1 a) $n \to p\, e$

angular momentum conservation

$$J_{int} = S_n = \frac{1}{2} = J_{final}$$

J_{final} has to be found composing $\underbrace{\vec{S}_p + \vec{S}_e}_{\vec{S}_{pe}} + \vec{L}_{pe}$

$S_e = S_p = 1/2$

$\Rightarrow |S_p - S_e| \leq S_{pe} \leq S_p + S_e$

$0 \leq S_{pe} \leq 1$

$\Rightarrow |S_{pe} - l_{pe}| \leq J_{fin} \leq S_{pe} + l_{pe}$ but both S_{pe} and l_{pe} are integers $\Rightarrow$ J_{fin} is integer

$\Rightarrow J_{fin} \neq 1/2$ $\Rightarrow$ no conservation of angular momentum

b) $n \to p\, e\, \nu$

now J_{final} must be obtained composing

$$\vec{J}_{fin} = \underbrace{\vec{S}_p + \vec{S}_e + \vec{S}_\nu}_{\vec{S}_{pe\nu}} + \vec{L}_{pe\nu}$$

$|S_{pe} - S_\nu| \leq S_{pe\nu} \leq S_{pe} + S_\nu$

$||S_{pe} - S_\nu| - l_{pe\nu}| \leq J_{fin} \leq S_{pe} + S_\nu + l_{pe\nu}$

take $S_{ep} = \cancel{1}$ and $l=0$ $\Rightarrow S_\nu = \text{int} = \frac{1}{2}$

take $S_{ep} = 1$ and $l=0$ $\Rightarrow S_\nu = 1/2, 3/2$

take $S_{ep} = 0$ and $l = 1$ $\Rightarrow S_\nu = 1/2$

take $S_{ep} = 1$ and $l = 1$ $\Rightarrow S_\nu = \frac{1}{2}, 3/2$

take $S_{ep} = 0$ and $l = 2$ $\Rightarrow S_\nu = 3/2$

in general if $S_{ep} = 0$ $\quad |l - S_\nu| \leq 1/2 \Rightarrow S_\nu \geq l - 1/2$

so all you say is that S_ν is half integer

(2) $\vec{S}_{BOUND} = \underbrace{\vec{S}_1 + \vec{S}_2}_{\vec{S}_{12}} + \vec{L}_{12}$

Since they are in BW-wave $l_{12} = 0 \Rightarrow S_{bound} = S_{12}$

with $0 = |S_1 - S_2| \leq S_{12} \leq S_1 + S_2 = 4$

$S_{boun} = 0, 1, 2, 3$

$m \equiv \langle S_z \rangle$

we know $m_{bound} = m_1 + m_2$

$\Rightarrow m_{bound} = 0$

looking at clebs Gordon lab

$$\left|C^{422}_{000}\right| = \sqrt{\frac{18}{35}}$$

$$\left|C^{222}_{000}\right| = \sqrt{\frac{2}{7}} \qquad \Rightarrow \left|C^{422}_{000}\right|^2 + \left|C^{222}_{000}\right|^2 + \left|C^{022}_{000}\right|^2$$

$$\left|C^{022}_{000}\right| = \sqrt{\frac{1}{5}} \qquad = \frac{18}{35} + \frac{2}{5} + \frac{1}{5} = 1$$

3 $\eta' \to \rho^0 \gamma$

$S_\rho = S_\gamma = 1$

we know $P_\rho = P_\gamma = -1$

$C_\rho = C_\gamma = -1$

$l_{\rho\gamma} = 1$ (p-wave)

em decay $\Rightarrow$ P and C are conserved

$$P_{\eta'} = P_{\rho^0\gamma} = P_\rho \times P_\gamma \times (-)^{l_{\rho\gamma}} = -1$$

$$C_{\eta'} = C_{\rho\gamma} = C_\rho \times C_\gamma = 1$$

angular momentum conservation

$$S_{\eta'} = J_{final} \text{ with } \vec{J}_{final} = \underbrace{\vec{S}_\rho + \vec{S}_\gamma}_{\vec{S}_{\rho\gamma}} + \vec{L}_{\rho\gamma}$$

$$0 = |S_\rho - S_\gamma| \leq S_{\rho\gamma} \leq S_\rho + S_\gamma = 2$$

$$0 \leq |S_{\rho\gamma} - 1| \leq J_{fin} \leq S_{\rho\gamma} + l_{\rho\gamma} \leq 3$$

so $J_{fin} \overset{\text{can be}}{=} 0, 1, 2, 3$

But η' is a bound state of $q\bar{q}$ and its C and P as well as its spin angular moment is defined by spin and orbital momentum of its components.

We derived in class that

$$\vec{S}_\eta = \overbrace{\vec{S}_q + \vec{S}_{\bar{q}}}^{S_{q\bar{q}}} + \vec{L}_{q\bar{q}}$$

$$P_\eta = (-1)^{l_{q\bar{q}}+1}$$

$$C_\eta = (-1)^{s_{q\bar{q}}+l_{q\bar{q}}}$$

and

$$0 = \left|\frac{1}{2} - \frac{1}{2}\right| \leq S_{q\bar{q}} \leq S_q + S_{\bar{q}} = \frac{1}{2} + \frac{1}{2} = 1 \Rightarrow S_{q\bar{q}} = 0, 1$$

So $P_\eta = -1 \Rightarrow l_{q\bar{q}}$ even

$C_\eta = 1 \Rightarrow S_{q\bar{q}} = \text{even} = 0$

now with $S_{q\bar{q}} = 0$ and $l_{q\bar{q}} = \text{even}$ we can make

$$|l_{q\bar{q}} - S_{q\bar{q}}| \leq S_\eta \leq S_{q\bar{q}} + l_{q\bar{q}}$$

since $S_{q\bar{q}} = 0 \Rightarrow S_\eta = l_{q\bar{q}} \equiv \text{even}$

So $\boxed{S_\eta = 0, 2}$ c.