Chapter 1

"Basics"

1) Introduction
2) Basic concepts of quantum physics for particle
3) Particle contents of the standard Model
4) Natural units

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1) Introduction

Particle physics addresses two fundamental questions

a) what is matter made of?

b) how does it behave? ≡ how particles interact

a) We know:

- matter is made of tiny "chunks" ≡ particles
- particles come in a small number of different types
- all particles of same type are "identical"

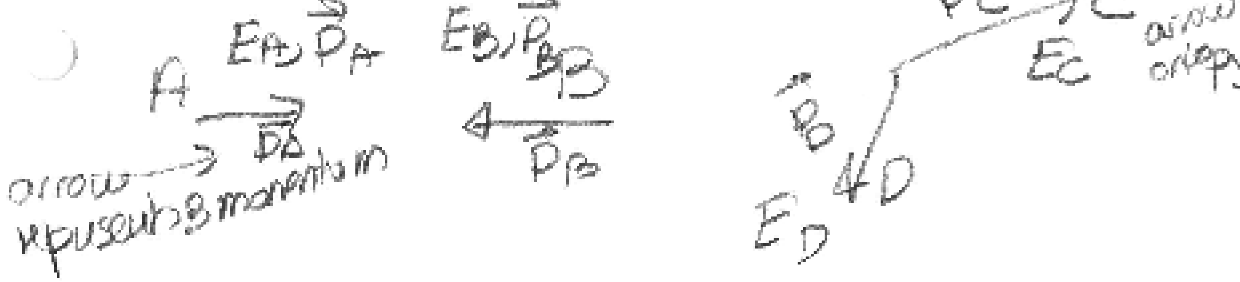
(≡ same intrinsic properties like mass, charge ...)

b) Interactions:

For these microscopic objects we can only obtain indirect information on their interactions from 3 probes

- scattering $A + B \to C + D + \ldots$

Initial state — Final state



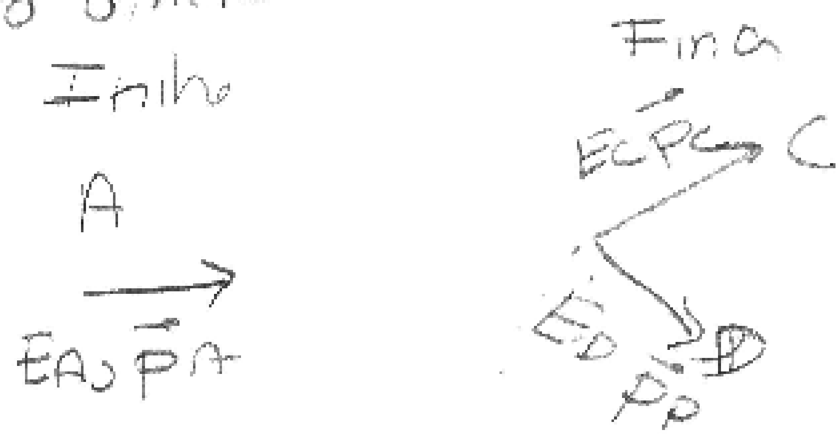
Notation
3 vectors

$$\vec{v} \equiv \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}$$

Studying what particles and how many come out and their energy and directions ($\equiv$ their kinematics) we learn about the forces among these particles

- Decay $A \to B + C \ldots\ldots$

In relativistic quantum systems (not classical) it is possible that a state disintegrates into others



again studying what particles and how often are produced and their kinematics we learn about their interactions

- Bound states made of these particles have masses which depend on the interaction responsible for their bounding

Standard approach:

- make a guess for some form of the interaction $\equiv$ "build a model"
- compute the prediction for something that we can measure on these probes ($\equiv$ observable) i.e. cross section of scattering, decay lifetime, mass spectrum of bound states
- compare expectation to data.

To make these constructions ($\equiv$ models) we need a framework which must verify two guiding principles
- special relativity because velocities of the particles in these probes are close to c $\Rightarrow$ chapter 2
- quantum mechanics because actions involved are comparable to $\hbar$

Mathematical framework employed in phys. (5)

macroscopic & slow object → small → microscopic & slow
Framework ≡ classical mechanics — Quantum mechan

↓ Fast

macroscopic & fast
Framework ≡ relativistic mechanics

microscopic & Fast
(particles)
Framework Quantum Field theory

Basics of QFT

- Particles are represented by "relativistic wave functions"
  ≡ fields
- A physics process (scattering or decay) is a transition between an initial and a final state consisting of some set (same or different) particles
- Given an initial and final state we can only predict and measure the probability of such transition to occur.

