}$$

$$(a) \leftrightarrow (b)$$

So if Parity was conserved in β decay then 50% of the times one should observe e^- emitted in direction of spin of nucleus and 50% opposite direction (ie 50% (a) and 50% (b))

But the experiment 100% of the times (b) and never (a)

$\Rightarrow$ Parity is <u>violated maximally</u> in β decay

Always (b) $\Rightarrow$ only the left handed spinors are involved in the decay so the amplitude was always

$$(\overline{u_{e_L}})\, \gamma^\alpha\, v_L = \overline{P_L u_e}\, \gamma^\alpha\, P_L\, v_\nu \qquad \text{with } P_L = \frac{1}{2}(1-\gamma_5)$$

since $\overline{P_L u_e} = u_e^+ P_L^+ \gamma^0 = u_e^+ \frac{(1-\gamma_5)}{2}\gamma^0 = u_e^+ \gamma^0 \left(\frac{1+\gamma_5}{2}\right) = \bar{u}_e P_R$

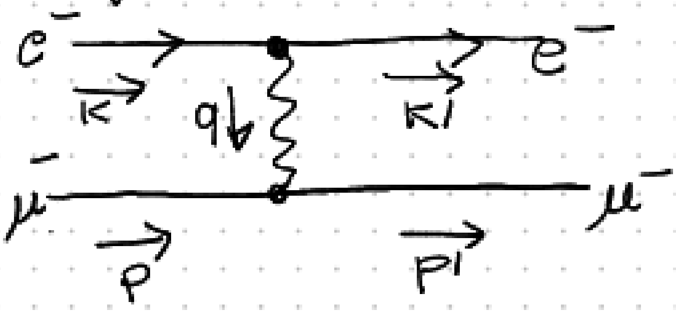
$$(\bar{u}_e)_L\, \gamma^\alpha v_\nu = \bar{u}_e P_R \gamma^\alpha P_L v_\nu = \bar{u}_e \gamma^\alpha P_L^2 v_\nu = \bar{u}_e \gamma^\alpha P_L v_\nu$$

(because $\gamma_5 \gamma^\mu = -\gamma^\mu \gamma_5$ and $\gamma^{5\,2} = I \Rightarrow P_R \gamma^\mu = \gamma^\mu P_L$
and $P_L^2 = \frac{1}{4}(1-2\gamma_5+\gamma_5^2) = \frac{1}{2}(1-\gamma_5)$)

$\Rightarrow$ weak int vertex for leptons is $\gamma^\alpha(1-\gamma_5)$
we will assume tha same holds for the hadronic vertex
$\Rightarrow$ this interaction violates both Parity and Charge Conjugation

② W-boson

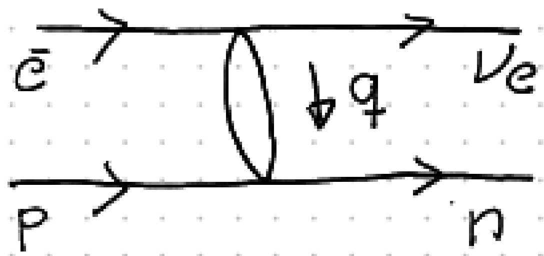
Again comparing with QED for $e^-\mu^- \to e^-\mu^-$



$$\mathcal{M}_{QED} = \frac{-e^2}{q^2}\, \bar{u}'_e \gamma^\alpha u_e \, \bar{u}'_\mu \gamma_\alpha u_\mu$$

q^2 ← from photon propagator

And for weak interactions



$$\mathcal{M}_{weak} = \frac{-G_F}{\sqrt{2}}\, \bar{u}_n \gamma^\alpha (1-\gamma_5) u_p \, \bar{u}_\nu \gamma_\alpha (1-\gamma_5) u_e$$

$\sqrt{2}$: for historical reasons

G_F: must have dimension E^{-2}

Furthermore experimentally we observe

$\mathcal{M}_{QED}$ grows at low $q^2 \to 0$

$\mathcal{M}_{weak} \to$ constant " $q^2 \to 0$

QED vertex does not exchange electric charge

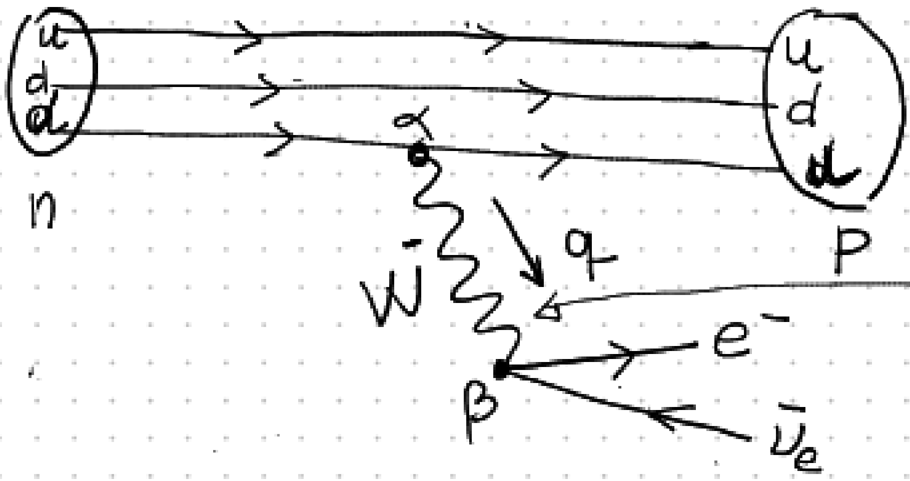
weak " does exchange " "

We are going to build a phenomenological model for these "charged" weak interactions.

Assumptions of model:

- only left-handed fermions interact in charged weak interactions
- charged weak interactions of hadrons are due to the interaction of their constituent quarks
- the interaction is due to the exchange of a charged vector boson: $W^\pm$.
- the $W^\pm$ has a mass M_W

So to lowest order β^- decay is due to



we use a wavy line also for the W boson as for the photon

(some book use a dashed line)

the interaction Lag for QED

$$\mathcal{L}_{QED,\,in} = -eQ\,\bar{\Psi}\gamma^\mu\Psi A_\mu$$

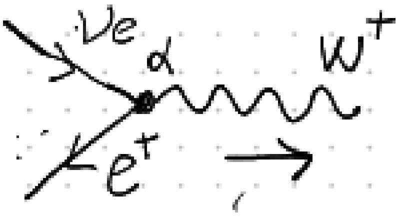
So we can write the phenomenological Lag for weak charged interactions (Let us call $W^\mu(x)$ the complex vector field for the $W^\pm$ particle)

$$\mathcal{L}_{\substack{int,\,weak\text{-}cc\\ leptons}} = -\underbrace{\frac{g_W}{\sqrt{2}}}_{\text{coupling constant}}\left[\bar{\Psi}_e\gamma^\mu\frac{(1-\gamma_5)}{2}\Psi_{\nu_e}W_\mu + \bar{\Psi}_{\nu_e}\gamma^\mu\frac{(1-\gamma_5)}{2}\Psi_e W_\mu^+\right]$$

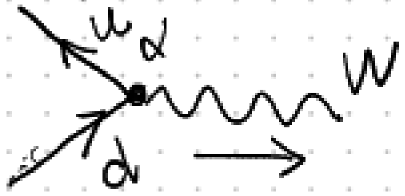
$$\mathcal{L}_{\substack{int,\,weak\text{-}cc\\ quarks}} = -\frac{g_W}{\sqrt{2}}\left[\bar{\Psi}_d\gamma^\mu\frac{(1-\gamma_5)}{2}\Psi_u W_\mu + \bar{\Psi}_u\gamma^\mu\frac{(1-\gamma_5)}{2}\Psi_d W_\mu^+\right]$$

this is for the 1st generation fermions.
For second and third we can make replicas of these Lag.
So the basic vertices for the corresponding Feynman

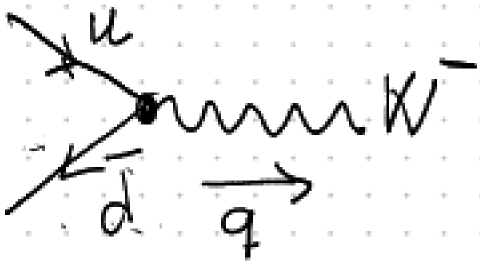
$$-\frac{ig_W}{\sqrt{2}}\,\gamma^\alpha\frac{(1-\gamma_5)}{2}$$



$$-\frac{ig_W}{\sqrt{2}}\,\gamma^\alpha\frac{(1-\gamma_5)}{2}$$



$$-\frac{ig_W}{\sqrt{2}}\,\gamma^\alpha\frac{(1-\gamma_5)}{2}$$



$$-\frac{ig_W}{\sqrt{2}}\,\gamma^\alpha\frac{(1-\gamma_5)}{2}$$

the FR for the W boson $\xrightarrow{p\,\alpha}$

W^- { incoming: $\epsilon^\alpha_\lambda(p)$
outgoing: $[\epsilon^\alpha_\lambda(p)]^*$

