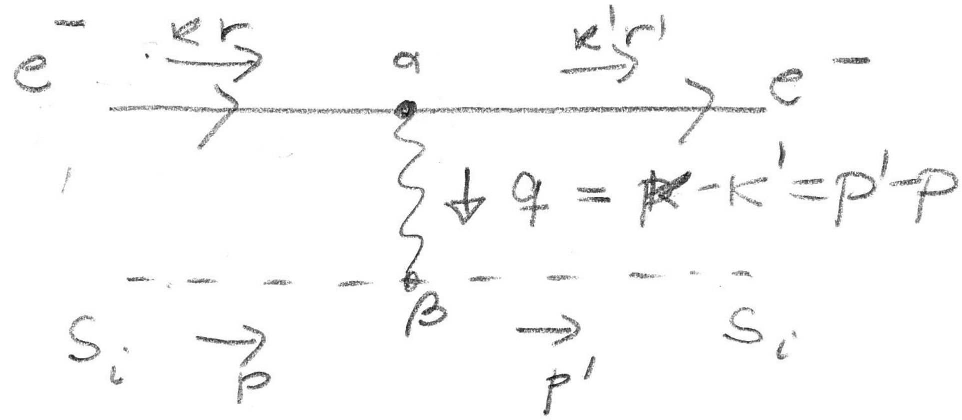
AHW7

1) For scalar partons the amplitude

$$e^-(k,r)\, S_i(p) \to e^-(k' r')\, S_i(p')$$



$$\mathcal{M} = \frac{e^2 e_i}{q^2}\, \bar{u}^{r'}(k')(\not{p}+\not{p}')u^{r}(k)$$

$$\overline{|\mathcal{M}|}^2 = \frac{1}{2} \sum_{rr'} \mathcal{M}\mathcal{M}^*$$

2 initial e^- helicity

$$= \frac{1}{2}\frac{e^4 e_i^2}{q^4}\, \mathrm{Tr}\left[\not{k}'(\not{p}+\not{p}')\not{k}(\not{p}+\not{p}')\right]$$

$$= \frac{1}{2}\frac{e^4 e_i^2}{q^4}\Big[\mathrm{Tr}(\not{k}'\not{p}\not{k}\not{p}) + \mathrm{Tr}(\not{k}'\not{p}\not{k}\not{p}') + \mathrm{Tr}(\not{k}'\not{p}'\not{k}\not{p}) + \mathrm{Tr}(\not{k}'\not{p}'\not{k}\not{p}')\Big]$$

$$= \frac{2e^4 e_i^2}{q^4}\Big[2(k'p)(kp) - 2(kk')m_s^2 + 2(k'p)(kp') + 2(k'p')(kp) - 2(kk')(pp') + 2(k'p')(kp')\Big]$$

using that

$$kp' = m_s E + \frac{q^2}{2}$$

$$k'p' = m_s E' - \frac{q^2}{2}$$

$$kk' = -\frac{1}{2}(k-k')^2 = -\frac{1}{2}q^2$$

$$pp' = \frac{-1}{2}(q^2 - 2m_s^2) = m_s^2 - \frac{1}{2}q^2$$

$$kp = m_s E$$

$$k'p = m_s E'$$

and $q^2 = -4EE' \sin^2\theta/2$ (in LAB)

$$|M|^2 = \frac{4e^4 e_i^2}{q^4} \Big[m_s E\left(m_s E' - \frac{q^2}{2}\right) - \left(-\frac{1}{2}q^2\right) m_s^2$$

$$+ m_s E'\left(m_s E + \frac{q^2}{2}\right) - \left(-\frac{1}{2}q^2 + m_s^2\right)\left(-\frac{1}{2}q^2\right)$$

$$+ \left(m_s E' - \frac{q^2}{2}\right)\left(m_s E + \frac{q^2}{2}\right) + m_s^2 EE' \Big]$$

