Chapter 9

"Weak interactions"

1) Weak decays and Parity violation: V–A charge current
2) W-boson
3) Tests: π-decay, μ-decay, β decay
4) Fermionic mixing matrix
5) CP violation
6) Neutral currents: Z boson

1) Weak decays

the lifetime of process like

$$\Delta^{++} \to p \pi^{+} \quad \tau_{\Delta} \sim 10^{-23} s$$

$$\pi^{0} \to \gamma\gamma \quad \tau_{\pi^0} \sim 10^{-6} s$$

can be understood as mediated by strong and em interactions since at lowest order

$$\tau \sim \frac{1}{\Gamma} \sim \frac{1}{|M|^2} \sim \frac{1}{g^4} \Rightarrow \tau_{strong} \ll \tau_{em}$$

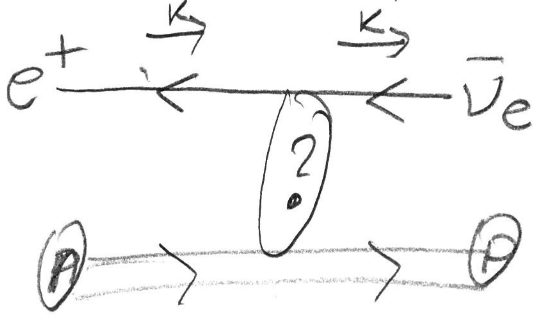
But there are decays such as

$$\left.\begin{array}{ll} \Sigma^{+} \to p \, \pi & \tau_{\Sigma} \sim 10^{-9} s \\ \pi^{\pm} \to \mu^{\pm} \overset{(-)}{\nu}_{\mu} & \tau_{\pi} \sim 10^{-8} s \\ n \to p \, e^{-} \bar{\nu}_{e} & \tau \sim 800 s \end{array}\right\} \text{much longer lifetimes}$$

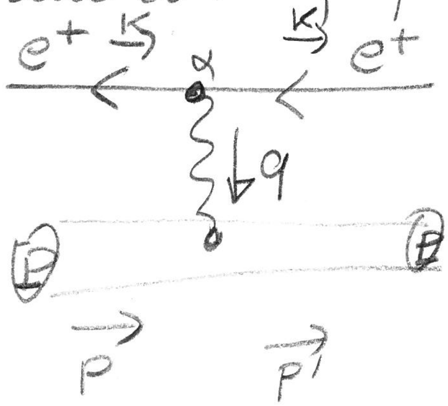
but $m_{\Sigma} \sim m_{\Delta}$ why $\tau_{\Sigma} \gg \tau_{\Delta}$

To explain why these decays were much slower a new and much weaker interaction was proposed.

If we want to build an amplitude for $n \to p e^- \bar{\nu}_e$ (let us take $n e^+ \to p \bar{\nu}_e$



We can take $e^+ p \to e^+ p$ in QED as guide



$$\mathcal{M}^{QED}_{e^+p \to e^+p} = -\frac{e^2}{q^2}\, j^{\alpha}_{e^+}\, j_{\alpha,had}$$

$$j^{\alpha}_{e} = \bar{v}_e(k)\gamma^{\alpha} v_e(k')$$

$$j_{\alpha,had} = \bar{u}_{Prot}(p')\gamma_{\alpha}\left[F_1(q^2) + \dots\right] u_{Prot}(p)$$

Form factor $F_1(0) = 1$

With this analogy we could write

$$\mathcal{M}_{n \to p e^- \bar{\nu}_e} = G\,[\bar{u}_e \gamma^{\alpha} v_{\nu}][\bar{u}_p \gamma_{\alpha} u_n]$$

must have dimension E^{-2} (like $\frac{1}{q^2}$ in QED)

Experimentally $G \sim 10^{-5}\,\mathrm{GeV}^{-2} \Rightarrow$ range of interaction

range $\sim \frac{1}{\sqrt{G}} \lesssim 10^{-1}$ fm while range in QED ∞

Also unlike QED in β-decay the vertex changed electric charge. We call this "weak charged interaction"

In homeworks we showed that the current $\bar{\psi}\gamma^{\mu}\psi$ conserves parity and charge conjugation.