W^+ { incoming: $[\epsilon^\alpha_\lambda(p)]^*$
outgoing: $\epsilon^\alpha_\lambda(p)$

where for W there are 3 physical polarizations
because $M_W \neq 0$ (see chapter 4)

Internal line  $-i\,\dfrac{g_{\alpha\beta}-\frac{q_\alpha q_\beta}{M_W^2}}{q^2-M_W^2+i\epsilon}$

So when $|q| \ll M_W$ the W propagator $\rightarrow i\,\dfrac{g_{\alpha\beta}}{M_W^2}$

$\Rightarrow$ constant as $q^2 \to 0$

unlike the photon progator or gluon propagator

$\Rightarrow$ Short range of weak interaction

With these FR we can write the amplitude

$$\mathcal{M}_{\beta\,\text{decay}} = \frac{g_W^2}{2}\left[\bar{u}_n \gamma^\alpha \frac{(1-\gamma_5)}{2} u_d\right]_{\substack{\text{in } p \\ n}} \frac{g_{\alpha\beta}-\frac{q_\alpha q_\beta}{M_W^2}}{q^2-M_W^2}\left[\bar{u}_e \gamma^\beta \frac{(1-\gamma_5)}{2} v_\nu\right]$$

$$\begin{pmatrix}\text{assuming}\\ q^2 \ll M_W^2\end{pmatrix} \simeq -\frac{g_W^2}{8M_W^2}\left[\bar{u}_n \gamma^\alpha (1-\gamma_5) u_d\right]\left[\bar{u}_e \gamma_\alpha (1-\gamma_5) v_\nu\right]$$

Comparing with the effective amplitude

$$\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8M_W^2}$$

From β decay $G_F \sim 10^{-5}\,\text{GeV}^{-2}$ so if $g_W \sim e$

$\Rightarrow g_W^2 \sim 4\pi\alpha$

$$M_W \sim \sqrt{\frac{4\pi\alpha\sqrt{2}}{8G_F}} \sim 50\,\text{GeV}$$

So $M_W \gg m_p, m_n$ which justifies $q^2 \ll M_W^2$

$\sim m_n^2, m_p^2$

3) Tests

1) $\pi^- \to \ell^- \bar\nu_\ell$ decay:

Looking at the decay modes of $\pi^- (\equiv \bar u\, d)$ we find

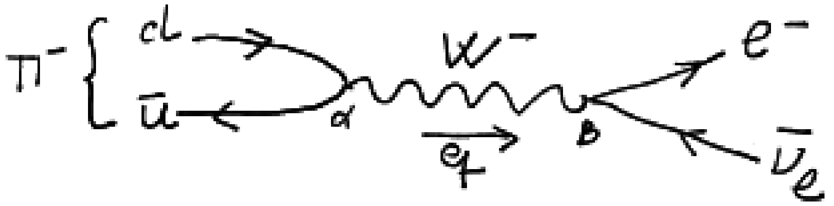
$\pi^- \to \mu^- \bar\nu_\mu$ 99.98% of times

$\pi^- \to e^- \bar\nu_e$ 10^{-4} of times

But $m_\mu \gg m_e \Rightarrow \pi^- \to \mu^- \bar\nu_\mu$ has less phase-space so why π^- decays mostly on the heavier lepton?

this can be explained by the $\frac{1-\gamma_5}{2}$ in the vertex

the lowest order diagram for this process



the corresponding amplitude

$$\mathcal{M} = \frac{g_W^2}{2}\left[\bar v_u \gamma^\alpha \frac{(1-\gamma_5)}{2} u_d\right]_{in\,\pi} \left[\frac{g_{\alpha\beta} - \frac{q_\alpha q_\beta}{M_W^2}}{q^2 - M_W^2}\right]\left[\bar u_\ell \gamma^\beta \frac{(1-\gamma_5)}{2} v_{\nu_\ell}\right]$$

Since $q^2 = (p_e + p_\nu)^2 = m_\pi^2 \ll M_W^2$

$$\mathcal{M} \simeq -\overbrace{\frac{g_W^2}{8M_W^2}}^{=\frac{G_F}{\sqrt{2}}} \left[\bar v_u \gamma^\alpha (1-\gamma_5) u_d\right]\left[\bar u_e \gamma_\alpha (1-\gamma_5) v_\nu\right]$$

the π^- is a pseudoscalar meson $\Rightarrow S_\pi = 0 \Rightarrow \vec J_{int} = 0$

$\Rightarrow \vec J_{final} = \vec S_\ell + \vec S_{\bar\nu_\ell} = 0$

the π^- is at rest

$$\Rightarrow \vec{P}_{\pi^-} = 0 = \vec{P}_{\ell} + \vec{P}_{\bar{\nu}}$$

So $\text{helicity}(\ell) = \frac{\vec{S}_e \cdot \vec{P}_e}{|\vec{P}_e|} = \frac{\vec{S}_{\bar{\nu}} \cdot \vec{P}_{\bar{\nu}}}{|\vec{P}_{\bar{\nu}}|} = \text{helicity}(\bar{\nu}_e)$

We know that the charged weak interaction only couples chirally left-handed $\bar{\nu}_e$ and e^-

If we neglect m_ν : chirally left $\bar{\nu}_e \simeq \text{helicity}(\bar{\nu}_e) = +\frac{1}{2}$

$\Rightarrow$ helicity of $e^- = +\frac{1}{2}$

But if we neglect m_e then chirality $e^- = \frac{1}{2} \Rightarrow e^-$ righthanded

So if we neglect m_e

$$\mathcal{M}(\pi^- \to e^- \bar{\nu}_e) = 0$$

If we keep m_e we get

$$\mathcal{M}(\pi^- \to e^- \bar{\nu}_e) \propto m_e$$

from phasespace integral

So $\quad \Gamma(\pi \to e^- \bar{\nu}_e) \propto m_e^2 (m_\pi^2 - m_e^2)$

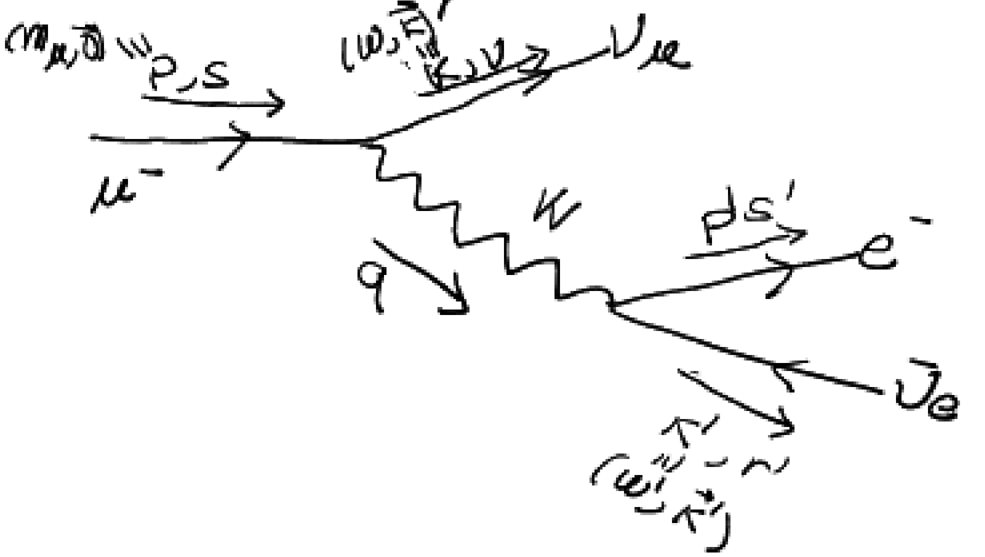
So we predict

$$\frac{\Gamma(\pi^- \to e^- \bar{\nu}_e)}{\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)} = \frac{m_e^2}{m_\mu^2} \frac{(m_\pi^2 - m_e^2)}{(m_\pi^2 - m_\mu^2)} = 1.23 \times 10^{-4}$$

and experimentally we observe $(1.23 \pm 0.002) \times 10^{-4}$

μ^- decay

For $\mu^- \to e^- \nu_\mu \bar{\nu}_e$ in weak interactions the only tree-level diagram



