Chapter 10

"Electroweak theory"

1) Weak interactions in isospin notation
2) Electroweak gauge theory
3) Spontaneous symmetry breaking
4) " " " of Electroweak symmetry
5) the Higgs boson

1) Weak interactions in isospin notation

So far we have written some effective Lag for weak CC and NC interactions in terms of coupling of massive vectors $W^{\pm}$ and Z to fermions

For the 1s generation ($\psi_f \equiv f$)

this ia d' but I am suppressin the

$$\mathcal{L}_{weak} = -\frac{g_W}{\sqrt{2}}\Bigg\{\Big[\overbrace{\bar{e}\gamma^{\mu}\frac{(1-\gamma_5)}{2}\nu_e + \bar{d}\gamma^{\mu}\frac{(1-\gamma_5)}{2}u}^{J^{\mu -}_{cc} \,\equiv}\Big]W_{\mu} + \Big[\overbrace{\bar{\nu}_e\gamma^{\mu}\frac{(1-\gamma_5)}{2}e + \bar{u}\gamma^{\mu}\frac{(1-\gamma_5)}{2}d}^{J^{\mu +}_{cc}}\Big]W_{\mu}^{\dagger}$$

dagger

$$+ g_Z\sum_{f=e\nu u d}\bar{f}\gamma^{\mu}\left(C_L^f\frac{(1-\gamma_5)}{2} + C_R^f\frac{(1+\gamma_5)}{2}\right)f\, Z_{\mu}$$

From Data

$$\frac{C_R^f}{Q^f} = -X \simeq -(0.22-0.23) \quad \text{for } f = e, u, d$$

$$C_R^{\nu} = 0$$

$$C_L^f = C_R^f + \frac{1}{2} \quad \text{for } \nu, u$$

$$C_L^f = C_R^f - \frac{1}{2} \quad \text{for } e, d$$

and $\dfrac{M_W^2}{M_Z^2} \simeq \dfrac{g_W^2}{g_Z^2} \simeq 1 - X$ and $\dfrac{g_W^2}{e^2} \simeq X$

We can write $J^{\mu\pm}_{cc}$ and J^{μ}_{NC} in matrix notation

$$J^{\mu +}_{cc} = \bar{\nu}_{e_L} \gamma^\mu e_L + \bar{u}_L \gamma^\mu d_L = (\bar{\nu}_{e_L}, \bar{e}_L)\gamma^\mu \underbrace{\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}}_{\equiv T_+} \begin{pmatrix} \nu_{e_L} \\ e_L \end{pmatrix}$$

$$+ (\bar{u}_L, \bar{d}_L)\gamma^\mu \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} u_L \\ d_L \end{pmatrix}$$

$$J^{\mu -}_{cc} = (J^{\mu +}_{cc})^\dagger = (\bar{\nu}_{e_L}, \bar{e}_L)\gamma^\mu \overbrace{\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}}^{T_-} \begin{pmatrix} \nu_{e_L} \\ e_L \end{pmatrix}$$

$$+ (\bar{u}_L, \bar{d}_L)\gamma^\mu \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} u_L \\ d_L \end{pmatrix}$$

$$\text{So } T_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \frac{1}{2}(\underset{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}{\sigma_1} + i \underset{\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}}{\sigma_2}) \equiv T_1 + i T_2$$

$$T_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \equiv \frac{1}{2}(\sigma_1 - i\sigma_2) = (T_1 - iT_2)$$

$T_i = \frac{\sigma_i}{2}$ are the generator of $SU(2)$ in the doublet representation

So if we define $L_L \equiv \begin{pmatrix} \nu_{e_L} \\ e_L \end{pmatrix}$ and $Q_L \equiv \begin{pmatrix} u_L \\ d_L \end{pmatrix}$

we can write

$$\mathcal{L}_{CC} = -\frac{g_W}{\sqrt{2}} \left[\bar{L}_L \gamma^\mu (T_- W_\mu + T_+ W_\mu^\dagger) L_L + \bar{Q}_L \gamma^\mu (T_- W_\mu + T_+ W_\mu^\dagger) Q_L \right]$$

$$\mathcal{L}_{CC} = -g_W \Big[\bar{L}_L \gamma^\mu (T_1 W_\mu^{(1)} + T_2 W_\mu^{(2)}) L_L + \bar{Q}_L \gamma^\mu (T_1 W_\mu^{(1)} + T_2 W_\mu^{(2)}) Q_L \Big]$$

with
$$\left. \begin{aligned} W_\mu^{(1)} &= \frac{1}{\sqrt{2}} (W_\mu + W_\mu^\dagger) \\ W_\mu^{(2)} &= \frac{i}{\sqrt{2}} (W_\mu - W_\mu^\dagger) \end{aligned} \right\} \Rightarrow \begin{aligned} W_\mu &= \frac{1}{\sqrt{2}} (W_\mu^{(1)} + i W_\mu^{(2)}) \\ W_\mu^\dagger &= \frac{1}{\sqrt{2}} (W_\mu^{(1)} - i W_\mu^{(2)}) \end{aligned}$$

For the NC

$$J^\mu_{NC} = \sum_f c_L^f \bar{f}_L \gamma^\mu f_L + c_R^f \bar{f}_R \gamma^\mu f_R$$

For $f \neq \nu$ (ie for $Q_f \neq 0$) $c_R^f \neq 0$

$$J^\mu_{NC} = +\frac{1}{2} \bar{\nu}_L \gamma^\mu \nu_L - \frac{1}{2} \bar{e}_L \gamma^\mu e_L + \frac{1}{2} \bar{u}_L \gamma^\mu u_L - \frac{1}{2} \bar{d}_L \gamma^\mu d_L$$
$$+ \sum_f (-x Q_f) \left[\bar{f}_R \gamma^\mu f_R + \bar{f}_L \gamma^\mu f_L \right] \qquad \sigma_3/2 = T_3$$
$$= (\bar{\nu}_L \; \bar{e}_L) \gamma^\mu \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} + (\bar{u}_L \; \bar{d}_L) \gamma^\mu \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} u_L \\ d_L \end{pmatrix}$$
$$+ \sum_{f = \ldots} (-x Q_f) (\bar{f}_R \gamma^\mu f_R + \bar{f}_L \gamma^\mu f_L)$$
$$= \bar{L}_L T_3 \gamma^\mu L_L + \bar{Q}_L T_3 \gamma^\mu Q_L + \sum_f (-x Q_f) (\bar{f}_R \gamma^\mu f_R + \bar{f}_L \gamma^\mu f_L)$$

