Elementary Particle Physics

Test, March 12th

NAME:

Use this approximate values for the constants:

$m_{proton} = 1\,\text{GeV}$ $\hbar c = 200\,\text{MeV fm} = 200 \times 10^{-18}\,\text{GeV m}$ $c = 3 \times 10^8\,\text{m}/s$

1 In a fix target experiment, a photon beam hits a target made of protons and produces a Δ^* resonance with mass m_{Δ^*}=3 GeV.

(1.1) Determine the energy of the incoming photon beam (10 points)

4 momentum consuahon $\Rightarrow S = (p_\gamma + P_p)^2 = P_\Delta^2 = m_\Delta^2$

in LAB frame $E_\gamma = |\vec{p}_\gamma|$ $E_p = m_p$ $\vec{P}_p = 0$

$$S = (E_\gamma + m_p)^2 - E_\gamma^2 = m_p^2 + 2E_\gamma m_p$$

$$\| \quad$$

$$m_\Delta^2 \Rightarrow E_\gamma = \frac{m_\Delta^2 - m_p^2}{2m_p} = \frac{9-1}{2} = 4\ \text{GeV}$$

(1.2) Determine the Energy and the β and γ of the produced Δ^* (10 points)

$E_\Delta = E_\gamma + m_p = 5\ \text{GeV}$

$P_\Delta = P_\gamma = 4\ \text{GeV}$

$$\Rightarrow \beta_\Delta = \frac{P_\Delta}{E_\Delta} = \frac{4}{5} \qquad \gamma_\Delta = \frac{E_\Delta}{m_\Delta} = \frac{5}{3}$$

nohce $\frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-\frac{16}{25}}} = \frac{5}{\sqrt{25-16}} = 5/3$

(1.3) The Δ^* decays as $\Delta^* \to p\gamma$ with decay width at rest of the $\Gamma_{\Delta^*} = 20$ MeV. How long will it travel before decaying in the Lab? (10 points)

$$\tau_\Delta = \frac{1}{\Gamma} = \frac{1}{20\,\text{MeV}} \times \frac{200\,\text{MeV}\,\underbrace{\text{fm}}_{=10^{-15}\,m}}{3\times10^{8}\ m/s} = \frac{1}{3}\times10^{-22}\ s$$

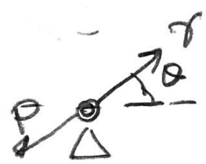
$$L_{lab} = \beta\gamma\tau_\Delta c = \frac{4}{3}\times\frac{1}{3}\times10^{-22}\times3\times10^{8} = 1.33\times10^{-14}\ m$$

$$= 13.3\ fm$$

Δ in RF

(1.4) Obtain the energy of emitted photon as a function of its emission angle and get its maximum and minimum value? (20 points)

In the Δ rest frame



$$P'^{RF}_{\gamma} = P'_{p} = \frac{M_\Delta^2 - M_p^2}{2M_\Delta} = \frac{4}{3}\,\text{GeV}$$

$$\| \quad E'_\gamma$$

in LAB take $\hat{x}$ the direction of Δ $\equiv$ direction of incoming γ

$$\text{So}\begin{pmatrix} E'_{LAB} \\ P'_{xLAB} \\ P'_{yLAB} \end{pmatrix} = \begin{pmatrix} \gamma_\Delta & \gamma_\Delta\beta_\Delta & 0 \\ \gamma_\Delta\beta_\Delta & \gamma_\Delta & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} E'^{RF}_\gamma \\ p'^{RF}_\gamma \cos\theta^R \\ p'^{RF}_\gamma \sin\theta^R \end{pmatrix} \qquad P'_{RF} = E'_{RF}$$

$$E'^{\gamma}_{LAB} = \gamma_\Delta E'^{RF}_\gamma (1 + \beta_\Delta \cos\theta^{RF}) \Rightarrow \text{maximum for } \cos\theta^{RF} = 1$$
$$\text{min for } \cos\theta^{RF} = -1$$

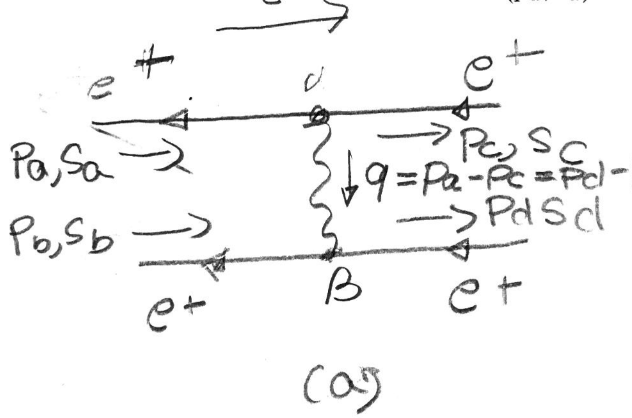
$$E'^{\gamma}_{max} = \frac{5}{3}\,\frac{4}{3}\left(1 + \frac{4}{5}\right)\text{GeV} = 4\ \text{GeV} \Rightarrow E^{\gamma}_{incoming} \Rightarrow P_n^{fin} = 0$$

$$E'^{\gamma}_{min} = \frac{5}{3}\,\frac{4}{3}\left(1 - \frac{4}{5}\right)\text{GeV} = \frac{4}{9}\ \text{GeV} \Rightarrow E_p^{fin} = 5 - \frac{4}{9} = \frac{41}{9}$$

