Elementary Particle Physics: Assignment # 9
Due April 23th

(1) 1.a) Knowing that $\alpha_s(M_Z^2) = 0.1180$ ($M_Z = 91$ GeV)obtain the value of α_s at $(2\text{ GeV})^2$ (use $N_f = 3$) and at $(100\text{ GeV})^2$ (use $N_f = 5$) and give their ratio. Compare with the ratio of the electromagnetic coupling constant α at those two energies

1.b) The QCD confinement scale $\Lambda_{\rm QCD}$ is defined as the scale at which the coupling constant becomes very large. So that one can approximate

$$0 = \frac{1}{\alpha_S(\Lambda^2_{\rm QCD})}$$

Show that in terms of this scale the strong coupling constant at any other scale can be written as

$$\alpha_s(q^2) = \frac{12\pi}{(33 - 2N_f)\ln\frac{q^2}{\Lambda^2_{\rm QCD}}}$$

1.c) What value of Λ_{QCD} would give the value of $\alpha_S(M_Z^2)$ given in (1.a) for $N_f = 5$?.

1.d) How can you explain that according to the PDB the corresponding extracted value for $N_f = 5$ is $\Lambda \simeq 210$ MeV
(see eq (9.24) in https://pdg.lbl.gov/2018/reviews/rpp2018-rev-qcd.pdf)?

(2) 2.a) Draw the Feyman diagram and write the amplitude for the QCD contribution to $u\overline{u} \to d\overline{d}$.

2.b) Obtain the colour factor for this amplitude in the colour singlet configuration. Reason the answer.

(3) Using the $SU(3)$ flavour model obtain the prediction for the mass of all the particles in the baryon decouplet and compare to data. (Hint: the quark constituent masses for baryons are not exactly the same than for mesons, see for example the table in the chapter of bounds states in Griffits)