Now in this notation

$$\mathcal{L}_{em} = -e \underbrace{\sum_f Q_f \bar{f} \gamma^\mu f}_{J^\mu_{em}} A_\mu$$

$$J^\mu_{em} = \sum_f Q_f \bar{f} \gamma^\mu f = \sum_f Q_f (\bar{f}_R \gamma^\mu f_R + \bar{f}_L \gamma_\mu f_L)$$

So in summary we have written $\mathcal{L}_{CC}$, $\mathcal{L}_{NC}$, and L_{em} in terms of 7 chiral fermions

$\nu_L, e_L, e_R, u_L, u_R, d_L, d_R$

and we have written the effective Lag in terms of two doublets of left-handed fermions and 3 right handed fermions which do with quantum #'s

hypercharge

	CC	$t_3^f = \langle T_3 \rangle$	Q_f	$\Rightarrow \frac{Y^f}{2} = Q^f - t_3^f$
$L = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$	SO(2) doublet	$1/2$ $-1/2$	0 -1	$-1/2$
$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	SU(2) "	$1/2$ $-1/2$	$2/3$ $-1/2$	$1/6$
e_R	" singlet	0	-1	-1
u_R	" "	0	$2/3$	$2/3$
d_R	" "	0	$-1/3$	$-1/3$

In summary the combinations L_L, Q_L, e_R, u_R, d_R

have well defined "charges" ($\equiv$ transformation properties) for $SU(2)_{Left}$

act only on left-handed fermions

and $U(1)_{\frac{Y}{2}}$

2 Electroweak gauge theory

We have written the Lag for weak cc and NC together with em of the fermions of each generation in terms of the interactions of 5 chiral combinations

$L_L \equiv \begin{pmatrix} \nu_L \\ e_L^- \end{pmatrix}$ $\quad Q_L \equiv \begin{pmatrix} u_L \\ d_L \end{pmatrix}$ $\quad e_R, u_R, d_R$

Which have well define charges under $SU(2)_{Left} \times U(1)_{\frac{Y}{2}}$

We can try to see how these interactions compare with what we would get from a gauge theory with those 5 combinations as matter content and $\underbrace{SU(2)_{Left} \times U(1)_{\frac{Y}{2}}}_{\shortparallel\, G_{EW}}$ gauge group

To build this Lag we start with the free Lag for these 5 combinations

$$\mathcal{L}_{free} = i \sum_{i=1}^{5} \bar{\Psi}_i \gamma^\mu \partial_\mu \Psi_i \,,$$

no mass for these fermions (more later)

$\Psi_1 = L_L$

$\Psi_2 = Q_L$

$\Psi_3 = e_R$

$\Psi_4 = u_R$

$\Psi_5 = d_R$

- Under $SU(2)_{Left}$ Ψ_i transform as

For $i = 1,2$

$$\Psi'_i(x) = \Big[\underbrace{e^{-i\sum_{a=1}^{3} \alpha_a(x)\, T_a}}_{2\times 2 \text{ Matrix}} \Big] \Psi_i$$

$\alpha_a(x)$: parameters of $SU(2)$ transformation; $T_a \equiv \frac{\sigma_a}{2}$ generator of $SU(2)$

$$\bar{\Psi}'_i(x) = \bar{\Psi}_i(x)\, e^{+i\sum \alpha_a(x) T_a}$$

$$T_a = \frac{\sigma_a}{2} \Rightarrow [T_a, T_b] = \sum_c i\epsilon_{abc} T_c$$

For $i = 3,4,5$ $\Psi_i(x)$ is a righ-handed fermion $\Rightarrow$ $\Rightarrow$ singlet of $SU(2)_{Left}$ $\Rightarrow$ remains invariant

$$\Psi'_i(x) = \Psi_i(x)$$

- Under $U(1)_{Y/2}$

$\beta(x)$: parameter of $U(1)$ transf

$$\Psi'_i(x) = e^{-i\beta(x)\frac{Y_i}{2}}\, \Psi_i(x) \Rightarrow \bar{\Psi}'_i(x) = \bar{\Psi}_i\, e^{i\beta(x)\frac{Y_i}{2}}$$

In order to build a gauge invariant Lag we need to substitute $\partial_\mu \Psi_i \to D_\mu \Psi_i$ and for that we need 3 gauge bosons for $SU(2)_{Left}$ $(W^{\mu,a})$ and one for $U(1)_{Y/2}$ (B^μ)

And the gauge invariant Lag for the fermions

$$\mathcal{L}_{EW}^{\text{fermions}} = \sum_{i=1}^{5} i\,\bar{\Psi}_i \gamma^\mu_i D_\mu \Psi$$

where for $i=1,2$

$$D^\mu \Psi_i = (\partial^\mu + ig \sum_{a=1}^{3} \overset{\sigma^a/2}{\overset{|||}{T^a}} W^{\mu,a} + ig' B^\mu \frac{Y_i}{2})\Psi_i$$

coupling constants

and for $i = 3, 4, 5$

$$D^\mu \Psi_i = (\partial^\mu + ig' B^\mu \frac{Y^i}{2})\Psi_i$$

and gauge invariance implies that under gauge trans

$$W_\mu^{\prime a} = W_\mu^a - \frac{1}{g}\partial_\mu \alpha^a - \sum_{b,c} \epsilon^{abc} \alpha^b W_\mu^c$$

$$B_\mu' = B_\mu - \frac{1}{g'}\partial_\mu \beta$$

Let us define $\quad W_\mu \equiv \frac{1}{\sqrt{2}}(W_\mu^{(1)} + i W_\mu^{(2)}) \equiv W_\mu^-$

$$W_\mu^+ = \frac{1}{\sqrt{2}}(W_\mu^{(1)} - i W_\mu^{(2)}) \equiv W_\mu^+$$

$W_\mu^{(3)} = \cos\theta_W Z^\mu + \sin\theta_W A^\mu$

$B^\mu = \cos\theta_W A^\mu - \sin\theta_W Z^\mu \Leftarrow \begin{cases} A_\mu \equiv \cos\theta_W B_\mu + \sin\theta_W W_\mu^{(3)} \\ Z_\mu \equiv -\sin\theta_W B_\mu + \cos\theta_W W_\mu^{(3)} \end{cases}$

as of now this is just a rotation of basis of new hat gauge bosons

We can write $\mathcal{L}_{EW}$ in this basis and see what we get

1) Pieces ~~in~~ ~~th~~ $W_\mu^{(1)}$ and $W_\mu^{(2)}$.