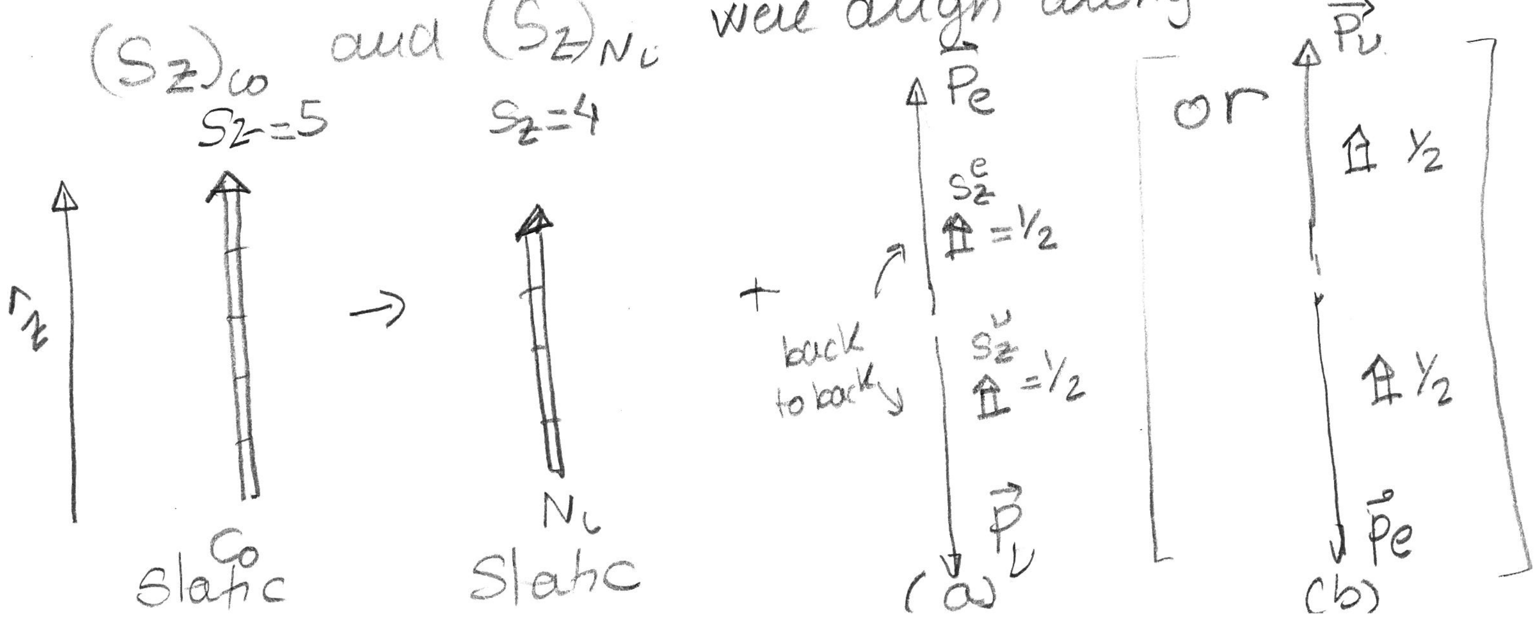
To verify if P was conserved in β decay Lee and Yang proposed an experiment. Wu made it

the proposal was to measure the direction of emission of e^- in Cobalt β decay

$$^{60}Co \longrightarrow {}^{60}Ni + e^- + \bar{\nu}_e$$

spin S	5	4	1/2	1/2

with respect to an external magnetic field. $\vec{B}$
to define a direction ($\hat{B} = \hat{z}$) to ensure that $(S_z)_{Co}$ and $(S_z)_{Ni}$ were align along $\hat{z}$ direction



So $J_Z = 5 = 4 + \frac{1}{2} + \frac{1}{2}$ is conserved as it should

there are two possible configurations allowed by angular momentum conservations

(b) e^- is emitted in direction of spin of the nucleus

helicity of $\bar{\nu}_e$ is $+1 \Rightarrow v_\nu \simeq v_L$ chirally
" " e^- is $-1 \Rightarrow u_e \simeq u_L$ left-handed