Neglecting $m_e, m_{\nu_e}, m_{\nu_\mu}$ and since $m_\mu^2 << M_W^2$

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} \left[\bar{u}^r(k)\gamma^\alpha(1-\gamma_5)u^s(p)\right]\left[\bar{u}^{s'}(p')\gamma_\alpha(1-\gamma_5)v^{r'}(k')\right]$$

$$\text{so } \overline{|\mathcal{M}|^2} = \frac{1}{2}\sum_{\substack{rs\\ r's'}} |\mathcal{M}|^2$$

$$= \frac{G_F^2}{4} \mathrm{Tr}\left[\not{k}\gamma^\alpha(1-\gamma_5)(\not{p}+m)\gamma^\beta(1-\gamma_5)\right]\mathrm{Tr}\left[\not{p}'\gamma_\alpha(1-\gamma_5)\not{k}'\gamma_\beta(1-\gamma_5)\right]$$

$$= \frac{G_F^2}{4} 16 \left[k^\alpha p^\beta + p^\alpha k^\beta - g^{\alpha\beta}(kp) + i\epsilon^{\mu\nu\alpha\beta}k_\mu p_\nu\right]$$
$$\left[k'_\alpha p'_\beta + k'_\beta p'_\alpha - g_{\alpha\beta}(k'p') + i\epsilon_{\rho\sigma\alpha\beta}p'^\rho k'^\sigma\right]$$

$$= 16 G_F^2 \left[2(kk')(pp') + 2(kp')(k'p) - \underbrace{\epsilon^{\mu\nu\alpha\beta}\epsilon_{\rho\sigma\alpha\beta}}_{\substack{\parallel \\ -2(g^\mu_\rho g^\nu_\sigma - g^\mu_\sigma g^\nu_\rho)}} k_\mu p_\nu p'^\rho k'^\sigma\right]$$

$$= 64\, G_F^2 (kp')(k'p)$$

$$\mathrm{Tr}(\gamma^\alpha\gamma^\beta\gamma^\rho\gamma^\sigma\gamma^5) = i\epsilon^{\alpha\beta\rho\sigma}$$

At μ^- rest frame

$$k'p = m\,\omega'$$

$$kp' = \frac{1}{2}\Big[(k+p')^2 - \underset{m_\nu^2=0}{k^2} - \underset{m_e^2=0}{p'^2}\Big] = \frac{1}{2}(k'-p)^2 = \frac{1}{2}\underset{M^2}{p^2} - (k'p)$$

$$= \frac{1}{2}(m^2 - 2\,m\,\omega')$$

$$\Rightarrow |\bar{M}|^2 = 32\,G_F^2\,m^2\,\omega'\,(m - 2\omega')$$

independent of direction of outgoing particles

the width

$$\Gamma_\mu = \frac{1}{2m}\int |\bar{M}|^2\, d\Phi_3$$

$$d\Phi_3 = (2\pi)^4\,\delta^4(p - p' - k' - k)\,\frac{d^3p'}{2E'(2\pi)^3}\,\frac{d^3k'}{2\omega'(2\pi)^3}\,\frac{d^3k}{2\omega(2\pi)^3}$$

We can use that $d^3k = 2\omega\,\delta(k^2)\,d^4k$
and integrate d^4k with δ^4

$$d^3\Phi_3 = \frac{1}{(2\pi)^5}\,\frac{1}{4E'\omega'}\,\delta\big[(p-k'-p')^2\big]\,d^3p'\,d^3k'$$

Let us define θ as the angle between $\vec{p}\,'$ and $\vec{k}\,'$

$$\Rightarrow (p-k'-p')^2 = m^2 - 2mE' - 2m\omega' + 2(p'k')$$

$$= m^2 - 2mE' - 2m\omega' + 2\omega'E'(1-\cos\theta)$$

So all angular variables in $d^3p'\,d^3k'$ can be integrated except θ

$$d^3k' \, d^3p' = \omega'^2 d\omega \, E'^2 dE' (4\pi)(2\pi) \, d\cos\theta$$

$$\Rightarrow d\phi_3 = \frac{1}{(2\pi)^3} \frac{1}{2} E'\omega' \delta[m^2 - 2mE' - 2m\omega' + 2\omega' E'(1-\cos\theta)] \, dE' d\omega' d\cos\theta$$

$$= \frac{1}{(2\pi)^3} \frac{1}{4} \delta\left[\cos\theta - \frac{m^2 - 2mE' - 2m\omega' + 2\omega' E'}{2\omega' E'}\right] dE' d\omega' d\cos\theta$$

We use the δ to integrate $d\cos\theta$

the condition $-1 \leq \cos\theta \leq 1$

$$\Rightarrow -1 \leq \frac{m^2 - 2mE' - 2m\omega' + 2\omega' E'}{2E'\omega'} \leq 1$$

$$\rightarrow \frac{1}{2}m - E' \leq \omega' \leq \frac{m}{2}$$

and $\quad 0 \leq E' \leq \frac{m}{2}$

So
$$\frac{d\Gamma}{dE'} = \int_{\frac{1}{2}m - E'}^{\frac{1}{2}m} \frac{1}{(2\pi)^3} \frac{1}{4} \frac{1}{2m} 32 G_F^2 m^2 \omega'(m - 2\omega') \, d\omega'$$

$$= \frac{G_F^2}{2\pi^3} m \int_{\frac{1}{2}m - E'}^{\frac{1}{2}m} \omega'(m - 2\omega') \, d\omega' = \frac{G_F^2}{12\pi^3} m^2 E'^2 \left(3 - \frac{4E'}{m}\right)$$

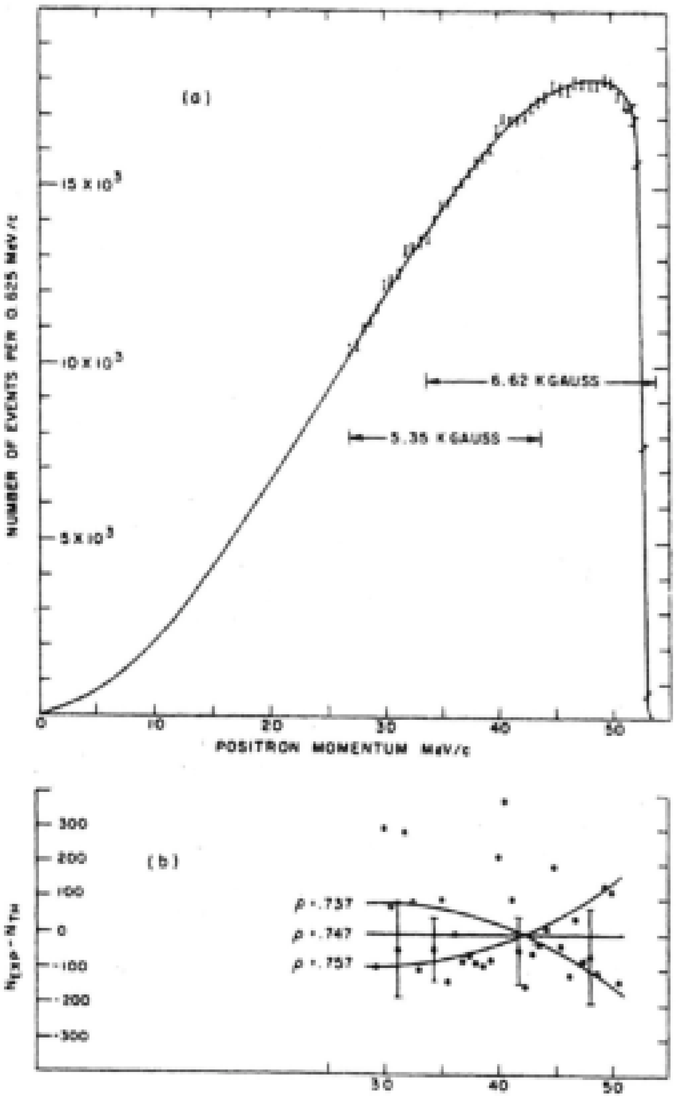
and
$$\Gamma = \int_0^{m/2} dE' \frac{d\Gamma}{dE'} = \frac{G_F^2 m^5}{192\pi^3}$$

For $\tau_\mu = 2.2 \times 10^{-6}$ s $\Rightarrow G_{F_{\mu\,\text{decay}}} = 1.16632 \times 10^{-5}\ \text{GeV}^{-2}$

of internal consistency of the data over various momentum and angular regions lead to larger uncertainty and a preliminary result of $\rho = 0.747 \pm 0.005$.

We wish to thank Dr. G. Sutter for help in the early phases of the experiment; F. Sippach for the design of the electronic system; G. Doremus for fabricating the chambers; J. Williams for computer programming; and C. Carlson, S. Herzka, B. Palatnick, and S. Stein for general assistance in the experiment.

*Work supported in part by the U. S. Office of Naval Research under Contract No. Nonr-266(72).



