(1) In Hadron - Hadron collisions

$$\sigma(A+B\to C)(S) = \sum_{ab} C_{ab} \int \left(f_a^A(x_a) f_b^B(x_b) + f_b^A(x_a) f_b^B(x_b)\right) dx_a dx_b$$
$$\times \sigma(ab\to c)(\hat{S} = x_a x_b . S)$$

For the case at hand $C = F\bar{F} \Rightarrow \hat{S} > 4M_F^2$

it is best to make a change of variables for x_a, x_b to x_a, $\tau = x_a x_b$ so $\tau > \tau_{min} = \frac{4M_F^2}{S}$
$= \hat{S}/S$

with this change $dx_a dx_b = \frac{1}{x_a} dx_a d\tau$

also $\sum_{ab} C_{ab}$ includes the sum over colours but but in QED colours in $q_a \bar{q}_b$ must be the same

so $\sum_{ab} C_{ab} = \frac{1}{9} \sum_{ij} \delta_{ij} \sum_{u,d} = \frac{1}{3} \sum_{u,d}$

So

$$\sigma(AB\to F\bar{F}x) = \frac{1}{3}\sum_{q=u,d} \int_{\tau_{min}}^{1} d\tau \int_{\tau}^{1} dx_a \frac{4\pi\alpha^2}{3S}\frac{1}{\tau}\sqrt{1-\frac{\tau_{min}}{\tau}}\, e_q^2$$
$$\times \left[f_q^A(x_a) f_{\bar{q}}^B\left(\frac{\tau}{x_a}\right) + f_{\bar{q}}^A(x_a) f_q^B\left(\frac{\tau}{x_a}\right)\right]$$

First of all

$$\sigma_0 = \frac{4\pi\alpha^2}{3S} = \frac{4\pi\left(\frac{1}{137}\right)^2}{3\times(7\,\mathrm{TeV})^2}\left(197\,\mathrm{MeV\,fm}\right)^2$$

$$= 0.18\times10^{-12}\,\underset{\overset{\shortparallel}{10^{-13}\,\mathrm{cm}}}{\mathrm{fm}}{}^2 = 1.8\times10^{-6}\,\underset{\overset{\shortparallel}{10^{-33}\,\mathrm{cm}^2}}{\mathrm{nb}}$$

- For $p\bar{p}\to F\bar{F}X$ using $f_q^p = f_{\bar{q}}^{\bar{p}}$

$$\sigma(p\bar{p}\to F\bar{F}) = \frac{\sigma_0}{3}\int_{\tau_{min}}^{1} d\tau\,\frac{1}{\tau}\sqrt{1-\frac{\tau_{min}}{\tau}}\int_{\tau}^{1}\frac{dx_a}{x_a}$$

$$\Bigg\{\Bigg[\frac{4}{9}\overset{q}{\left(u_v(x_a)+u_s(x_a)\right)}\overset{\bar{q}}{\left(u_v\left(\frac{\tau}{x_a}\right)+u_s\left(\frac{\tau}{x_a}\right)\right)} + \frac{1}{9}\left(d_v(x_a)+d_s(x_a)\right)\left(d_v\left(\frac{\tau}{x_a}\right)+d_s\left(\frac{\tau}{x_a}\right)\right)\Bigg]$$

$$+\left[\frac{4}{9}\,\overset{\bar{q}}{u_s(x_a)}\,\overset{q}{u_s\left(\frac{\tau}{x_a}\right)} + \frac{1}{9}\,d_s(x_a)\,d_s\left(\frac{\tau}{x_a}\right)\right]\Bigg\}$$

$$= \frac{\sigma_0}{3}\int_{\tau_{min}}^{1} d\tau\,\sqrt{1-\frac{\tau_{min}}{\tau}}\,\frac{1}{\tau}\int_{\tau}^{1}\frac{dx_a}{x_a}$$

$$\Bigg\{\frac{1}{9}\left(4+\frac{1}{4}\right)u_v(x_a)\,u_v\left(\frac{\tau}{x_a}\right) + \frac{1}{9}\left(4+\frac{1}{2}\right)\left(u_v(x_a)\,u_s\left(\frac{\tau}{x_a}\right) + u_v\left(\frac{\tau}{x_a}\right)u_s(x_a)\right) +$$

$$+\frac{2}{9}(4+1)\, u_s(x_a)\, u_s\left(\frac{\tau}{x_a}\right)\Big\}$$

using

$$\Rightarrow \int_\tau^1 \frac{dx_a}{x_a} u_v(x_a)\, u_v\left(\frac{\tau}{x_a}\right) = 36\int_\tau^1 \frac{dx_a}{x_a} (1-x_a)^2\left(1-\frac{\tau}{x_a}\right)^2$$