$$-g\,[\bar{L}_L \gamma^\mu (T_1 W_\mu^{(1)} + T_2 W_\mu^{(2)}) L_L$$
$$+\bar{Q}_L \gamma^\mu (T_1 W_\mu^{(1)} + T_2 W_\mu^{(2)}) Q_L$$
$$= -\frac{g}{\sqrt{2}} \{ [\bar{L}_L \gamma^\mu T^+ L_L + \bar{Q}_L \gamma^\mu T^+ Q_L] W_\mu^+$$
$$+ [\bar{L}_L \gamma^\mu T^- L_L + \bar{Q}_L \gamma^\mu T^- Q_L] W_\mu^- \}$$
$$\equiv \mathcal{L}^{CC}_{weak} \quad \text{with } g_W = g$$

2) Pieces with $W_\mu^{(3)}$ and B_μ

$$-(\bar{\nu}_e\ \bar{e}_L)\gamma^\mu (g T_3 W_\mu^{(3)} + g' \frac{Y^L}{2} B_\mu) \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$
$$-(\bar{u}_L\ \bar{d}_L)\gamma^\mu (g T_3 W_\mu^{(3)} + g' \frac{Y^Q}{2} B_\mu) \begin{pmatrix} u_L \\ d_L \end{pmatrix}$$
$$-\bar{e}_R \gamma^\mu \frac{Y^{e_R}}{2} B_\mu e_R - \bar{u}_R \gamma^\mu \frac{Y^{u_R}}{2} B_\mu u_R - \bar{d}_R \gamma^\mu \frac{Y^{d_R}}{2} B_\mu d_R$$

$$= \sum_{f=\nu e u d} -\bar{f}_L \gamma^\mu (g\, t_3^{f_L} W_\mu^{(3)} + g' \frac{Y^{f_L}}{2} B_\mu) f_L \qquad t_3^{f_L} = \frac{1}{2} \text{ or } -\frac{1}{2}$$
$$\sum_{f=eud} -\bar{f}_R \gamma^\mu \frac{g'}{2} Y^{f_R} B_\mu$$

$$= \sum_f -A_\mu \Big[\bar{f}_L \gamma^\mu \left(g\, t_3^{f_L} \sin\theta_W + \frac{g'}{2} Y^{f_L} \cos\theta_W \right) f_L$$

$$+ \bar{f}_R \gamma^\mu \frac{g'}{2} Y^{f_R} \cos\theta_W\, f_R \Big]$$

$$- Z_\mu \Big[\bar{f}_L \gamma^\mu \left(g\, t_3^{f_L} \cos\theta_W - \frac{g'}{2} Y^{f_L} \sin\theta_W \right) f_L$$

$$- \bar{f}_R \gamma^\mu \frac{g'}{2} Y^{f_R} \sin\theta_W\, f_R \Big]$$

Let us make $\tan\theta_W = \frac{g'}{g} \Rightarrow g' \cos\theta_W = g \sin\theta_W$

$\Rightarrow$ piece in A^μ

$$= A_\mu g \sin\theta_W \Big[\bar{f}_L \gamma^\mu \underbrace{\left(t_3^{f_L} + \frac{Y^{f_L}}{2} \right)}_{\equiv Q_f} + \bar{f}_R \gamma^\mu \frac{Y^{f_R}}{2} f_R$$

$$= - A_\mu g \sin\theta_W Q_f \left(\bar{f}_L \gamma^\mu f_L + \bar{f}_R \gamma^\mu f_R \right)$$

$$= - A_\mu\, g \sin\theta_W Q_f\, \bar{f} \gamma^\mu f \equiv \mathcal{L}_{em} \text{ with } e = g \sin\theta_W$$

using $g' \sin\theta_W = \frac{g \sin^2\theta_W}{\cos\theta_W}$

$\Rightarrow$ piece in Z_μ

$$- Z_\mu \frac{g}{\cos\theta_W} \Big[\bar{f}_L \gamma^\mu \left(t_3^{f_L} \cos^2\theta_W - \frac{Y^{f_L}}{2} \sin^2\theta_W \right) f_L$$

$$- \bar{f}_R \gamma^\mu \underbrace{\frac{Y^{f_R}}{2}}_{\equiv Q_f} \sin^2\theta_W\, f_R$$

notice that

$$t_3^{f_L}\cos^2\theta_W - \frac{Y^{f_L}}{2}\sin^2\theta_W$$

$$= t_3^{f_L}\cos^2\theta_W + (t_3^{f_L} - Q^f)\sin^2\theta_W$$

$$= t_3^{f_L} - Q^f\sin^2\theta_W \equiv C_L^f \quad \text{with } x = \sin^2\theta_W$$

So the piece in $\mathcal{L}^\mu$ is

$$-Z^\mu \frac{g}{\cos\theta_W}\Big[\bar{f}_L\gamma^\mu(t_3^{f_L} - \sin^2\theta_W Q^f) f_L - \bar{f}_R\gamma^\mu \underbrace{\sin^2\theta\, Q^f}_{C_R^f} f_R\Big]$$

$= \mathcal{L}_{NC}$ with

$$g_Z = \frac{g}{\cos\theta_W} = \frac{e}{\sin\theta_W\cos\theta_W}$$

($g \equiv g_W$)

and $x = \sin^2\theta_W \simeq 0.23$

$$\Rightarrow \frac{e^2}{g^2} = \sin^2\theta_W \simeq 0.23$$

In summary: starting with a gauge theory with gauge group $SU(2)_{Left} \times U(1)_{Y/2}$ we have been able to construct

$$\mathcal{L}_{EW}^{fermions} = \mathcal{L}_{free}^{fermion} + \mathcal{L}_{weak\,CC}^{int} + \mathcal{L}_{weak\,NC}^{int} + \mathcal{L}_{em}^{int}$$