(a) e^- is emitted opposite of spin of nucleus

helicity of $\bar{\nu}_e$ is $-1 \Rightarrow v_{\bar{\nu}} \simeq v_R$ chirally
" " " e^+ is $+1 \Rightarrow u_e \simeq u_R$ right-handed

under Parity

$$\text{spin } \vec{S} \xrightarrow{\mathcal{P}} \vec{S} \quad \text{(spin is a pseudo vector)}$$
$$\vec{p}_e \longrightarrow -\vec{p}_e$$
$$\vec{p}_\nu \longrightarrow -\vec{p}_\nu$$
$$(a) \longrightarrow (b)$$

So if Parity is conserved in β decay we should observe 50% times (a) and 50% (b).
But the experiment observed 100% (b) and never (a)

$\Rightarrow$ Parity is <u>maximally violated</u> in β decay

Alway (b) $\Rightarrow$ only chirally left-handed spinors are included in the decay. So the amplitude is

$$(\bar{u}_e)_L \gamma^\alpha v_L = \overline{P_L u_e}\, \gamma^\alpha (P_L v_\nu) \quad \text{with } P_L = \tfrac{1}{2}(1-\gamma^5)$$

since $\overline{P_L u_e} = u_e^+ P_L \gamma^0 = u_e^+ \gamma^0 P_R = \bar{u}_e P_R$

$$(\bar{u}_e)_L \gamma^\alpha v_L = \bar{u}_e P_R \gamma^\alpha P_L v_\nu = \bar{u}_e \gamma^\alpha \overset{P_L^2}{\overset{\shortparallel}{P_L}} v_\nu$$

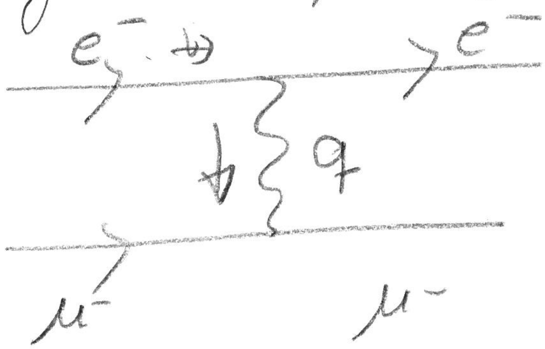
$\Rightarrow$ weak interaction charge vertex for leptons is $\gamma^\alpha(1-\gamma^5)$

We will assume the same holds for hadronic vertex

$\Rightarrow$ this interactions violates both Parity and Charge Conjug.

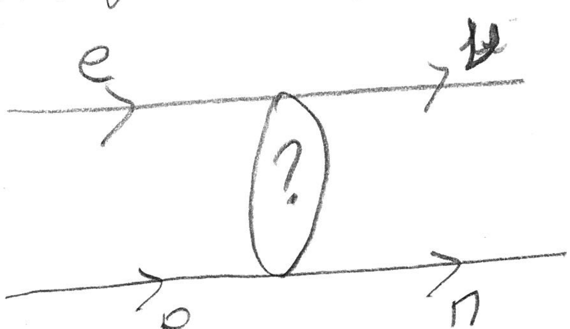
2 W-boson

Again comparing with QED



$$\mathcal{M}_{QED} = -\frac{e^2}{q^2}\, \bar{u}_e' \gamma^\alpha u_e\, \bar{u}_\mu' \gamma_\alpha u_\mu$$

And for charge weak int



$$\mathcal{M}_{weak} = -\frac{G_F}{\sqrt{2}}\, \bar{u}_n \gamma^\alpha (1-\gamma_5) u_p\, \bar{u}_\nu \gamma_\alpha (1-\gamma_5) u_e$$

must have dimension E^{-2}

So we observe that

$\sigma^{QED}_{e\mu \to e\mu}$ grows at $q^2 \to 0$

while $\sigma^{weak}_{ep \to \nu n}$ → constant at $q^2 \to 0$

also QED vertex does not change electric charge

weak charge " change " "