$$= \frac{4e^4 e_i^2}{q^4} \Big\{ 2m_s^2 EE' - 2\frac{m_s E}{2} q^2 + \frac{1}{2} m_s^2 q^2 + 2m_s E' \frac{q^2}{2}$$

$$+ \frac{1}{4}q^4 + \frac{1}{2}q^2 m_s^2 - \frac{q^4}{4} \Big\}$$

$$= \frac{4e^4 e_i^2}{q^4} \Big\{ 4m_s^2 EE' - m_s(E-E')q^2 - \frac{q^4}{2} - m_s^2 q^2 \Big\}$$

③

the phase space integral is the same as for $e^-\mu \to e^-\mu$ and we will get

a $\delta(\nu + \frac{q^2}{2m_s})$ with $\nu = E - E'$

$$\Rightarrow m_s(E-E')\, q^2 = m_s \nu\, q^2 = -\frac{q^4}{2}$$

so with this constraint

$$\overline{|M|}^2 = \frac{4e^4 e_i^2}{q^4}\left\{4m_s^2 EE' - m_s^2 q^2\right\}$$

LAB

$$= \frac{4e^4 e_i^2}{q^4}\left\{4EE' m_s^2\left(1 - \sin^2\frac{\theta}{2}\right)\right\}$$

$$= \frac{4e^4 e_i^2}{q^4}\, 4EE' m_s^2 \cos^2\frac{\theta}{2}$$

$q^4 \to 16E^2E'^2\sin^4\frac{\theta}{2}$

So since in LAB

$$d\sigma = \frac{1}{4m_s E}\, \frac{\overline{|M|}^2}{(2\pi)^2}\, \underbrace{\frac{d^3k'}{2E'}\, \frac{d^3p'}{2p'_0}\, \delta^4(p+k-p'-k')}_{= \delta(\nu + \frac{q^2}{2m_s})\, \frac{E'\, dE'\, d\Omega}{4m_s}}$$

=

$$\left.\frac{d\sigma(e^- s_i \to e^- s_i)}{dE' d\Omega}\right|_{LAB} = \frac{\overset{4(2\pi\alpha)^2}{\overset{\shortparallel}{16 e^4}} e_i^2 \, m_s^2 \, E'^2 \cancel{E} \cos^2\theta/2}{q^4 \, 64\pi^2 \, m_s^2 \, \cancel{E}} \, \delta\left(\nu + \frac{q^2}{2M}\right)$$

$$= \frac{(2\alpha E')^2}{q^4} e_i^2 \cos^2\frac{\theta}{2} \quad \delta\left(\nu + \frac{q^2}{2M}\right)$$

So for scalar partons the fundamental inelastic predicts $Q^2 = -q^2)$

$$W_2^{es\to es} = \delta\left(\nu - \frac{Q^2}{2m_s}\right) = \frac{1}{\nu}\,\delta\left(1 - \frac{Q^2}{2m_s\nu}\right)$$

$$W_1^{es\to es} = 0$$ ← different from inelastic with spin $\frac{1}{2}$ parton

$$\nu W_2^{e^-s\to e^-s} = \delta\left(1 - \frac{Q^2}{2m_s\nu}\right)$$ ← same as for inelastic with spin 1/2 parton

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So in an the parton model for the proton with scalar partons we would predict

$$W_1^{ep\to eX}(Q^2,\nu) \xrightarrow{Q^2\to \text{large}} F_1^{ep}(x) = 0$$

$$W_2^{ep\to eX)}(Q^2\nu) \xrightarrow[i]{Q^2\to \text{large}} F_2^{ep}\left(x = \frac{1}{\omega} = \frac{2M_p\nu}{Q^2}\right) = \sum_i e_i^2 x f_i^p(x)$$

So the Callan-Gross relation for scalar partons woul be

$$\frac{F_1^{DIS}}{F_2^{DIS}} = 0$$

which clearly contradicts the data