To do so we need to combine the T^3 part of $SU(2)_{left}$ with $U(1)_{Y/2}$ in two linear combinations with a rotation angle θ_W (with $\sin^2\theta_W \simeq 0.23$) and this rotation must be related to the ratio of the coupling constants as

$$\tan\theta = \frac{g'}{g} \quad \text{and} \quad e = g\sin\theta_W = g'\cos\theta_W$$

the full Lag should also contain the Lag for the gauge bosons

$$\mathcal{L}_{EW}^{TOT} = \sum_{i=1}^{5} \bar{\psi}\gamma^\mu D_\mu \psi - \frac{1}{4}\sum_{a=1}^{3} W_{\mu\nu}^{(a)} W^{\mu\nu(a)} - \frac{1}{4} B_{\mu\nu}B^{\mu\nu}$$

with $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$

and $W_{\mu\nu}^{a} = \partial_\mu W_\nu^a - \partial_\nu W_\mu - g\sum_{bc} \epsilon_{abc} W_\mu^b W_\nu^c$

But:

- the four gauge bosons are massless because GI ⇒ massless gauge bosons ($M_V^2 V^\mu V_\mu \neq M_V^2 V'^\mu V'_\mu$)
- the fermions are massless because a fermion mass

$$m \bar{f} f = m(\underbrace{\bar{f}_R}_{SU(2)\ \text{singlet}} \underbrace{f_L}_{\text{part of a } SU(2)\ \text{doublet}} + \bar{f}_L f_R) \Rightarrow \text{not } SU(2)_{\text{Left}} \text{ gauge inv.}$$

- We do not know what is the reason for the rotation

$$(W_3^\mu, B^\mu) \rightarrow (Z^\mu, A^\mu)$$

The solution to all these will arise from the spontaneous breaking of EW symmetry

(3) Spontaneous symmetry breaking

To give mass to the $W^{\pm}$ and Z and the fermions we need to break the gauge symmetry. $SU(2)_L \times U(1)_{\frac{Y}{2}}$

We could break it "explictly" just adding those mass terms to the $\mathcal{L}$ag. But then we would lose all bonuses of gauge theory, in particular the theory would not be renormalizable and we would have divergences when going to higher order in perturbation theory.

In order to break the symmetry in the mass spectrum of the particles but not in the $\mathcal{L}$ag we need to notice that particles are quantum excitations above the ground state ($\equiv$ the vacuum). So if the ground state above which we define the particle breaks the EW symmetry then $\mathcal{L}_{EW}$ in terms of their wave functions will "apparently" be not gauge inv.

We call this form of breaking the symmetry $\equiv$ spontaneous symmetry breaking (SSB)

To introduce in our theory the possibility of SSB we need to add "something" to the theory which allows to describe such non-trivial ground state. The most straightforward is to add a field with a potential which is minimize by a symmetry breaking ground state.

Lorentz invariance $\Rightarrow$ ground state cannot have spin $\Rightarrow$ field added must be scalar

Let us take a complex scalar field

$$\phi(x) = \frac{1}{\sqrt{2}}\left(\phi_1(x) + i\,\phi_2(x)\right)$$

real

The most general Lag for this field

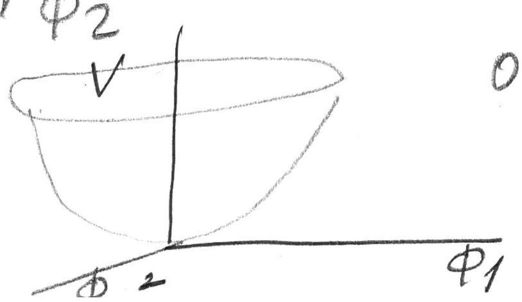
$$\mathcal{L}_\phi = (\partial^\mu \phi^*)(\partial_\mu \phi) - \underbrace{\left(\mu^2 \phi^* \phi + \lambda(\phi^*\phi)^2\right)}_{V(\phi)}$$

$\rightarrow$ invariant under global U(1) $\phi \to e^{i\alpha}\phi$, $\mathcal{L} \to \mathcal{L}$

It must be $\lambda > 0$ for $V(\phi)$ to be bounded (so V does not become infinitely negative when $|\phi| \to$ large)

If we plot $V(\phi) = \mu^2|\phi|^2 + \lambda|\phi|^4$ as function of ϕ_1 and ϕ_2

• For $\mu^2 > 0$



Notice that the minimum

$$0 = \left.\frac{dV}{d|\phi|}\right|_{min} = 2\mu^2|\phi|_{min} + 4\lambda|\phi|^3_{min} = 0$$

$$\Rightarrow |\phi|_{min} = 0$$

$$\Rightarrow V_{min} = 0$$

• If $\mu^2<0 \Rightarrow \mu^2=-|\mu^2|$ $V(\phi) = -|\mu^2||\phi|^2 + \lambda|\phi|^4$

$$0=\frac{dV}{d|\phi|}\Big|_{min} = -2|\mu^2||\phi|_{min} + 4\lambda|\phi|^3_{min}$$

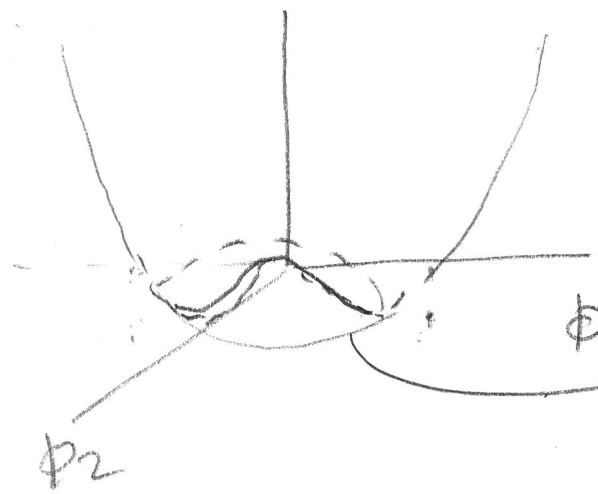
$$\Rightarrow 2|\phi|^2_{min} = \frac{|\mu^2|}{\lambda} \equiv v^2 > 0$$ $\equiv$ vacuum expectation value (vev) of Φ

or $|\phi|=0$ (which is a local maximum)

$$V_{min}\left(\frac{|\mu|^2}{2\lambda}\right) = -\frac{|\mu^2|^2}{4\lambda} < 0 \quad \text{w.r.t.} \quad V(0)=0$$