$\sim \frac{d\Gamma}{dE'}$

FIG. 4. (a) Experimental points for magnetic-field settings, normalized to the overlap region. The solid line is the theoretical spectrum for $\rho = 0.75$. The Michel spectrum,[4]

$$\rho(x)dx = 4[12x^2 - 12x^3 + \rho(32/3x^3 - 8x^2)]\,dx$$

Similarly one can obtain for β-decay (more complicated because of nuclear effects and $m_p \simeq m_n$)

For $^{14}O \longrightarrow {}^{14}N^* + e^+ + \nu_e$

$\begin{matrix} 8p \\ 6n \end{matrix}$ $\quad$ $\begin{matrix} 7p \\ 7n \end{matrix}$

[so this is $p \to n\, e^+ \nu_e$ which can only happen in a nucleus because $m_p < m_n$ but in a nucleus because of binding energy $E_p > E_n$]

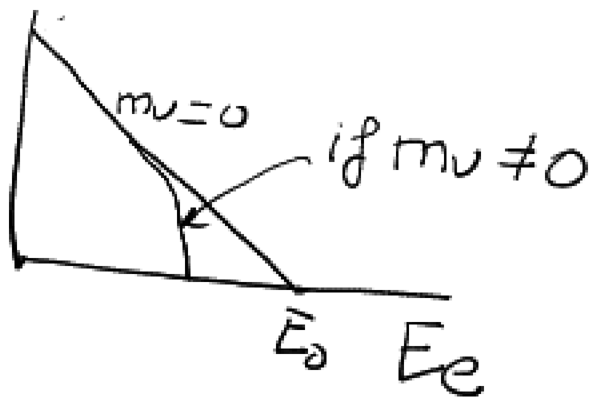
One get

$$\frac{d\Gamma}{dE_e} = \frac{G_F^2}{\pi^3} E_e |\vec{p}_e| (E_0 - E_e)^2 \qquad \text{with } E_0 = M_O - M_N \text{ and } |\vec{p}_e| = \sqrt{E_e^2 - m_e^2}$$

notice that if $m_\nu \neq 0$ $\quad E_0 = E_e + E_\nu \Rightarrow E_e = E_0 - E_\nu \leq E_0 - m_\nu$

we can reconstruct

$$\sqrt{\frac{d\Gamma}{dE} \frac{1}{E_0} \frac{1}{|\vec{p}_e|}}$$



this is the Kurie plot and this end of the energy spectrum of e^- in β-decay is our most sensitive probe of m_ν. We want E_0 to be as low as possible. Best case is tritium β decay $^3H \to {}^3He + e^- + \bar{\nu}_e$

From data for ^{16}O decay

$$G_F\Big|_{\beta\text{-decay}} = 1.136 \times 10^{-5}\ \text{GeV}^{-2}$$

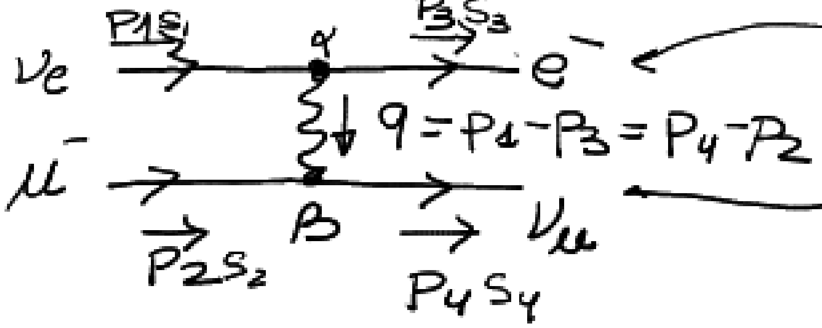
Comparing with

$$G_F\Big|_{\mu\text{-decay}} = 1.166 \times 10^{-5}\ \text{GeV}^{-2}$$

$$\Rightarrow \frac{G_F|_{\beta\text{-dec}}}{G_F|_{\mu\text{-dec}}} \simeq 0.975$$

So the coupling of weak charged interactions of quarks as "almost" the same as the coupling of weak charged interactions of leptons.
In fact the couplings are equal and this small difference can be explained as we will see in Sec 4.

- $\nu_e \mu^- \to \nu_\mu e^-$ scattering



Notice vertex does not mix generation

$$\mathcal{M} = \frac{g_W^2}{2}\left[\bar{u}_{\nu_\mu}^{s_4}(p_4)\gamma^\beta \frac{(1-\gamma_5)}{2} u_m^{s_2}(p_2)\right]\left[\frac{g_{\alpha\beta} - \frac{q_\alpha q_\beta}{M_W^2}}{q^2 - M_W^2}\right]\left[\bar{u}_{e^-}^{s_3}(p_3)\gamma^\alpha \frac{(1-\gamma_5)}{2} u_{\nu_e}^{s}(p_1)\right]$$

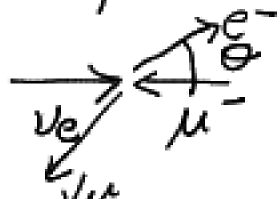
for $q^2 \ll M_W^2$

$$\simeq -\frac{G_F}{\sqrt{2}}\left[\bar{u}_\nu^{s_4}(p_4)\gamma^\beta(1-\gamma_5)^2 u_m^{s_2}(p_2)\right]\left[\bar{u}_e^{s_3}(p_3)\gamma_\beta(1-\gamma_5)\, u_{\nu_e}^{s_1}(p_1)\right]$$

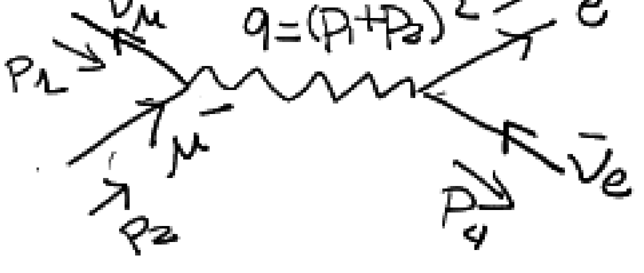
$$\overline{|\mathcal{M}|^2} = \frac{1}{2}\sum_{\substack{s_1 s_2\\ s_3 s_4}} |\mathcal{M}|^2 = 64\, G_F^2\, (p_4 p_3)(p_1 p_2) = 16\, G_F^2\, s^2$$

$$\Rightarrow \left.\frac{d\sigma}{d\Omega}\right|_{CoM} = \frac{G_F^2}{4\pi^2}\, s$$

independent of θ



For $\bar{\nu}_\mu \mu^- \to \bar{\nu}_e e^-$



Same amplitude with

$$p_1 \to -p_4$$

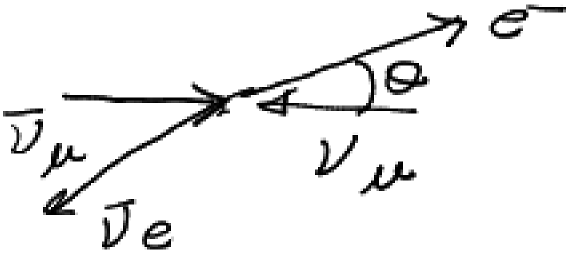
$$p_4 \to -p_1$$

$$\Rightarrow s = (p_1+p_2)^2 \to (p_2 - p_4)^2 = t$$

So in this case

$$\frac{d\sigma}{d\Omega}\Big|_{CM} = \frac{G_F^2}{4\pi^2}\frac{t^2}{s} = \frac{G_F^2}{4\pi^2}\frac{s}{4}(1-\cos\theta)^2$$

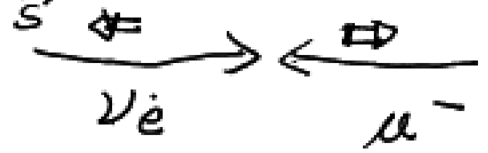
So in this case $\frac{d\sigma}{d\Omega}(\theta=0)=0$



the difference can be understood from the V-A form of the vertex

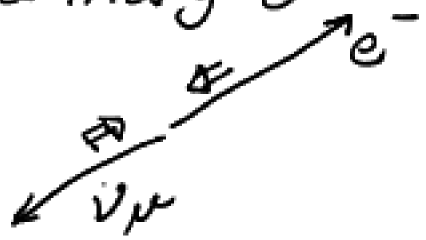
- In $\nu_e \mu^- \to e^- \nu_\mu$ neglecting masses all spinors left-handed $\Rightarrow$ all u fermions negative helicity

