Particle Physics: Assignment # 5

Due Wed 03/05 before class

(1) Using the transformation properties of the fermion spinors given in class which I repeat here ($P = \gamma^0$ and $C = i\gamma^2\gamma^0$),

$$\begin{array}{ll} \psi_P(x) = P\,\psi(x_P) & \overline{\psi}_P(x) = \overline{\psi}(x_P)\,P \\ \psi^C(x) = C\,\bar{\psi}^T(x) & \overline{\psi}^C(x) = -\psi^T(x)C^{-1} \end{array}$$

a) Derive the transformation properties under Parity and Charge Conjugation of the following four bilinears (a and b are two type of fermions)

$$\begin{array}{ll} 1)\ \bar{\psi}_a(x)\psi_b(x) & 2)\ \bar{\psi}_a(x)\gamma_5\psi_b(x) \\ 3)\ \bar{\psi}_a(x)\gamma^\nu\psi_b(x) & 4)\ \bar{\psi}_a(x)\gamma^\nu\gamma^5\psi_b(x) \end{array}$$

Hint: Do not forget that there is a $(-)$ sign which you have to include when exchanging fermions.

b)With the resuts above check whether the following Lagrangiangs are invariant under Parity and Charge Conjugation

$$\begin{aligned} \mathcal{L}_A &= -B\;\bar{\psi}_a(x)\gamma^\mu\psi_a(x)A_\mu(x) \\ \mathcal{L}_Z &= -B\;\bar{\psi}_a(x)\gamma^\mu(1-\gamma^5)\psi_a(x)Z_\mu(x) \\ \mathcal{L}_{W1} &= -D\;\bar{\psi}_a(x)\gamma^\mu\psi_b(x)W_\mu(x) + h.c. \\ \mathcal{L}_{W2} &= -D\;\bar{\psi}_a(x)\gamma^\mu(1-\gamma^5)\psi_b(x)W_\mu(c) + h.c. \end{aligned}$$

A^μ and Z^μ are real vector field, defined as odd under charge conjugation. while W^μ is a complex vector field which under charge conjugation changes to $-W^{\dagger\mu}$. B is a real constant (as required by reality of the Lagrangian). D is also a constant but can be complex.

What does this tell you about the Charge Conjugation and Parity properties of electromagnetic interactions?

(2) Draw the Feynman diagram for

$$e^+e^- \to \mu^+\mu^-$$

and using the Feynman rules given in class obtain the expression of the Feynman amplitude.

(3) Using the chiral representation of the 4-spinors given in homework 4, (neglect fermion masses) compute the Feynman amplitudes generated by QED for the process

$$e^+e^- \to \mu^+\mu^-$$

for the 16 helicity combinations in the COM. You should find that only some of the 16 helicity amplitudes are non-zero. Reason why (think of conservation of angular momentum). Compare with the results of question (2) of HW4.

Hint: define $\hat{z}$ axis as the collisions axis and define the $y = 0$ plane as the plane where the 4 particles are. Call θ the scattering angle of the μ. Remember you have to use the γ matrices in the chiral representation.