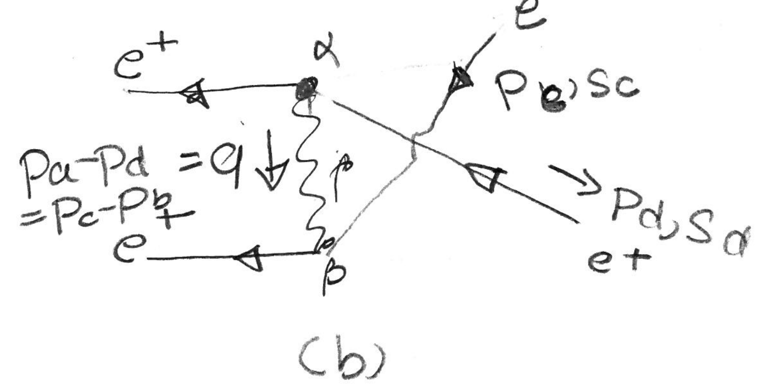
$$\Rightarrow P_p^{fin} = \sqrt{\frac{41^2}{9^2} - 1} = \frac{40}{9}$$

2 Draw the Feynman diagrams at lowest order in QED for the process with time running left to right (10 points)

$$e^+(p_a, r_a)\, e^+(p_b, r_b) \to e^+(p_c, r_c)\, e^+(p_d, r_d)$$



(a)



(b)

(2.2) Write the corresponding Feynman amplitudes (20 points)

$$\mathcal{M}_a = \left[\bar{v}^{s_a}(p_a)(ie\gamma^\alpha)v^{s_c}(p_c)\right] \frac{-ig_{\alpha\beta}}{(p_a-p_c)^2} \left[\bar{v}^{s_b}(p_b)\, ie\gamma^\beta v^{s_d}(p_d)\right]$$

$$= \frac{-ie^2}{(p_a-p_c)^2}\left[\bar{v}^{s_a}(p_a)\gamma^\alpha v^{s_c}(p_c)\right]\left[\bar{v}^{s_b}(p_b)\gamma_\alpha v^{s_d}(p_d)\right]$$

(b) is as (a) with exchange $c \leftrightarrow d$

(−) relative sign for exchange of identical fermions in final state

$$\mathcal{M}_b = (-)\frac{+ie^2}{(p_a-p_d)^2}\left[\bar{v}^{s_a}(p_a)\gamma^\alpha v^{s_d}(p_d)\right]\left[\bar{v}^{s_b}(p_b)\gamma_\alpha v^{s_c}(p_c)\right]$$

$P_\rho = -1$, $S_\rho = 1$ $\quad P_\pi = -1$, $S_\pi = 0$

3 The a_1^0 is a meson with $C = 1$. The ρ^+ is a vector meson, and the π^- is a psedudoscalar meson

(3.1) The decay $a_1^0 \to \rho^+ \pi^-$ is allowed by strong interactions for final particles in either S-wave or D-wave (this with $\ell = 0, 2$). What is the spin and parity of the a_1^0? (10 points)

- Angular momentum conservation
  $j_{init} = S_a$
  since $S_\pi = 0$: $|\ell_{\pi\rho} - j_\rho| \leq j_{fin} \leq j_\rho + \ell_{\pi\rho}$ and since $\ell_{\pi\rho} = 0$ is observed $\Rightarrow j_{fin} = j_\rho = 1 \Rightarrow S_a = 1$
- Parity conservation : $P_a = P_\rho \times P_\pi \times (-)^{\ell_{\pi\rho}} = (-1)(-1)(1) = 1$

(3.2) Explain the s, C and P of the a_1^0 in terms of its quark constituents (10 points)

we know that $|S_{q\bar{q}} - \ell_{q\bar{q}}| \leq \underset{=1}{S_a} \leq S_{q\bar{q}} + \ell_{q\bar{q}}$

with $S_{q\bar{q}} = 0, 1$ So $\ell_{q\bar{q}} = 0, 1, 2$ possible

$P_a = (-)^{\ell_{q\bar{q}} + 1} = 1 \Rightarrow \ell_{q\bar{q}} = 1$

$C = (-)^{S_{q\bar{q}} + \ell_{q\bar{q}}} \Rightarrow S_{q\bar{q}} = 1$

So a_1^0 is an pseudo vector meson in a $S_{q\bar{q}} = \ell_{q\bar{q}} = 1$ configuration