Initial



$\Rightarrow \vec{J}_{int}=0$

Final in any θ



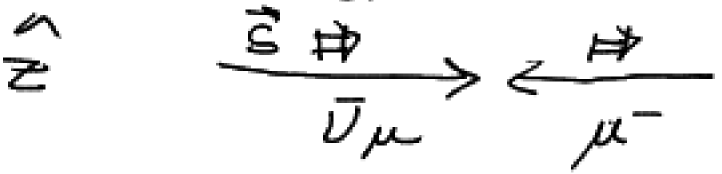
$\vec{J}_f=0$ for any θ

- In $\bar{\nu}_\mu \mu^- \rightarrow \bar{\nu}_e e^-$

For $\bar{\nu}_\mu$ and $\bar{\nu}_e$ left handed $\Rightarrow$ positive helicity

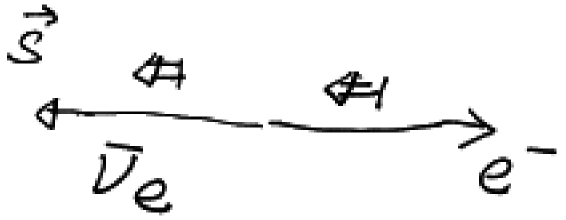
$\Rightarrow$

Initial



$\Rightarrow J_z^{int} = +1$

In final state for $\theta = 0$ (e^- forward wrt $\bar{\nu}_\mu$)



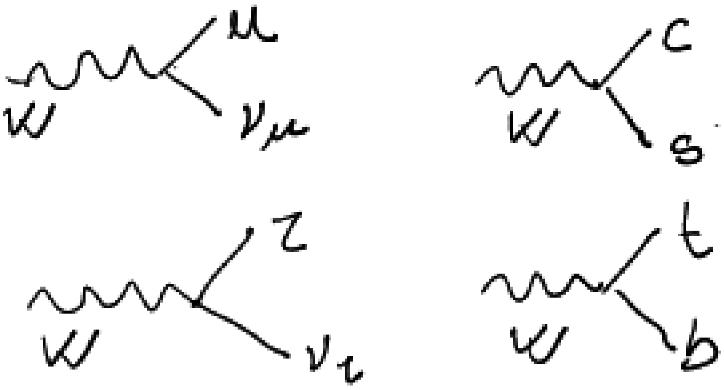
$J_z^{fin} = -1$

So for $\theta = 0$ no possible angular momentum conservation

4 Fermionic Mixing

We have written the vertices for the first generation

And we can make replicas for the other generations



Doing this we assume that the W does not couple to fermions of different generations -

However we have observed the decay $K^+ \to \mu^+ \nu_\mu$
$\quad\;\; \parallel\!\!\!\!\!\;\; (u\bar{s})$

vertex with u (≡ 1st generation)
and s (≡ 2nd ")

To understand how these generation mixing appears in the cc weak interaction we need to clarify how we assign flavour to the fermions

We have two observables to define what we call

u vs c vs $t \Rightarrow$ have same $Q = \frac{2}{3}$ and colours

d vs s vs $b \Rightarrow$ " " $Q = -1/3$ " "

Let us call $\begin{pmatrix} u' \\ d' \end{pmatrix}$ $\begin{pmatrix} c' \\ s' \end{pmatrix}$ $\begin{pmatrix} t' \\ b' \end{pmatrix}$

the quark generations which do not mix in the weak CC (so you can put a prime in all veltices in previous page)

Let us call u, c, t the quarks with $Q = 2/3$ and $m_u < m_c < m_t$

and d, s, b the quarks with $Q = -1/3$ and $m_d < m_s < m_b$

We can always chose $u = u'$, $c = c'$ and $t = t'$

but in general

unitary matrix

$$\begin{pmatrix} d \\ s \\ b \end{pmatrix} = U_{3\times3} \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}$$

Let us consider only 1st and 2nd generations

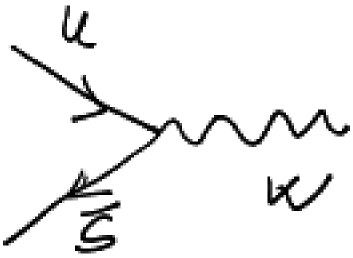
$$\begin{pmatrix} d \\ s \end{pmatrix} = U_{2\times2} \begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d' \\ s' \end{pmatrix}$$

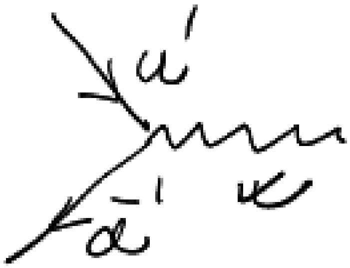
$$d = \cos\theta_c \, d' + \sin\theta_c \, s'$$
$$s = -\sin\theta_c \, d' + \cos\theta_c \, s'$$

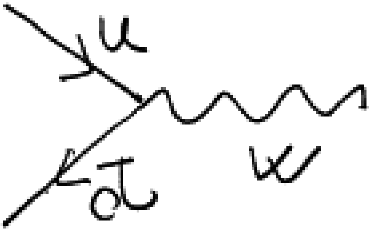
non-relativistic mass eigenstates

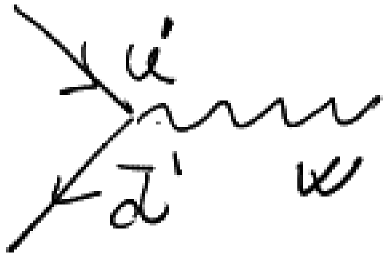
So $K^{+} \equiv (u\bar{s})$ (not $u'\bar{s}'$)

$\pi^{+} \equiv (u\bar{d})$ (not $u'\bar{d}'$)

So 

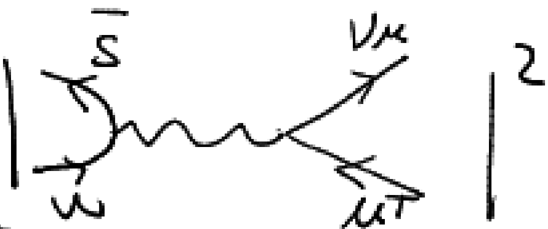
$\equiv \sin\theta_c$ 

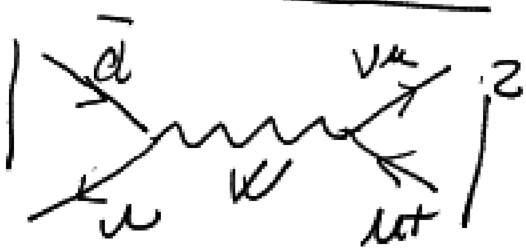


$\equiv \cos\theta_c$ 

So comparing

$$\frac{\Gamma(K^{+}\to\mu^{+}\bar{\nu}_{\mu})}{\Gamma(\pi^{+}\to\mu^{+}\bar{\nu}_{\mu})} \propto$$

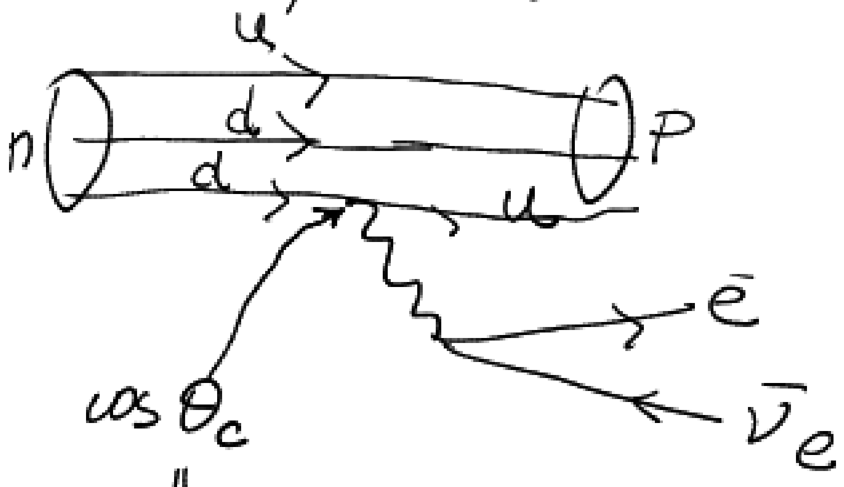




$\propto \tan^2\theta_c$

From data $\sin\theta_c \simeq 0.12$ $(\theta_c \simeq 13^\circ)$

So in β-decay



$$\Downarrow$$

$$G_{F\,\beta\text{-dec}} = \cos\theta_c \; G_{F\,\mu\text{-dec}}$$

$$\Rightarrow \frac{G_{F\,\beta\text{-dec}}}{G_{F\,\mu\text{-dec}}} = \sqrt{1-0.22^2} \simeq 0.975 \;!!\; ☺$$