In this course we are going to omit the technicalities of QFT. But we need to remind ourselves some basic concepts relevant to quantum relativistic systems

(2) Basic Concepts of QM for particles

I.) the properties of a quantum system φ are completely specified by its vector state $|\varphi\rangle$

- $|\varphi\rangle \equiv$ mathematical representation of physical state

  $(|\varphi\rangle^{+} \equiv \langle\varphi|)$

- $|\varphi\rangle$ is an element of the (Hilbert) space of physical states

- We chose the state vectors to be normalized to 1 (as of today)

  $$\|\varphi\|^2 = \langle\varphi|\varphi\rangle = 1$$

- If $|\varphi\rangle$ and $|\chi\rangle$ are vectors of 2 physical states
  $\Rightarrow$ linear combination $\alpha|\varphi\rangle + \beta|\chi\rangle$ is a vector of a physical state

II) Given state vectors $|\varphi\rangle$ and $|\chi\rangle$ there is a probability of finding φ in state χ

$$a(\varphi \rightarrow \chi) \equiv \langle\chi|\varphi\rangle$$

~~And~~ the probability of measuring the state φ

as x is

$$P(\varphi \to x) = |a(\varphi \to x)|^2 = |\langle x|\varphi\rangle|^2$$

$\Rightarrow$ vectors $|\varphi\rangle$ and $|\varphi'\rangle \equiv e^{i\alpha}|\varphi\rangle$ represent

the same physical state because we can only

measure probabilities and

$$P(\varphi' \to x) = |\langle x|\varphi'\rangle|^2 = \underbrace{|e^{i\alpha}|^2}_{1} |\langle x|\varphi\rangle|^2$$

$$= P(\varphi \to x)$$

so we cannot distinguish $|\varphi\rangle$ from $|\varphi'\rangle$

III) For every measurable physical property ($\equiv$ observable)

A (ie energy, charge ...) of the states there is

an associated hermitian operator $\hat{A}$ ($\equiv \hat{A}^+$)

which acts on the space of physical states

- If the eigenvalues of A (a_n) are not degenerate

$\Rightarrow$ eigenstates of A $|n\rangle$ (so $A|n\rangle = a_n|n\rangle$)
a_n: number

can be taken as basis of space of physical

states

- If the eigenvalues are degenerate (≡ several states have same eigenvalue a_n) we need another operator B verifying $[A,B] = AB - BA = 0$ (≡ A, B are simultaneously diagonalizable) so we can chose the basis of space of physical state as the eigenstates of both operator $|n_a, m_b\rangle$

$$A|n_a, m_b\rangle = a_{n_a}|n_a, m_b\rangle$$
$$B|n_a, m_b\rangle = b_{m_b}|n_a, m_b\rangle$$

- If a_{n_a}, b_{m_b} are still degenerate then we need to find another operator C / $[A,C] = [B,C] = 0$ to characterize the basis $|n_a, m_b, s_c\rangle \ldots$

- For states $|\varphi\rangle$ which are not eigenstates of ~~some~~ observable A we define the "expectation value of A in φ" $\langle A\rangle_\varphi \equiv \langle\varphi|A|\varphi\rangle$

IV) the time evolution of state vector $|\varphi(t)\rangle$ is governed by ev. eq

$$i\hbar \frac{d|\varphi(t)\rangle}{dt} = H|\varphi(t)\rangle \Rightarrow \left[-i\hbar \frac{d\langle\varphi|}{dt} = \langle\varphi|H\right]$$