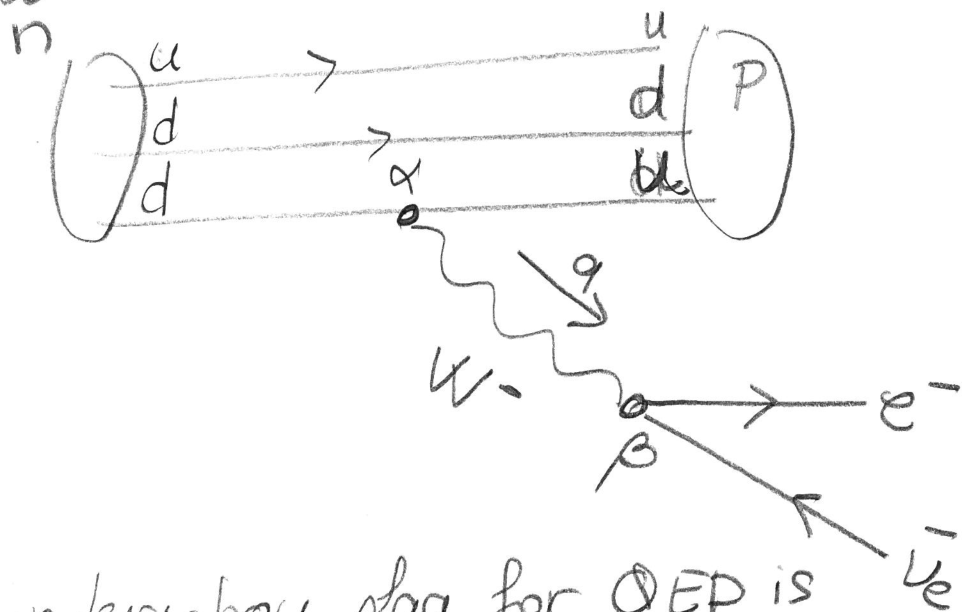
We are going to build some phenomenological model for these charge weak int

Assumptions of model:

- only left-handed fermions interact.
- charged weak interactions of hadrons can be understood as interactions of their constituent quarks
- the interaction is due to exchange of a charge vector $W^{\pm\mu}$
- the $W^{\pm}$ has a mass M_W

$n \to p\, e^- \bar{\nu}_e$

So at lowest order β^- decay is due



the interaction Lag for QED is

$$\mathcal{L}_{QED,int} = -e Q_f \overline{\psi}_f \gamma^\mu \psi_f A_\mu$$

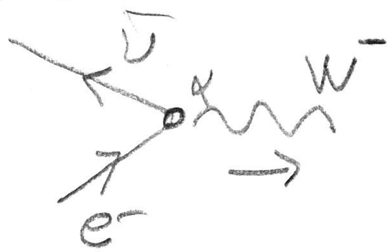
So we can write the phenomenological Lag for weak charged interactions. Let us call W_i^μ the complex vector field for the $W^{\pm}$ particle coupling constant →

$$\mathcal{L}_{\substack{\text{int. weak cc}\\ \text{leptons}}} = \frac{-g_W}{\sqrt{2}}\left[\bar{\psi}_e \gamma^\mu \frac{(1-\gamma_5)}{2}\psi_{\nu_e} W_\mu + \bar{\psi}_{\nu_e}\gamma^\mu \frac{(1-\gamma_5)}{2}\psi_e W_\mu^+\right]$$

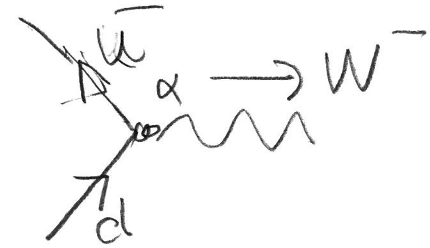
$$\mathcal{L}_{\substack{\text{int weak cc}\\ \text{quarks}}} = \frac{-g_W}{\sqrt{2}}\left[\bar{\psi}_d \gamma^\mu \frac{(1-\gamma_5)}{2}\psi_u W_\mu + \bar{\psi}_u\gamma^\mu \frac{(1-\gamma_5)}{2}\psi_d W_\mu^+\right]$$