For 3 generations we can write $\left[\overset{\text{notation}}{\psi_q(x) \equiv q}\right]$

$$\mathcal{L}_{cc}^{\text{quarks}} = -\frac{g_W}{\sqrt{2}}\left[\bar{u}'\gamma^\alpha P_L d' + \bar{c}'\gamma^\alpha P_L s' + \bar{t}'\gamma^\alpha P_L b'\right] W_\alpha^+ + \text{h.c}$$

$$= -\frac{g_W}{\sqrt{2}}\left[\bar{u}_L' \gamma^\alpha d_L' + \bar{c}_L' \gamma^\alpha s_L' + \bar{t}_L' \gamma^\alpha b_L'\right] W_\alpha^+ + \text{hc}$$

$$= -\frac{g_W}{\sqrt{2}} \left(\bar{u}_L', \bar{c}_L', \bar{s}_L'\right) \gamma^\alpha \begin{pmatrix} d_L' \\ s_L \\ b_L \end{pmatrix} W_\alpha^+ + \text{h.c}$$

$$\mathcal{L}^{quarks}_{mass} = -m_u \bar{u}u - m_c \bar{c}c - m_t \bar{t}t - m_d \bar{d}d - m_s \bar{s}s - m_b \bar{b}b$$

Since $P_R^2 + P_L^2 = P_R + P_L = I$

$$\bar{q}q = \underbrace{\bar{q}P_L}_{\bar{q}_R}\underbrace{P_L q}_{q_L} + \underbrace{\bar{q}P_R}_{\bar{q}_L}\underbrace{P_R q}_{q_R} = \bar{q}_R q_L + \bar{q}_L q_R = \bar{q}_R q_L + h.c$$

$$\mathcal{L}^{quark}_{mass} = -m_u \bar{u}_R u_L - m_c \bar{c}_R c_L - m_t \bar{t}_R t_L - m_d \bar{d}_R d_L - m_s \bar{s}_R s_L - m_b \bar{b}_R b_L + h.c$$

$$= -(\bar{u}_R, \bar{c}_R, \bar{t}_R) \underbrace{\begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix}}_{\equiv M_U} \begin{pmatrix} u_L \\ c_L \\ t_L \end{pmatrix}$$

$$-(\bar{d}_R, \bar{s}_R, \bar{b}_R) \underbrace{\begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix}}_{\equiv M_D} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} + h.c$$

In general

$$\begin{pmatrix} u'_R \\ c'_R \\ t'_R \end{pmatrix} = U^U_R \begin{pmatrix} u_R \\ c_R \\ t_R \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} u'_L \\ c'_L \\ t'_L \end{pmatrix} = U^U_L \begin{pmatrix} u_L \\ c_L \\ t_L \end{pmatrix}$$

and equivalently for the down quarks

So

$$\mathcal{L}^{quarks}_{cc} = -\frac{g_W}{\sqrt{2}} (\bar{u}_L, \bar{c}_L, \bar{t}_L) \, U^{U+}_L \gamma^\alpha U^D_L \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} W^+_\alpha + h.c$$

We define the CKM (Cabbibo, Kobayashi, Maskawa) matrix

$$V_{CKM} \equiv \left(U^U_L\right)^+ U^D_L$$

So

$$\mathcal{L}^{quarks}_{cc} = -\frac{g_W}{\sqrt{2}} \sum_{ij} V_{ij} \, \bar{u}_i \, \gamma^\alpha \frac{(1-\gamma_5)}{2} d_j W^+_\alpha + h.c$$

How many parameters has V_{CKM} ?

It is a unitary 3x3 complex matrix:

- A 3x3 complex matrix has 9 complex elements

  $\Rightarrow$ 9 real (modulus) and 9 phases

- unitarity $\Rightarrow \sum_{i=1}^{3} |U_{ji}|^2 = 1$ for $j=1,2,3$ (3 real conditions)

$$\sum_{i=1}^{3} U^*_{ij} U_{ij'} = 0 \quad \text{for } j \neq j'$$

(3 complex conditions

$\Rightarrow$ 9 - 6 = 3 real parameters

9 - 3 = 6 phases

Of those 6 phases 5 can be absorbed in the quark fields (only 5 because $\mathcal{L}$ is hermitian)

So when we write the vertex for the CC interaction for quarks in the mass basis we have a mixing V_{ij} between generation "i" and "j" the parametrization of V_{CKM} in the PPG $\begin{bmatrix} c_{ij} \equiv \cos\theta_{ij} \\ s_{ij} \equiv \sin\theta_{ij} \end{bmatrix}$

$$V_{CKM} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and from experiments we find

$$\theta_{12} \simeq \theta_C \sim 13^\circ$$

$$\theta_{23} \simeq 2.3^\circ$$

$$\theta_{13} \sim 0.2^\circ$$

$\Rightarrow$ mixing between 1st and 2nd generation is small and between 2nd and 3rd is smaller and between 1st and 3rd even smaller

$$V_{CKM} \sim \begin{pmatrix} O(1) & O(\lambda) & O(\lambda^3) \\ O(\lambda) & O(1) & O(\lambda^2) \\ O(\lambda^3) & O(\lambda^2) & O(1) \end{pmatrix} \text{ with } \lambda \sim 0.2$$

We say that V_{CKM} is very hierarchical.

Also we have measured $\delta \sim 68^\circ$.

How about leptons?

In principle we can do the same with $u \to \nu_e$, $d \to e^-$

$$\begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix} = U^e_L \begin{pmatrix} e'_L \\ \mu'_L \\ \tau'_L \end{pmatrix} \qquad \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} = U^\nu_L \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

But if ν's are massless $m_{\nu_1} = m_{\nu_2} = m_{\nu_3} = 0$ the U^ν_L is arbitrary and we can chose it

$$U^\nu_L = U^e_L \Rightarrow U_{LEP} = U^{\nu\dagger}_L U^e_L = I_{3\times 3}$$

$\Rightarrow L_e, L_\mu, L_\tau$ (lepton flavours) are not mixed

But from ν oscillation experiments we know that $m_{\nu_1} \neq m_{\nu_2} \neq m_{\nu_3}$ so when written in mass basis

$$\mathcal{L}^{leptons}_{CC} = -\frac{g_W}{\sqrt{2}} \sum_{ij} U^{LEP}_{ij} \bar{\nu}_i \gamma^\alpha \frac{(1-\gamma_5)}{2} l_j W^+_\alpha + h.c.$$

and we have measured

$\theta_{12}^{LEP} \sim 30^\circ$ ← solar ν's

$\theta_{23}^{LEP} \sim 45^\circ$ ← atmospheric and long baseline accelerator ν's

$\theta_{13}^{LED} \sim 8^\circ$ ← reactor ν's

δ_{LEP} still not clearly determined

But clearly U^{LED} very different from V_{CKM}

Why??

(5) CP Violation

In the homeworks we saw that a Lagrangian

$$\mathcal{L} = C\, \bar{\Psi}_a \gamma^\mu (1-\gamma_5) \Psi_b V_\mu + h.c$$

violates $\mathcal{C}$ (charge conjugation) and $\mathcal{P}$ (Parity).
But how about $\mathcal{CP}$?

To verify experimentally if cc weak interactions conserve CP one studies the system of neutral kaons

$K^0 \equiv (d\bar{s}) \qquad \bar{K}^0 \equiv (\bar{d}s)$

which we saw that they were pseudoscalar mesons ⇒
spin $\to S=1$ and $P=-1$
on systems made of pions: $\pi^+ = (u\bar{d})$, $\pi^- = (\bar{u}d)$, $\pi^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$
which are also pseudoscalar mesons

$$\Rightarrow \quad \phi_{meson} \xrightarrow{\mathcal{P}} -\phi_{meson}$$

So under Parity

$$\phi_{K^0} \xrightarrow{\mathcal{P}} -\phi_{K^0}$$
$$\phi_{\bar{K}^0} \rightarrow -\phi_{\bar{K}^0}$$

under charge conjugation

$$\phi_{K^0} \xrightarrow{\mathcal{C}} \phi_{\bar{K}^0}$$
$$\phi_{\bar{K}^0} \xrightarrow{\mathcal{C}} \phi_{K^0}$$

So if we make the combination

$$\phi_{K_1} \equiv \frac{1}{\sqrt{2}} (\phi_{K^0} + \phi_{\bar{K}^0}) \xrightarrow{\mathcal{CP}} \frac{1}{\sqrt{2}} (-\phi_{\bar{K}^0} - \phi_{K^0}) = -\phi_{K_1}$$

$$\phi_{K_2} \equiv \frac{1}{\sqrt{2}} (\phi_{K^0} - \phi_{\bar{K}^0}) \longrightarrow \frac{1}{\sqrt{2}} (-\phi_{\bar{K}^0} + \phi_{K^0}) = \phi_{K_2}$$