↑ Hamiltonian operator

≡ hermitian operator representing the Energy

If a time independent operator A verifies

$[A, H] = 0$

$$\Rightarrow \frac{d\langle A\rangle_\varphi}{dt} = \frac{d\langle\varphi|A|\varphi\rangle}{dt} = \frac{d\langle\varphi|}{dt}A|\varphi\rangle + \langle\varphi|A|\frac{d|\varphi\rangle}{dt}$$

$$= -i\hbar\langle\varphi|HA|\varphi\rangle + i\hbar\langle\varphi|AH|\varphi\rangle$$

$$= -i\hbar\langle\varphi|[H, A]|\varphi\rangle = 0$$

$\Rightarrow$ the expectation value of A in φ is a constant

$\Rightarrow$ We call it the A quantum # of φ

If we call $|\vec{x}\rangle$ the eigenstate of position op. $\vec{X}$

$|\vec{p}\rangle$ " " & 3-momentum op. $\vec{P}$

$$\Rightarrow \vec{X}|\vec{x}\rangle = \vec{x}|\vec{x}\rangle \quad ; \quad \vec{P}|\vec{p}\rangle = \vec{p}|\vec{p}\rangle$$

we define the wavefunctions for $|\varphi\rangle$ in position space

$$\varphi(x) \equiv \langle\vec{x}|\varphi\rangle$$

and in momentum space

$$\tilde{\varphi}(p) = \langle\vec{p}|\varphi\rangle$$

$|\varphi(\vec{x})|^2$ represents the probability density of finding φ at position $\vec{x}$

- $|\tilde{\varphi}(\vec{p})|^2$ represents the probability density of finding φ with 3-momentum $\vec{p}$

they verify
$$\tilde{\varphi}(\vec{p}) \sim \int d^3\vec{x}\, e^{-i\frac{\vec{p}\vec{x}}{\hbar}}\, \varphi(\vec{x})$$
$$\varphi(\vec{x}) \sim \int d^3\vec{p}\, e^{i\frac{\vec{p}\vec{x}}{\hbar}}\, \tilde{\varphi}(\vec{p})$$

- A particle with well defined 3-momentum $\vec{p}_0$ ($|\vec{p}_0\rangle$) has a wave function in momentum space
$$\tilde{\varphi}_{p_0}(p) \equiv \langle p | p_0 \rangle \sim \delta^3(\vec{p}-\vec{p}_0)$$

and its wave function in position space
$$\varphi_{p_0}(\vec{x}) = e^{i\vec{p}_0\vec{x}} \quad \equiv \text{plane wave}$$

- [remember $\int_{-\infty}^{\infty} dx\, \delta(x-x_0)\, f(x) = f(x_0)$

the equation for the time evolution of the wave function

$$i\hbar \frac{\partial \varphi(\vec{x},t)}{\partial t} = \langle x | H | \varphi(t) \rangle$$

$\Rightarrow$ a particle of well defined 3-momentum $\vec{p}_0$ and energy E_0 (f.e for a non-relativistic free particle

$$E_0 = \frac{|\vec{p}_0|^2}{2m}$$

verifies

$$H|\vec{p}_0\rangle = E_0|\vec{p}_0\rangle$$

$$\Rightarrow i\hbar \frac{\partial \varphi_{p_0}(\vec{x})}{\partial t} = E_0 \langle x | p_0 \rangle = E_0 \varphi_{p_0}(\vec{x})$$

$$\Rightarrow \varphi_{p_0}(\vec{x},t) = e^{\frac{-iE_0 t}{\hbar}} \; e^{\frac{i\vec{p}_0 \vec{x}}{\hbar}}$$

③ Particle contents of Standard Model

Matter is composed of fermions (≡ spin $\frac{1}{2}$ particles)
which we classify in two groups

- quarks (which are bounded inside nucleus)
- leptons which are not

they come in 3 generation each one with two type of quarks and two type of leptons

	1st gen	2nd	3rd	Q	B	L
Quarks	up (u)	charm (c)	top (t)	$2/3$	$1/3$	0
	down (d)	strange (s)	bottom (b)	$-1/3$	$1/3$	0
leptons	electron (e^-)	muon (μ^-)	tau (τ^-)	-1	0	1
	electron neutrino (ν_e)	muon neutrino (ν_μ)	tau neutrino (ν_τ)	0	0	1

additive Quantum #'s: Q, B, L

Q ≡ electric charge in units of "e"

B ≡ baryon #

L ≡ lepton #

Note: t, c, u have same Q, B, L distinguish by $m_t > m_c > m_u$

d, s, b " " $m_b > m_s > m_d$

$e^- \mu^- \tau^-$ " " $m_\tau > m_\mu > m_e$

Special relativity + quantum mechanics $\Rightarrow$
for each particle there must be an "antiparticle"
with same mass and spin but opposite Q, L, B ..

