Elementary Particle Physics: Assignment # 1

Due WEDNESDAY 02/05 11:00 am in class

Neutrinos are produced on the top of the atmosphere by the collision of cosmic rays. Both ν_e and ν_μ are produced, but while ν_e are detected below the Earth surface in ammounts compatible with the expectations in the Standard Model, ν_μ show a clear deficit. We will construct a simplified new physics model which can account for this.

- We assume that only two type of neutrinos are relevant, they cannot be $\nu_e's$ because they come in the expected numbers, so let's assume a quantum system formed with ν_μ and ν_τ
- The entire phenomenon takes place during the propagation from production to detection but the effect of the atmosphere and Earth matter can be neglected.

In the Standard Model neutrinos are massless. If, on the contrary, they are massive, we can place them in their rest frame and write down the Hamiltonian in the ν_μ , ν_τ basis (we are working in natural units):

$$|\nu_\mu\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |\nu_\tau\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad H = \begin{pmatrix} m_{\mu\mu} & m_{\mu\tau} \\ m_{\mu\tau} & m_{\tau\tau} \end{pmatrix}$$

1. Show that the states of definite mass are $|\nu_{1,2}\rangle$;

   $$|\nu_1\rangle = \cos\theta\,|\nu_\mu\rangle + \sin\theta\,|\nu_\tau\rangle \qquad\qquad |\nu_2\rangle = \cos\theta\,|\nu_\tau\rangle - \sin\theta\,|\nu_\mu\rangle$$

   with

   $$\tan 2\theta = \frac{2m_{\mu\tau}}{m_{\mu\mu} - m_{\tau\tau}}$$

   and that their masses are

   $$m_{1,2} = \frac{m_{\mu\mu} + m_{\tau\tau}}{2} \pm \sqrt{m_{\mu\tau}^2 + \left(\frac{m_{\mu\mu} - m_{\tau\tau}}{2}\right)^2}$$

2. Neutrinos propagate with speed close to c. From above it follows that a ν_μ produced at $t = 0$ has state vector

   $$|\phi(t = 0)\rangle = |\nu_\mu\rangle = \cos\theta\,|\nu_1\rangle - \sin\theta\,|\nu_2\rangle$$

   Show that after a time t the amplitude to detect it as a ν_μ is (E_i is the energy of ν_i)

   $$a(\phi(t) \to \nu_\mu) = \exp(-iE_1 t)\cos^2\theta + \exp(-iE_2 t)\sin^2\theta$$

3. Show that the probability of detecting a ν_μ at time t is

   $$p(\nu_\mu \to \nu_\mu) \equiv p(\phi(t) \to \nu_\mu) = 1 - \sin^2(2\theta)\sin^2\left(\frac{\Delta E\, t}{2}\right)$$

   with $\Delta E = E_2 - E_1$ (This transformation is called neutrino oscillation).

4. In the expressions above $E_i = \sqrt{|\vec{p}|^2 + m_i^2}$. Show that if $|\vec{p}| \gg m_{1,2}$ (in this case neutrinos travel very close to the speed of light so one can set $t = L$)

   $$\frac{\Delta E\, t}{2} = \frac{(m_2^2 - m_1^2)L}{4|\vec{p}|} = 1.27\frac{(m_2^2 - m_1^2)}{\mathrm{eV}^2}\frac{L}{\mathrm{km}}\frac{\mathrm{GeV}}{|\vec{p}|}$$

5. Assume that $\theta = 45°$ and $\Delta m^2 \equiv m_2^2 - m_1^2 = 2.5 \times 10^{-3}$ eV^2. Plot the probability as a function of the distance traveled by neutrinos with $|\vec{p}| = 1$ GeV, from those coming vertically down ($L \simeq 10$ km) to those coming vertically up ($L \simeq 10^4$ km) (plot the distance in log scale)

6. Which fraction of the predicted flux of ν_μ do you expect to find coming from below?.