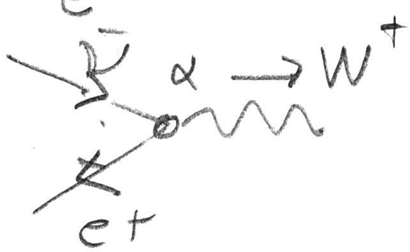
this is for the 1st generation fermions
(for 2nd and 3d generation we can make replicas for these Lag)

So the basic vertices are

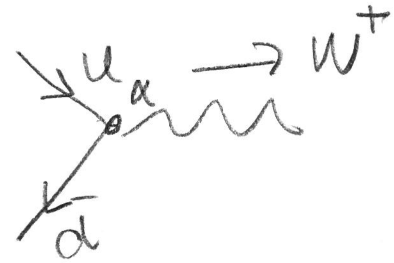


$$-i\frac{g_W}{\sqrt{2}}\gamma^\alpha\frac{(1-\gamma_5)}{2}$$

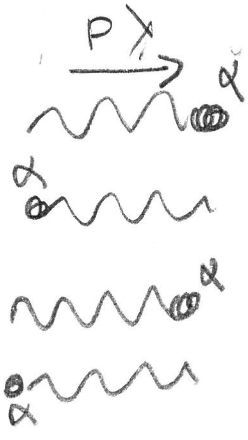




$$-i\frac{g_W}{\sqrt{2}}\gamma^\alpha\frac{(1-\gamma_5)}{2}$$



and For the W



W^- incoming $\quad \epsilon^\alpha_\lambda(p)$

outgoing $\quad \epsilon^{\alpha *}_\lambda(p)$

W^+ incoming $\quad \epsilon^{\alpha *}_\lambda(p)$

outgoing $\quad \epsilon^\alpha_\lambda(p)$

$\lambda = 0, \pm 1$

3 physical polarisations because $M_W \neq 0$

W propagator

$$\frac{-i\left(g_{\alpha\beta} - \frac{q_\alpha q_\beta}{M_W^2}\right)}{q^2 - M_W^2 + i\epsilon}$$

So when $|q| \ll M_W \Rightarrow$ W propagator $\sim \frac{i g_{\alpha\beta}}{M_W^2}$

$\Rightarrow$ constant at $q^2 \to 0$

unlike photon (or gluon propag)

$\Rightarrow$ short range of weak interactions

with these FR we can write the amplitude

$$\mathcal{M}_{\beta\,decay} = \frac{g_W^2}{2}\left(\bar{u}_u \gamma^\alpha \frac{(1-\gamma_5)}{2} u_d\right)_{\substack{\text{in } p \\ n}} \frac{g_{\alpha\beta} - \frac{q_\alpha q_\beta}{M_W^2}}{q^2 - M_W^2}\left[\bar{u}_e \gamma^\beta \frac{(1-\gamma_5)}{2} v_\nu\right]$$

$\left(\begin{matrix}\text{assuming} \\ q^2 \ll M_W^2\end{matrix}\right) \simeq \frac{-g_W^2}{8M_W^2}\left[\bar{u}_u \gamma^\alpha (1-\gamma_5) u_d\right]\left[\bar{u}_e \gamma_\alpha (1-\gamma_5) v_\nu\right]$

lifetimes

From β decay we find $G_F \sim 10^{-5}\,\text{GeV}^{-2}$

$$\Rightarrow \quad \frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8M_W^2}$$

$\Rightarrow$ if $g_W \sim e \Rightarrow g_W^2 \sim 4\pi\alpha$

$$M_W \sim \sqrt{\frac{4\pi\alpha\sqrt{2}}{8G_F}} \sim 50\,\text{GeV}$$

So $M_W \gg m_n, m_p$ which justifies $q^2 \ll M_W^2$

$\sim m_n^2, m_p^2$

3 Tests

1. $\pi^- \to \ell \bar{\nu}_e$:

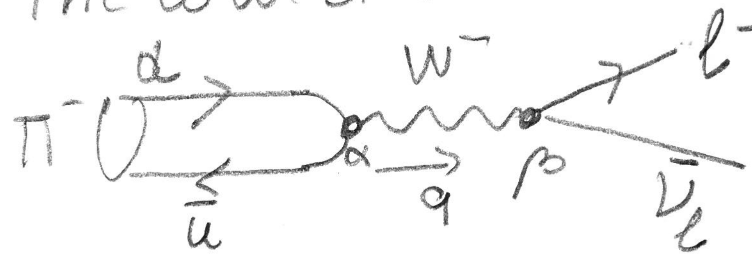
Looking at this decay of the $\pi^-(\equiv \bar{u}d)$ we find that $\pi^- \to \mu^- \bar{\nu}_\mu$ 99.98% of times

$\pi^- \to e^- \bar{\nu}_e$ $\sim 10^{-4}$ of times

But $m_\mu >> m_e \Rightarrow \pi^- \to \mu^- \bar{\nu}_\mu$ has much less phase space. So why π^- decays mostly to the heavier lepton.

This can be understood as due to the $\frac{(1-\gamma_5)}{2}$ in the vertex.

The lowest order diagram for this decay



$$\mathcal{M} = \frac{g_W^2}{2}\left[\bar{v}_u \gamma^\alpha \frac{(1-\gamma_5)}{2} u_d\right]_\pi \left[\frac{g_{\alpha\beta} - \frac{q_\alpha q_\beta}{M_W^2}}{q^2 - M_W^2}\right]\left[\bar{u}_e \gamma^\beta \frac{(1-\gamma_5)}{2} v_{\nu_e}\right]$$

since $q^2 = (p_\nu + p_e)^2 = m_\pi^2 << M_W^2$

$$\mathcal{M} \simeq \frac{-g_W^2}{8 M_W^2}\left[\bar{v}_u \gamma^\alpha (1-\gamma_5) u_d\right]_\pi \left[\bar{u}_e \gamma_\alpha (1-\gamma_5) v_\nu\right]$$

π^- is a pseudoscalar $\Rightarrow S_\pi = 0 \Rightarrow \vec{J}_{init} = 0$

$\Rightarrow \vec{J}_{final} = \vec{S}_\ell + \vec{S}_\nu = 0$

the π^- is at rest $\Rightarrow \vec{P}_{int} = 0 = \vec{P}_{final} = \vec{P}_e + \vec{P}_\nu$

$\Rightarrow \vec{P}_e = -\vec{P}_\nu$

So helicity of $\ell^- = \dfrac{\vec{S}_e \cdot \vec{P}_e}{|\vec{P}_e|} = \dfrac{\vec{S}_\nu \cdot \vec{P}_\nu}{|\vec{P}_\nu|}$ = helicity of $\bar{\nu}_e$

We know that charged weak interactions only couple chirally left-handed $\bar{\nu}_e$ and ℓ^-

If we neglect m_ν chiral left $\bar{\nu}_e \Rightarrow$ helicity $\bar{\nu}_e = +\frac{1}{2}$

$\Rightarrow$ helicity of $\ell^- = 1/2$

But if we neglect $m_\ell \Rightarrow$ helicity $1/2$ $\ell^- \equiv$ right-handed chiral ℓ^-

$\Rightarrow$ if $m_e = 0$ $\quad M_{\pi^- \to e^- \bar{\nu}_e} = 0$

If we keep the mass

$$M_{\pi^- \to e^- \bar{\nu}} \propto m_e$$

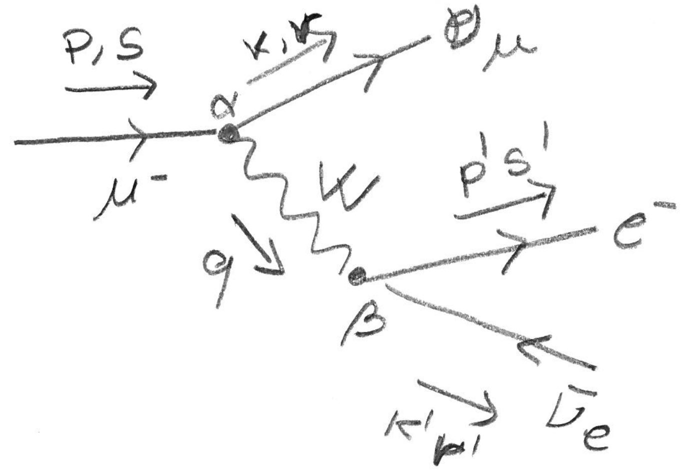
So $\quad \Gamma(\pi^- \to e^- \bar{\nu}_e) \propto m_\ell^2 (m_\pi^2 - m_\ell^2)$ from phase space integral