I.e for 1st generation

	Q	L	B
antiquark $u \equiv \bar{u}$	$-2/3$	0	$-1/3$
" $d \equiv \bar{d}$	$1/3$	0	$-1/3$
antielectron = positron $\equiv e^+$	$+1$	-1	0
anti-neutrino $\equiv \bar{\nu}$	0	-1	0

Quarks are never observed as free states.
they are always in bound states $\equiv$ hadrons.
Hadrons come in two types

- mesons $\equiv$ bound state $q\bar{q}'$ $\Rightarrow B = 0$
  and spin is integer (chapter 3)
  examples: pions $\pi^\pm\ \pi^0$, kaons $K^\pm\ K^0$ ...
- baryons $\equiv$ bound states of 3 quarks $qq'q''$
  $\Rightarrow B = 1$
  examples proton (p), neutron (n)

Interactions between these fermions are mediated by spin 1 particles $\equiv$ gauge bosons
$\underbrace{\text{spin 1 particles}}_{\equiv \text{ vectors}}$

In the SM there are 3 type of interactions

Int.	Gauge boson	Q
electromagnetic	photon γ	0
strong	gluons g	0
Weak → neutral	Z^0	0
Weak → charged	$W^\pm$	± 1

All have $B = L = 0$

In addition the model contains a spin = 0 (≡ scalar) particle ≡ Higgs boson (h).

We will describe the state and evolution of these particles in terms of "relativistic wave functions"

the form of the particle wave function depends on its spin ⇒ chapter 4

4 Natural units

In physics there are 3 independent dimension full quantities, i.e

Quantity	Dimension	Units in SI
Mass	[M]	Kg (kilogram)
Length	[L]	m (meter)
time	[T]	s (seconds)

the dimension of any other quantity is expressed as powers of these 3

units in SI

i.e velocity $[v] \equiv \left[\frac{L}{T}\right]$ $\quad m/s$

Energy $[E] \equiv \left[M\frac{L^2}{T^2}\right]$ $\quad Kg\frac{m^2}{s^2} \equiv J$ (Joule)

Action $[S] \equiv [E\,T] = \left[M\frac{L^2}{T}\right]$ $\quad Kg\frac{m^2}{s} \equiv J \cdot s$

In particle physics we need to use a much smaller unit for energy. We use the electron volt

$$eV \equiv 1.602\times10^{-19}\,J$$

and their multiples

$KeV \equiv 10^3 eV$, $MeV \equiv 10^6 eV$, $GeV \equiv 10^9 eV$, $TeV \equiv 10^{12} eV$

...

Two universal constants

IS

- speed of light $c \stackrel{\downarrow}{=} 2.999 \times 10^{8}$ m/s

- reduced plank constant $\hbar = 1.055 \times 10^{-34}$ J S
$= 6.582 \times 10^{-16}$ eV · s

Most particles move with velocities close to c and for quantum systems actions are order $\hbar$ So it is convenient to define a system of units ≡ natural units in which velocities are given in units of c and action in units of $\hbar$

⟹ In NU speed of ligh = 1
Plank reduced constant = 1

⟹ In NU we can express any quantity in units of a single dimension full quantity We use energy and expres any quantity in units of powes of eV

Quantitatively

$$\hbar \overset{IS}{=} 6.58\times10^{-16}\,eV\,s \overset{\text{in NU}}{\equiv} 1 \Rightarrow 1s = 1.52\times10^{15}\,eV^{-1} \quad \text{in NU}$$

$$\hbar c = 1.97\times10^{-7}\,eV\,m \equiv 1 \Rightarrow 1m = 5.08\times10^{6}\,eV^{-1}$$

$\Rightarrow$ we can express time and length in units of eV^{-1}

$$\frac{\hbar}{c^2} \overset{IS}{=} 0.117\times10^{-50}\,kg\,s \Rightarrow 1\,kg = 8.52\times10^{50}\,s = 5.6\times10^{35}\,eV$$

$\Rightarrow$ we can express mass in units of eV

i.e. proton mass $= 1.673\times10^{-27}\,kg = 938$ MeV

From now on we will use NU and we will not write "c's" nor "$\hbar$'s" in our equations. So we are implicitly giving velocities in units of c, actions in units of $\hbar$ and when we want to change a time from eV^{-1} to s or a length from eV^{-1} to m or a mass from eV to kg we just use the relations above.