$$= 36\int_\tau^1 \frac{dx_a}{x_a} \frac{1}{x_a^2} (1-x_a)^2 (x_a-\tau)^2 = 36\, I_1(\tau)$$

$$\Rightarrow \int_\tau^1 \frac{dx_a}{x_a}\left[u_v(x_a)\, u_s\left(\frac{\tau}{x_a}\right) + u_v\left(\frac{\tau}{x_a}\right) u_s(x_a)\right]$$

$$= \frac{6}{4}\int_\tau^1 \frac{dx_a}{x_a}\left[(1-x_a)^2\left(1-\frac{\tau}{x_a}\right)^3 \frac{x_a}{\tau} + \left(1-\frac{\tau}{x_a}\right)^2 \frac{(1-x_a)^3}{x_a}\right.$$

$$= \frac{3}{2}\left[\int_\tau^1 \frac{(1-x_a)^2 (x_a-\tau)^3}{\tau\, x^3} + \int_\tau \frac{(1-x_a)^3 (x_a-\tau)^2}{x^4}\right.$$

$$= 3\, I_2(\tau)$$

$$\Rightarrow \frac{1}{16}\int_\tau^1 \frac{dx_a}{x_a} \frac{(1-x_a)^3}{x_a} \frac{\left(1-\frac{\tau}{x_a}\right)^3}{\tau} x_a = \frac{1}{16}\frac{1}{\tau}\int_\tau^1 \frac{dx_a (1-x_a)^3 (x_a-\tau)^3}{x_a^4}$$

$$\frac{1}{16}\frac{1}{\tau} I_3(\tau)$$

Defining $J_i = \int_{\tau_{min}}^{1} \frac{d\tau}{\tau} \sqrt{1-\frac{\tau_{min}}{\tau}} \; \frac{I_i}{\tau}$ for $i=3$

$$\sigma(p\bar{p} \to F\bar{F}) = \frac{\sigma_0}{3}\left(\frac{1}{9}\,\frac{17}{4} \times 36\; J_1 + \frac{3}{2} J_2 + \frac{5}{9} \times \frac{2}{16} J_3\right)$$

$$= \frac{\sigma_0}{3}\left(17\, J_1 + \frac{3}{2}\, J_2 + \frac{5}{72}\, J_3\right)$$

$$= \frac{\sigma_0}{4}\left(2\,J_2 + \frac{5}{52} J_3 + \frac{68}{3}\, J_1\right)$$

For $pp \to F\bar{F}$

$$\sigma(pp \to F\bar{F}) = \frac{\sigma_0}{3} \int_{\tau_{min}}^{1} \frac{d\tau}{\tau} \sqrt{1-\frac{\tau_{min}}{\tau}} \int_{\tau}^{1} \frac{dx_a}{x_a}$$

$$\Big\{ \frac{4}{9}\Big[\big(u_v(x_a)+u_s(x_a)\big)\, u_s\!\left(\frac{\tau}{x_a}\right) + u_s(x_a)\left(u_v\!\left(\frac{\tau}{x_a}\right) + u_s\!\left(\frac{\tau}{x_a}\right)\right)\Big]$$

$$+ \frac{1}{9}\Big[\big(d_v(x_a) + d_s(x_a)\big)\, d_s\!\left(\frac{\tau}{x_a}\right) + d_s(x_a)\left(d_v\!\left(\frac{\tau}{x_a}\right) + d_s\!\left(\frac{\tau}{x_a}\right)\right)\Big]$$

$$= \frac{\sigma_0}{3}\left[\frac{1}{9}\left(4+\frac{1}{2}\right) 3\, J_2 + \frac{1}{9}(4+1)\frac{1}{16} J_3\right]$$

$$= \frac{\sigma_0}{3}\left(\frac{3}{2} J_2 + \frac{5}{72} J_3\right) = \frac{\sigma_0}{4}\left(2\,J_2 + \frac{5}{52} J_3\right)$$

Numerically

$M_F =$	J_1	J_2	J_3
100	7.64	235	1985
1000	0.11	0.21	18

So

$$\sigma(p\bar{p} \to F\bar{F}X) = \begin{cases} M_F = 100 & 3.7 \times 10^{-4} \\ M_F = 1000 & 2.1 \times 10^{-6} \end{cases}$$

$$\sigma(pp \to F\bar{F}X) = \begin{cases} M_F = 100 & 2.9 \times 10^{-4} \\ M_F = 1000 & 9.39 \times 10^{-7} \end{cases}$$

So
$$\frac{\sigma(pp \to F\bar{F}X)}{\sigma(p\bar{p} \to FFX)} = \begin{cases} M_F = 100 & 0.78 \\ M_F = 1000 & 0.44 \end{cases}$$

the cs for $p\bar{p}$ is larger than for pp because of the valence quark - valence antiquark contribution this contribution is relatively larger at larger x which is more needed for larger $\tau \Rightarrow$ for larger m_F