Graphically

$$V(\phi) = V_{min} + \lambda\left(|\phi|^2 - \frac{v^2}{2}\right)^2$$



notice that there is a continuous of states ϕ all with $|\phi|_{min}$ all with V_{min}. they are all

$$\phi = |\phi|_{min}\, e^{i\alpha} \text{ with } 0\leq\alpha\leq 2\pi$$

They are related by a global U(1) symmetry as it should be because $\mathcal{L}_\phi$ has that symmetry

$$\mathcal{L}_\phi \xrightarrow{\phi\to e^{i\alpha}\phi} \mathcal{L}_\phi$$

When we quantize this theory we need to fix the ground state above which we define our quantum excitations (ie our particles) $\Rightarrow$ chose some α_0

For example we can chose $\alpha_0 = 0$

$$\phi_0 = \frac{1}{\sqrt{2}} v + i0 \Rightarrow \left(\phi_{1min} = \frac{v}{\sqrt{2}} , \phi_{2min} = 0 \right.$$

So our quantized field will be scalar "particle" wave func.

$$\phi(x) = \phi_0 + \tilde{\phi}(x)$$

In terms of $\tilde{\phi}$

$$\mathcal{L}_\phi = (\partial_\mu \tilde{\phi}^*)(\partial^\mu \tilde{\phi}) - \overbrace{v^2 \lambda}^{-|\mu|^2} \left(\frac{v^2}{2} + |\tilde{\phi}|^2 + 2 \frac{v}{\sqrt{2}} \mathrm{Re}\, \tilde{\phi} \right)$$

$$+ \lambda \left(\frac{v^2}{2} + |\tilde{\phi}|^2 + 2 \frac{v}{\sqrt{2}} \mathrm{Re}\, \tilde{\phi} \right)^2$$

this is not inv when $\tilde{\phi} \to e^{i\alpha} \tilde{\phi}$

$\Rightarrow$ global U(1) symmetry has been "spontaneously broken" by chosing to quantize above an specific ϕ_0 which fixes the "direction" in the (ϕ_1, ϕ_2) plane for the ground state

(19)

We can alway write instead of $\phi = \frac{v}{\sqrt{2}} \tilde{\phi}$

complex $\equiv$ 2 compon.

$$\phi(x) = \frac{1}{\sqrt{2}}(v + h(x))\, e^{i\frac{\xi(x)}{v}}$$

real

$\mathcal{L}_\phi$ in terms of h and ξ [using $\partial_\mu \phi = \frac{1}{\sqrt{2}}\left(\partial_\mu h + i\frac{\partial_\mu \xi}{v}(v+h)\right)e^{\frac{i\xi}{v}}$]

$$\mathcal{L}_\phi = \frac{1}{2}(\partial_\mu h)(\partial^\mu h) + \frac{1}{2}(\partial_\mu \xi)(\partial^\mu \xi)\left(1+\frac{h}{v}\right)^2$$

$$-\frac{\mu^2}{2}(v+h)^2 - \frac{\lambda}{4}(v+h)^4$$

$$= \boxed{\frac{1}{2}(\partial_\mu h)(\partial^\mu h) - h^2\underbrace{\left(\frac{\mu^2}{2} + 6\frac{v^2}{4}\lambda\right)}_{-\mu^2 = |\mu^2|}} + \boxed{(\partial^\mu \xi)(\partial^\mu \xi)}$$

$-\mu^2/\lambda = v^2$

$$\underbrace{+(\partial^\mu \xi)(\partial_\mu \xi)\frac{h}{v} + (\partial_\mu \xi)(\partial^\mu \xi)\frac{h^2}{v^2}}$$

Free Lag for a neutral scalar particle of mass $m_h = \sqrt{2|\mu^2|} = \sqrt{2\lambda}\, v$

interactions terms

Lag for massless neutral scalar $\equiv$ goldstone boson

Notice that the terms linear in h

$$-\frac{\mu^2}{2}(2vh) - \frac{\lambda}{4}(4v^3h) = (-\mu^2 v + \mu^2 v)h = 0$$
$$v \overset{\parallel}{\cdot} \lambda \frac{|\mu^2|}{\lambda}$$

Let us repeat now but let us take $\mathcal{L}_\phi$ to be gauge invariant under U(1) and $\phi(x)$ charged under U(1) with charge Q_ϕ :

$\Rightarrow$ under U(1) transf $\phi \to e^{-i\alpha(x)Q_\phi}$

$$\mathcal{L}_\phi = (D_\mu\phi)^\dagger(D^\mu\phi) - \underbrace{[\mu^2\phi^*\phi + \lambda(\phi^*\phi)^2]}_{V(\phi)}$$

with $D_\mu\phi = \partial_\mu\phi + ie A_\mu Q_\phi \phi$ $\leftarrow$ U(1) gauge vector boson

The introduction of A^μ does not affect $V(\phi)$ so we still have the continuous of ground states for $\mu^2 < 0$

And again we can chose to quantize above one specific ground state

$$\phi_0 = \frac{v}{\sqrt{2}}(1 + i0)$$

and the quantize field can still be written as

$$\phi(x) = \frac{v + h(x)}{\sqrt{2}} e^{i\frac{\xi(x)}{v}}$$

$$\Rightarrow D_\mu \phi = \frac{1}{\sqrt{2}}\left(\partial_\mu h + \frac{i(v+h)}{v}\partial_\mu \xi + ieQ_\phi A_\mu (v+h)\right) e^{\frac{i\xi}{v}}$$