So K_1 and K_2 are eigenstates of $\mathcal{CP}$

Now for the pions

$$\phi_{\pi^{\pm,0}} \xrightarrow{\mathcal{P}} -\phi_{\pi^{\pm,0}}$$

$$\phi_{\pi^+} \xrightarrow{\mathcal{C}} \phi_{\pi^-}$$

$$\phi_{\pi^-} \xrightarrow{\mathcal{C}} \phi_{\pi^+}$$

$$\phi_{\pi^0} \xrightarrow{\mathcal{C}} \phi_{\pi^0}$$

So if we take a system $\pi^+\pi^-$ in $\underset{\text{orbital ang momentum}}{\ell} = 0$

$\pi^+\pi^-\pi^0$ " " "

$$\phi_{\pi^+\pi^-} \xrightarrow{\mathcal{P}} (-)^2(-)^\ell \phi_{\pi^+\pi^-} = \phi_{\pi^+\pi^-}$$

$$\phi_{\pi^+\pi^-} \xrightarrow{\mathcal{C}} \overset{\text{by } \mathcal{B}}{\phi_{\pi^-\pi^+}} = \phi_{\pi^+\pi^-}$$

$$\Rightarrow \phi_{\pi^+\pi^-} \xrightarrow{\mathcal{CP}} \phi_{\pi^+\pi^-}$$

while $\phi_{\pi^+\pi^-\pi^0} \xrightarrow{\mathcal{P}} (-)^\ell(-)^3 \phi_{\pi^+\pi^-\pi^0} = -\phi_{\pi^+\pi^-\pi^0}$

$$\phi_{\pi^+\pi^-\pi^0} \xrightarrow{\mathcal{C}} \phi_{\pi^-\pi^+\pi^0} = \phi_{\pi^+\pi^-\pi^0}$$

$$\Rightarrow \phi_{\pi^+\pi^-\pi^0} \xrightarrow{\mathcal{CP}} -\phi_{\pi^+\pi^-\pi^0}$$

So if CP is conserved in weak decays

K_1 should only decay in $\pi^+\pi^-\pi^0$

K_2 " " " " $\pi^+\pi^-$

since two body decays are faster than 3 body decays $\Rightarrow \tau_1 \gg \tau_2$

So if we start with a beam of $|K^0\rangle \equiv \frac{1}{\sqrt{2}}(|K_1\rangle + |K_2\rangle)$ and put a detector at d with $c\tau_1 \ll d \sim c\tau_2$ at distance d all the K_2 component will have decay and only K_1 is left in the beam and one should only observe decay in 3 pions

But the experiment found that about 1‰ of times a decay into $\pi^+\pi^-$ was observed.
So the long-lived state was not a pure K_L but it had a small component of K_2

$$|K_{LONG}\rangle \simeq |K_L\rangle + \epsilon |K_2\rangle$$

with $\epsilon \simeq 2 \times 10^{-3}$ $\Rightarrow$ CP is violated in CC weak interactions

Also it was observed that

$$\Gamma(K_{LONG} \to e^+\pi^-\nu_e) \neq \Gamma(K_{LONG} \to e^-\pi^+\bar{\nu}_e)$$

by about 3‰

Notice that this CP allows us to define in "absolute" manner and e^- vs an e^+
(e^- is the ony produced 3‰ times more in K_{LONG} decay)

We say that CP allows for particle-antiparticle asymmetry this is one of the 3 conditions which Sakharov found were required to generate the matter-antimatter asymmetry we find in the Universe.
(the other two are $\not{C}$ and departure from thermal equilibrium)

Where is the $\mathcal{L}_{cc}$ is encoded the posibility of $\mathcal{CP}$?

For quarks

$$\mathcal{L}^{\mathcal{CC}} = -\frac{g_W}{\sqrt{2}} \sum_{i,j=1}^{3} \Big\{ V_{ij}^{CKM}\, \bar{u}_i \gamma^\alpha \frac{(1-\gamma_5)}{2} d_j W_\alpha^+$$

$$+ V_{ij}^{*CKM}\, \bar{d}_j \gamma^\alpha \frac{(1-\gamma_5)}{2} W_\alpha \Big\}$$

Under $\mathcal{C}$ and $\mathcal{P}$

$$\mathcal{P}\, [\bar{u}_i \gamma^\alpha (1-\gamma_5) d_j]\, \mathcal{P}^{-1}$$

$$= \bar{u}_i\, \gamma^\beta (1+\gamma_5)\, d_j \quad P^\alpha_\beta \equiv \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

$$\mathcal{C}\, [\bar{u}_i \gamma^\beta (1+\gamma_5) d_j]\, \mathcal{C}^{-1}$$

$$= -\bar{d}_j \gamma^\beta (1-\gamma_5) u_i$$

$$\mathcal{P}\, W_\alpha^+\, \mathcal{P}^{-1} = P_\alpha^\gamma W^+$$

$$\mathcal{P}\, W_\alpha\, \mathcal{P}^{-1} = P_\alpha^\gamma W^+$$

$$\mathcal{C}\, W_\alpha\, \mathcal{C}^{-1} = -W_\alpha^+$$

$$\mathcal{C}\, W_\alpha^+\, \mathcal{C}^{-1} = -W_\alpha$$

$$\Rightarrow \mathcal{CP}\, [\bar{u}_i \gamma^\alpha (1-\gamma_5) d_j W_\alpha^+] (\mathcal{CP})^{-1}$$

$$= \bar{d}_j \gamma^\beta (1-\gamma_5) u_i W_\gamma \underbrace{P^\alpha_\beta P^\gamma_\alpha}_{= \delta^\gamma_\beta}$$

$$= \bar{d}_j \gamma^\alpha (1-\gamma_5) u_i W_\alpha$$

$$\Rightarrow (CP)\mathcal{L}^{cc}(CP)^{-1}$$

$$= -\frac{g_W}{\sqrt{2}}\sum_{ij}\left\{V_{ij}^{CKM}\,\bar{d}_j\gamma^\alpha\frac{(1-\gamma_5)}{2}u_i W_\alpha + V_{ij}^{*CKM}\,\bar{u}_i\gamma^\alpha\frac{(1-\gamma_5)}{2}d_j W_\alpha^\dagger\right\}$$

$$= \mathcal{L}^{cc} \iff V_{ij}^{CKM} = V_{ij}^{*CKM}$$

We have seen that considering mixing among the 3 generations of quarks V_{CKM} can contain a complex phase. $\Rightarrow$ CP̸ is possible in the SM.

For $m_\nu = 0$ this is the only source of CP̸ in the SM and in fact we have measured $\delta_{CKM} \neq 0$

But now we know that $m_\nu \neq 0$ $\Rightarrow$ we can also have $\delta_{LEP} \neq 0$ $\Rightarrow$ CP̸ in the lepton sector

$\Rightarrow$ new mechanism to generate the matter-antimatter asymmetry

there is a huge experimental programme devoted to determine δ_{LEP}

6 Weak Neutral currents

So far we have discussed weak interactions in which there is a change of electric charge ± 1 in the interaction vertex ≡ weak cc

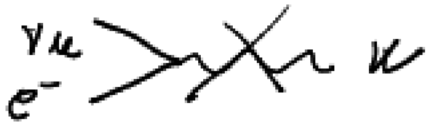
In 1973 some processes involving ν's were observed

$$\nu_\mu \, e^- \longrightarrow \nu_\mu \, e^-$$

$$\nu_\mu \underset{p,n}{\underset{|||}{N}} \longrightarrow \nu_\mu \, N$$

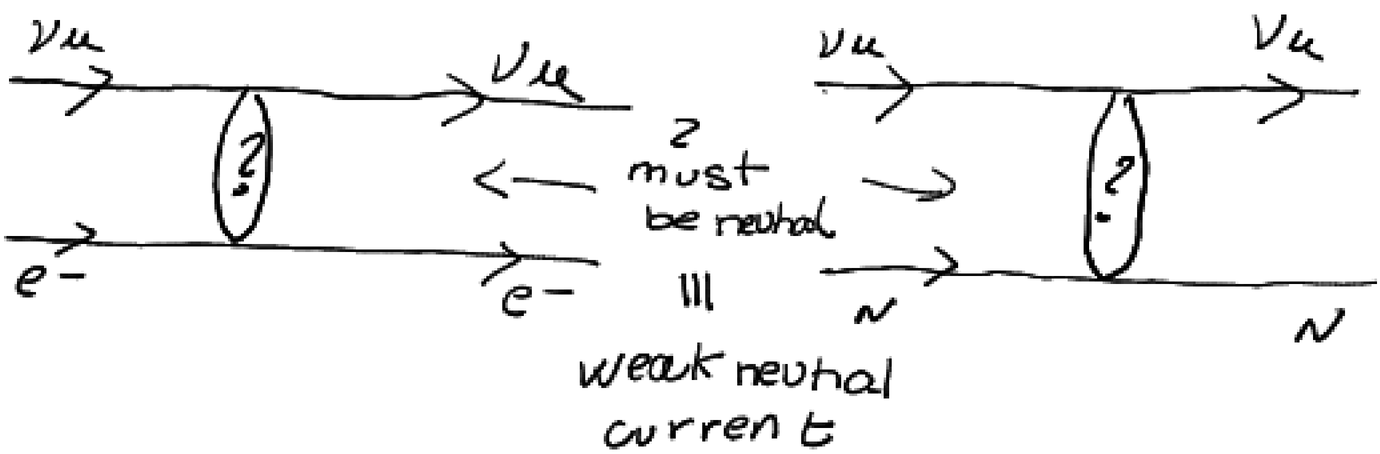
Since ν has no electric charge nor colour these had to be weak interactions.