GI $\Rightarrow$ physics described by A_μ and by $A'_\mu = A_\mu - \frac{1}{e}\partial_\mu \xi$ is the same. And in terms of A'_μ ($\equiv$ unitary gauge)

$$D_\mu \phi = \frac{1}{\sqrt{2}}\left(\partial_\mu h + ieQ_\phi A'_\mu (v+h)\right) e^{i\frac{\xi}{v}}$$

$$\Rightarrow (D_\mu \phi)^* (D^\mu \phi) = \frac{1}{2}\left[(\partial_\mu h)(\partial^\mu h) + e^2 A'_\mu A'^\mu (v+h)^2 Q_\phi^2\right]$$

$\Rightarrow$ ξ disappears from Lag

instead we have a term $\frac{Q_\phi^2}{2} e^2 v^2 A'_\mu A'^\mu \equiv \frac{1}{2} \underset{e^2 v^2}{\underset{\shortparallel}{M_A^2}} A^\mu A_\mu$

$\Rightarrow$ a mass term for the gauge boson has been generated

We say that the would-be goldstone boson ξ has been "eaten" by the gauge boson ☺

In this gauge

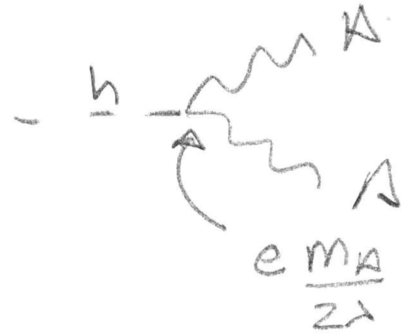
$$\mathcal{L}_{\phi,A} = \mathcal{L}_\phi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \qquad F^{\mu\nu} \to \partial^\mu A^\nu - \partial^\nu A^\mu$$

$$= \frac{1}{2}(\partial_\mu h)(\partial^\mu h) - \Big(\overbrace{\frac{\mu^2}{2} + \frac{6}{2}\lambda v^2}^{-\mu^2 = |\mu^2|}\Big) h^2 \to \text{Lag for a real massive scalar of mass } m_h^2 = 2|\mu^2| = 2\lambda v^2$$

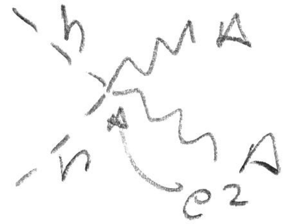
$$- \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{Q_\phi^2}{2} e^2 v^2 A^\mu A_\mu \leftarrow \text{Lag for real massive vector of mass } M_A^2 = e^2 v^2 Q_\phi^2$$

$+ \ldots$

$$+Q_\phi e \underset{\underset{\displaystyle e\frac{M_A}{2\lambda}}{|||}}{v} A^\mu A_\mu h \longrightarrow \text{vertex}$$



$$+Q_\phi^2 \frac{e^2}{2} A^\mu A_\mu h^2 \longrightarrow \text{vertex}$$



$$-\lambda v h^3 \longrightarrow \text{vertex}$$

$$-\frac{\lambda}{6} h^4 \longrightarrow \text{"}$$

So after expressing the Lag in terms of the particle fields $h(x)$ and $A^\mu(x)$ (in the unitary gauge) defined as quantum excitations above the specific Φ_0 we have generated mass for the gauge boson to which ϕ couples to. The generated mass is proportional to the vev of the scalar and the gauge coupling constant

Counting degrees of freedom

- Before SSB: complex scalar (2) + massless vector (2)
- After SSB: real scalar (1) + massive vector (3)

(9) Spontaneous breaking of the EW symmetry

Summarizing

- SSB ≡ breaking of symmetry by choiced ground state above which the theory is quantized
- simplest realization ≡ add a complex scalar with a potential which has a minimum at non-zero value of the scalar
- symmetry ⇒ degenerate ground state
- after chosing one of the ground states as the state above which we define the particles the Lag written in terms of the particle fields is apparently not symmetric
- the spectrum of particles contains a massive real scalar and massless scalars (≡ goldstone bosons)
- if the symmetry is local (≡ gauge) and the complex scalar is charged under that gauge symmetry, the spectrum of physical particles after SSB contains the massive real scalar and the gauge boson has acquired a mass which is proportional to its charge (squared) and the vev of the scalar

To go for $\mathcal{L}_{EW} \longrightarrow$ real world we need

$M_{W,Z} \neq 0$ but $M_A = 0$

So we need to break

$SU(2)_L \times U(1)_Y \longrightarrow U(1)_{em}$

- To break $SU(2)_L \Rightarrow \phi$ must not be a $SU(2)$ singlet

The lowest representation possible is that ϕ is a $SU(2)_L$ doublet

$$\Phi \equiv \begin{pmatrix} \phi_3 + i\phi_4 \\ \phi_1 + i\phi_2 \end{pmatrix} \qquad \phi_i \equiv \text{real}$$

- We want to break $U(1)_Y \Rightarrow Y_\phi \neq 0$
- we want $U(1)_{em}$ to be unbroken $\Rightarrow$ some component of ϕ must have $Q_{\phi_i} = 0$

$$\text{Since } Q_{\phi_i} = \frac{Y^\phi}{2} + t_3^{\phi_i} \Rightarrow \frac{Y^\phi}{2} = \mp \frac{1}{2} \qquad \left(t_3^{\phi_i} = \pm\frac{1}{2}\right)$$

the usual choice is $\frac{Y^\phi}{2} = \frac{1}{2} \Rightarrow \phi_1 + i\phi_2$ has $Q = 0$

the Lag for Φ

$$\mathcal{L}_\Phi = (D^\mu \Phi)^\dagger (D_\mu \Phi) - [\mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2]$$

with $D_\mu \Phi = (\partial_\mu + i g \sum_{a=1}^{3} T^a W_\mu^{(a)} + i \frac{g'}{2} B_\mu) \Phi$

$$= \left\{ \begin{bmatrix} \partial_\mu & 0 \\ 0 & \partial_\mu \end{bmatrix} + \frac{i}{2} \begin{bmatrix} g W_\mu^3 + g' B_\mu & g(W_\mu^1 - i W_\mu^2) \\ g(W_\mu^{(1)} + i g' W_\mu^{(2)}) & -g W_\mu^3 + g' B_\mu \end{bmatrix} \right. \begin{bmatrix} \phi_3 + i\phi_4 \\ \phi_1 + i\phi_2 \end{bmatrix}$$

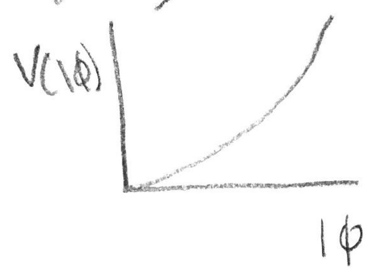
and $V(\phi) = +\mu^2 |\phi|^2 + \lambda |\phi|^4$ with $|\phi|^2 = \phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2$

So for $\mu^2 > 0$

$$\frac{dV}{d|\phi|} = |\phi|(2\mu^2 + 4\lambda|\phi|^2) = 0$$

$\Rightarrow |\phi|_{min} = 0 \Rightarrow \phi_i = 0 \; i=1,2,3,4$

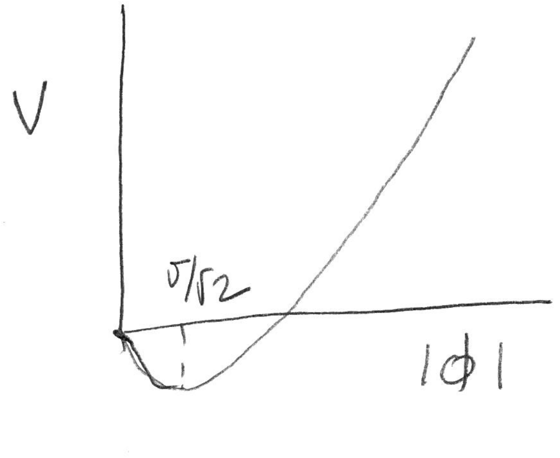
$\Rightarrow V_{min} = 0$



For $\mu^2 < 0$

$$\frac{dV}{d|\phi|} = 0 \quad \begin{cases} |\phi|_m = 0 \rightarrow V_{min} = 0 \\ |\phi|^2_{m} = -\frac{\mu^2}{2\lambda} \equiv \frac{v^2}{2} \Rightarrow V_\mu < 0 \end{cases}$$

$\rightarrow$ minimum



So again there is a continuous of ground states all with

$$|\Phi_k|^2 = \phi_1^2+\phi_2^2+\phi_3^2+\phi_4^2 = \frac{v^2}{2}$$

We spontaneously break the symmetry when we chose to quantize above one of these states. Since we want the ground state to have $Q=0$. We chose we chose it with $\phi_{3\,min} = \phi_{4\,min} = 0$.

$$\Phi_0 = \begin{pmatrix} 0+i0 \\ \frac{v}{\sqrt{2}}+i0 \end{pmatrix}$$

So the quantum field is

$$\phi(x) = \begin{pmatrix} h_3(x)+ih_4(x) \\ \frac{v}{\sqrt{2}}+h_1(x)+ih_2(x) \end{pmatrix}$$

$h_i \equiv$ 4 real fields

We can always write it as $T_a \equiv \sigma^a/2$

$$\equiv \underbrace{\frac{1}{\sqrt{2}}\, e^{i\sum_{a=1}^{3} T_a\, \xi^a(x)}}_{\substack{\equiv \\ U}} \underbrace{\begin{pmatrix} 0 \\ v+h(x) \end{pmatrix}}_{\substack{\equiv \\ \Phi_U}}$$

real scalar

U: 3 × 3 unitary matrix

3 real scalar fields

So

$$(D_\mu \overline{\Phi}) = \left\{ \partial_\mu \left[U \phi_v \right] + \left(ig \sum_a T_a W_\mu^a + i\frac{g'}{2} B_\mu \right) U \phi_v \right\}$$

$$= \left\{ U \partial_\mu \phi_v + (\partial_\mu U)\phi_v + i\frac{g'}{2} B_\mu U \phi_v + ig \sum_a T_a W_\mu^a U \phi_v \right\}$$

If we define a vetor $W_\mu'^a$ which verifies

$$U\left(ig \sum_a T^a W_\mu'^a\right) = ig \sum_a T^a W_\mu^a U + \partial_\mu U \quad (*)$$

then

$$D_\mu \phi = U \left\{ \partial_\mu \phi_v + \left(ig \sum_a T^a W_\mu'^a + i\frac{g'}{2} B_\mu \right) \phi_v \right\}$$

$$\equiv U(D'_\mu \phi_v)$$

$$\Rightarrow (D_\mu \phi)^\dagger (D^\mu \phi) = (D'_\mu \phi_v)^\dagger \underbrace{(U^\dagger U)}_{I} (D'^\mu \phi_v)$$

But the condicion (*) is just a SU(2) gauge transf.

To see this explicitly ~~we~~ expand at ξs ordering

$$\sum_a T^a W_\mu'^a = U^{-1} \sum_a T^a U W_\mu^a - \frac{i}{g} U^{-1} \partial_\mu U$$

expanding exponentials

$$\simeq \left(1 - i\sum_b \frac{\xi^b}{v} T^b\right) \sum_a T^a \left(1 + i\sum_b \frac{\xi^b T^b}{v}\right) W_\mu^a + \frac{1}{g} \sum_c \frac{\partial_\mu \xi^c}{v} T^c$$

~~leading~~ ordering in ξ

$$\simeq \sum_a T^a W_\mu^a - \frac{i}{v} \sum_{ab} \underbrace{[T^a, T^b]}_{i\sum_c \epsilon_{abc} T^c} \xi^b W^b + \frac{1}{gv} \sum_c \partial_\mu \xi^c T^c$$

multiplying by T^d and using $T^a T^d = \delta^{ad}$

$$W_\mu'^{d} = W_\mu^{d} + \frac{1}{gv}\partial_\mu \xi^d + \sum_{ab} \epsilon_{abd}\, \xi^a W_\mu^{(c)}$$

this is exactly the gauge transformation for the gauge bosons of SU(2)

Gauge invariance $\Rightarrow$ the physics described by $W_\mu^{(a)}$ and $W_\mu^{(a)\prime}$ as the same