But cc with leptons does not change flavour (for $m_\nu \Rightarrow 0$)



Furthermore in $\nu_\mu N \longrightarrow \nu_\mu N$ there is no charge lepton

So the amplitudes for these processes had to be



Experimentally

$$R_\nu \equiv \frac{\sigma^{NC}(\nu_\mu N \to \nu_\mu X)}{\sigma^{CC}(\nu_\mu N \to \mu^- X)} = 0.31$$

$$R_{\bar\nu} \equiv \frac{\sigma^{NC}(\bar\nu_\mu N \to \bar\nu_\mu X)}{\sigma^{CC}(\bar\nu_\mu N \to \mu^+ X)} = 0.38$$

these results could be explain with amplit.

same as in CC

constant $\simeq 1$

$$\mathcal{M}(\nu_\mu q \to \nu_\mu q) = \frac{-8G_F}{\sqrt{2}}\rho\left[\bar u_{\nu_\mu}\gamma^\alpha\frac{(1-\gamma_5)}{2}c_L^\nu u_\nu\right]\left\{\bar u_q\gamma_\alpha\left[c_L^q\frac{(1-\gamma_5)}{2}+\frac{c_R^q}{2}\frac{(1+\gamma_5)}{2}\right]u_q\right\}$$

$$\mathcal{M}(\nu_\mu e^- \to \nu_\mu e^-) = \frac{-8G_F}{\sqrt{2}}\rho\left[\bar u_{\nu_\mu}\gamma^\alpha\frac{(1-\gamma_5)}{2}c_L^\nu u_\nu\right]\left\{\bar u_e\gamma_\alpha\left[c_L^e\frac{(1-\gamma_5)}{2}+c_R^e\frac{(1+\gamma_5)}{2}\right]u_e\right\}$$

with

Sine

f	C_L^f	C_R^f	Q_f
ν	$1/2$	0	0
e	-0.27	0.23	-1
u	0.35	-0.15	$2/3$
d	-0.38	0.11	$1/3$

$\Rightarrow$

$$C_R^f = -Q_f x$$

$$C_L^{\nu,u} = \frac{1}{2} - Q_f x$$

$$C_L^{d,e} = -\frac{1}{2} - Q_f x$$

with $x \simeq 0.22 - 0.23$

these amplitudes can be understood in terms of an effective interaction with

- interaction is mediated by neutral vector $\equiv Z^\mu$
- Z has a mass M_Z

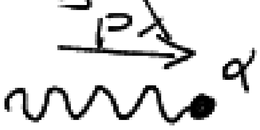
coupling constant ↘

$$\mathcal{L}^{NC} = -\sum_{f=\nu,e,u,d} g_Z \, \bar{\psi}_f \gamma^\mu \Big[C_L^f \overbrace{P_L}^{\frac{1-\gamma_5}{2}} + C_R^f \overbrace{P_R}^{\frac{1+\gamma_5}{2}} \Big] \psi_f \, Z_\mu(x)$$

So the FR for this interaction

$$-i g_Z \gamma^\alpha \left[C_L^f \frac{(1-\gamma_5)}{2} + C_R^f \frac{(1+\gamma_5)}{2} \right]$$

- Z^μ external lines

Z incoming 

$\epsilon^\alpha_\lambda(p)$

Z outgoing 

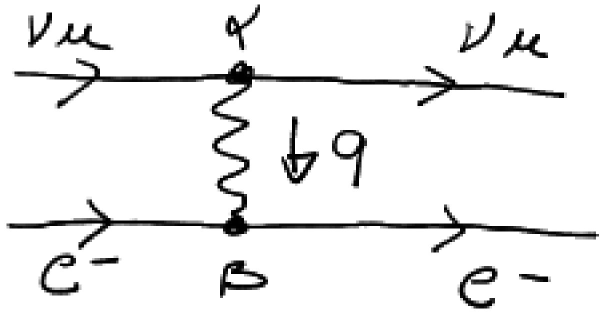
$[\epsilon^\alpha_\lambda(p)]^*$

- Z propagator

$$-i \, \frac{g_{\alpha\beta} - \frac{p_\alpha p_\beta}{M_Z^2}}{p^2 - M_Z^2 + i\epsilon}$$

Since $M_Z \neq 0$ λ can take 3 values (3 polarizations)

For example



$$\mathcal{M} = g_Z^2 \left[\bar{u}_{\nu_\mu} C_L^{\nu} \gamma^\alpha \frac{(1-\gamma_5)}{2} u_{\nu_\mu}\right] \frac{g_{\alpha\beta} - \frac{q_\alpha q_\beta}{M_Z^2}}{q^2 - M_Z^2} \left\{\bar{u}_e \gamma^\beta \left[C_L^e \frac{(1-\gamma_5)}{2} + C_R^e \frac{(1+\gamma_5)}{2}\right] u_e\right\}$$

$$\xrightarrow{q^2 \ll M_Z^2} = -\frac{g_Z^2}{M_Z^2} \left[\bar{u}_{\nu_\mu} \ldots u_{\nu_\mu}\right]\left[\bar{u}_e \ldots u_e\right]$$

$$\Rightarrow \frac{8 G_F}{\sqrt{2}} \rho = \frac{g_Z^2}{M_Z^2} \Rightarrow \frac{g_W^2}{M_W^2} \overset{\simeq 1}{\rho} = \frac{g_Z^2}{M_Z^2}$$

$$\frac{g_W^2}{M_W^2} \equiv \frac{8 G_F}{\sqrt{2}}$$

So the masses and couplings of the CC and NC weak bosons come in the same ratio (?)

We have written $\mathcal{L}^{NC}$ for one generation.

For 3 generations we make 3 copies.
What opens the issue of generation mixing.

Let us write $\mathcal{L}^{NC}$ in the weak (prime) basis in which there is no mixing

same for u, c, b

$$\mathcal{L}^{NC}_{quarks} = -g_Z \sum_{i=1}^{3} \Big\{ \bar{u}'_i \gamma^\alpha \left[C_L^u P_L + C_R^u P_R \right] u'_i \, Z_\alpha$$
$$\bar{d}'_i \gamma^\alpha (C_L^d P_L + C_R^d P_R) d'_R \, Z_\alpha$$

same for d, s, b

$$= -g_Z \sum_i \left[\bar{u}'_{i_L} \gamma^\alpha C_L^u u'_{i_L} + \bar{u}'_{i_R} \gamma^\alpha C_R^u u'_{i_R} \right] Z_\alpha$$
$$+ \left[\bar{d}'_{i_L} \gamma^\alpha C_L^d d'_L + \bar{d}'_R \gamma^\alpha C \, d'_R \right] Z_\alpha$$

If we rotate to the mass (unprime) basis

$$u'_{\substack{L\\R}i} = \sum_j [U^u_{\substack{L\\R}}]_{ij} u_j \qquad \bar{u}_{\substack{L\\R}i} = \sum_k \bar{u}_{\substack{L\\R}k} (U^{u+}_{\substack{L\\R}})_{ki}$$

(and equivalently for d's)

$$\Rightarrow \sum_i \bar{u}'_{L_i} \gamma^\alpha C_L^u \, u'_{L_i} = \sum_i \sum_{jk} \bar{u}_{L_k} \gamma^\alpha C_L (U_L^+)_{ki} U_{ij} u_j$$

$$= (U^+ U)_{kj} = \delta_{kj}$$

$$= \sum_j \bar{u}_j \gamma^\alpha C_L^u u_j$$

$\Rightarrow$ no generation mixing

Same for the righ-handed components.

So there is no generation mixing in weak NC in mass basis $\equiv$ GIM mechanism
$\parallel$
Glashow Iliopoulos Maiani

Notice that for this to happen we need to consider full generations each with an up-type and a down-type quark.

But when NC were found only u, d, s were known. So the observation of no flavour changing NC could not be easily explained.

GIM realized that they could explain it if they predicted the existence of a 4th quark up-type.

In 1974 the J/ψ $\equiv$ meson made of $c\bar{c}$ state was discovered.