In the "1" gauge $\equiv$ unitary gauge

$$(D_\mu \phi)^\dagger (D^\mu \phi) = (D_\mu \phi_0)^\dagger (D_\mu \phi_0)$$

$$= \frac{1}{2}\left[\begin{pmatrix} 0 \\ \partial_\mu h \end{pmatrix} + \frac{i}{2}\begin{bmatrix} g W_\mu^3 + g' B_\mu & \overbrace{g(W_\mu^1 - i W_\mu^2)}^{\sqrt{2}W_\mu^+} \\ \underbrace{g(W_\mu^1 + i W_\mu^2)}_{\sqrt{2}W_\mu} & -g W_\mu^3 + g' B_\mu \end{bmatrix}\begin{pmatrix} 0 \\ v+h \end{pmatrix}\right]^+$$

$$\left[\quad " \quad \right]$$

$$= \frac{1}{2}(\partial_\mu h)(\partial^\mu h) + \left(\frac{g}{2}(v+h)\right)^2 W_\mu W^{\mu +}$$

$$+ \frac{1}{8}\left[g' B_\mu - g W_\mu^{(3)}\right]\left[g' B^\mu - g W^{\mu(3)}\right](v+h)^2$$

(and all ξ^a dissappear)

Counting degrees of freedom

- Before SSB
  - 1 complex scalar doublet $= 4$
  - 4 massless gauge bosons $= 8$
  - $\Big\} = 12$

- After SSB
  - 1 real scalar $= 1$
  - 3 massive gauge bosons $= 9$
  - 1 massless " " 2
  - $\Big\} = 12$

(5) The Higgs boson

The Lag for the scalar in the unitary gauge

$$\mathcal{L} = (D^\mu \phi_0)^\dagger (D_\mu \phi_0) - V(\phi_0) :$$

with $(D^\mu \phi_0)^\dagger (D_\mu \phi) = \frac{1}{2}(\partial^\mu h)(\partial_\mu h) + M_W^2 W_\mu^\dagger W^\mu (1 + \frac{h}{v})^2$

$$+ \frac{1}{2} M_Z^2 Z_\mu Z^\mu (1 + \frac{h}{v})^2$$

$$V(\phi) = \mu^2 (v+h)^2 + \frac{\lambda}{4}(v+h)^4$$

$$= h(\underbrace{\cancel{\mu^2 v + \lambda v^3}}_{0}) + h^2 \underbrace{\left(\frac{\mu^2}{2} + 6\lambda v^2\right)}_{= -\mu^2 = |\mu^2|} + \lambda h^3 v + \frac{\lambda}{4} h^4$$

$$+ \frac{1}{2}\mu^2 v^2 + \frac{\lambda}{4} v^4 = -\frac{\lambda v^4}{4} \equiv V_{min}$$

So altogether the Lag for the higgs boson $h(x)$

$$\mathcal{L}_h = \frac{1}{2}\left[(\partial_\mu h)^2 + 2\mu^2 h^2\right] \to$$ massive real scalar with mass $m_h = \sqrt{-2\mu^2} = \sqrt{2\lambda}\, v$.

$+ h\, W_\mu^+ W^\mu \frac{2M_W^2}{v} \equiv g M_W \to$ vertex

$-i\frac{2M_W^2}{v} g^{\alpha\beta} = -i g M_W g^{\alpha\beta}$

$+ h^2\, W_\mu^+ W^\mu \frac{M_W^2}{v^2} \equiv \frac{g^2}{4} \longrightarrow$ vertex

$-i\frac{g^2}{2} g^{\alpha\beta}$

$+ h\, Z_\mu Z^\mu \frac{M_Z^2}{v} \equiv 2g\frac{M_Z}{c_W} \longrightarrow$ vertex

$-i\frac{2M_Z^2}{v} g^{\alpha\beta} = -i\frac{g}{c_W} M_Z g^{\alpha\beta}$

$+ h^2\, Z_\mu Z^\mu \frac{M_Z^2}{2v^2} \longrightarrow$ vertex

$-i\frac{g_Z^2}{2} g^{\alpha\beta}$

$- h^3 \lambda v \longrightarrow$ vertex

$-i 6\lambda v = -i3\sqrt{\frac{\lambda}{2}}\, m_h$

$- h^4 \frac{\lambda}{4} \longrightarrow$ "

$-i6\lambda$

How about fermion mases?

We saw that we could not construct a fermion mass term because

$$m_f \bar{f} f = m_f (\bar{f}_L f_R + \bar{f}_R f_L) \quad \text{not } SU(2)_L \text{ gauge int}$$

But now we have another $SU(2)_L$ doublet Φ

so we can build

$$\mathcal{L}_{YUK} = -\lambda_e \bar{L}_L \Phi e_R - \lambda_d \bar{Q}_L \phi d_R$$
$$- \lambda_u \bar{Q}_L (i\sigma_2 \phi) u_R \quad + h.c$$

Which is $SU(2)_L$ gauge inv.

in the unitary gauge

$$\lambda_e \bar{L}_L \Phi_0 e_R + h c = \frac{\lambda e}{\sqrt{2}} (\bar{\nu}_{eL}, \bar{e}_L) \begin{pmatrix} 0 \\ v+h \end{pmatrix} e_R + hc$$

$$= \underbrace{\frac{\lambda_e v}{\sqrt{2}}}_{\equiv m_e} \underbrace{(\bar{e}_L e_R + h.c)}_{\bar{e} \cdot e} + \underbrace{\frac{\lambda_e}{\sqrt{2}}}_{\equiv \frac{m_e}{v}} h \underbrace{(\bar{e}_L e_R + h.c)}_{\bar{e} e.}$$

As we have seen

$$M_W = \frac{1}{2} g v$$

$$M_Z = \frac{1}{2} \frac{g v}{\cos\theta_W}$$

$$\frac{G_F}{\sqrt{2}} = \frac{g}{8 M_W^2}$$

$$\alpha = \frac{e^2}{4\pi} = g^2 \frac{\sin^2\theta_W}{4\pi}$$

$$C^f_{L,R} = \pm\frac{1}{2} - Q^f \sin^2\theta_W$$

Redundant ways to measure 3 independent quantities
$g, v, \cos\theta$ i.e
So we can check loop contributions which depend on m_H.

i.e

corrects the relation betwem M_W and M_Z
$\Rightarrow$ measuring both very precisely we can get
indirect inf of m_H.