so we predict $\dfrac{\Gamma(\pi^- \to e^- \bar{\nu}_e)}{\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)} = \dfrac{m_e^2}{m_\mu^2} \dfrac{(m_\pi^2 - m_e^2)}{(m_\pi^2 - m_\mu^2)} = 1.23 \times 10^{-4}$

experimentally $(1.23 \pm 0.002) \times 10^{-4}$

$\mu^- \to e^- \nu_\mu \bar{\nu}_e$

the tree level diagram



$k = (\omega, \vec{k})$
$k' = (\omega', \vec{k}')$

$p = k + k' + p'$

Neglecting $m_e, m_{\nu_e}, m_{\nu_\mu}$ and since $m_\mu^2 \ll m_W^2$

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} \left[\bar{u}^r(k)\gamma^\alpha(1-\gamma_5)u^s(p)\right]\left[\bar{u}^{s'}(p')\gamma_\alpha(1-\gamma_5)v^{r'}(k')\right]$$

So $$|\overline{\mathcal{M}}|^2 = \frac{1}{2}\sum_{\substack{rs\\r's'}} |\mathcal{M}|^2$$

$$= \frac{G_F^2}{4}\,\mathrm{Tr}\Big[\underbrace{\not{k}\gamma^\alpha(1-\gamma_5)\not{p}\gamma^\beta(1-\gamma_5)}_{2\not{k}\gamma^\alpha\not{p}\gamma^\beta(1-\gamma_5)}\Big]\,\mathrm{Tr}\Big[\underbrace{\not{p}'\gamma_\alpha(1-\gamma_5)\not{k}'\gamma_\beta(1-\gamma_5)}_{2\not{p}'\gamma_\alpha\not{k}'\gamma_\beta(1-\gamma_5)}\Big]$$

$$= G_F^2 \times 16\left[k^\alpha p^\beta + k^\beta p^\alpha - g^{\alpha\beta}(kp) + i\epsilon^{\mu\nu\alpha\beta}k_\mu p_\nu\right]$$
$$\left[p'_\alpha k'_\beta + p'_\beta k'_\alpha - g_{\alpha\beta}(k'p') + i\,\epsilon_{\rho\sigma\alpha\beta}\,p'^\rho k'^\sigma\right]$$

$4i\epsilon^{\alpha\beta\rho\sigma} = \mathrm{Tr}(\gamma^\alpha\gamma^\beta\gamma^\rho\gamma^\sigma\gamma^5)$

$$16G_F^2\Big[2(kp')(k'p) + \cancel{2(kk')(pp')} - \underbrace{\epsilon^{\mu\nu\alpha\beta}\epsilon_{\rho\sigma\alpha\beta}}_{-2(g^\mu{}_\rho g^\nu{}_\sigma - \cancel{g^\mu{}_\sigma g^{\nu}{}_\rho})} k_\mu p_\nu p'^\rho k'^\sigma\Big]$$

$$= 64\,G_F^2\,(kp')(k'p)$$

In μ rest frame

$$k'p = m\omega'$$

$$kp' = \frac{1}{2}\left[(k+p')^2 - \underset{m_\nu^2=0}{k^2} - \underset{m_e^2=0}{p'^2}\right] = \frac{1}{2}(k'-p)^2 = \frac{1}{2}p^2 - k'p$$

$$= \frac{1}{2}(m^2 - 2m\omega)$$

$$\Rightarrow |\bar{M}|^2 = 32\, G_F^2\, m^2 \omega' (m - 2\omega') \leftarrow \text{independent of directions of outgoing particles}$$

the decay width

$$\Gamma_\mu = \frac{1}{2m}\int |\bar{M}|^2 \, d\Phi_3$$

with

$$d\Phi_3 = (2\pi)^4 \delta^4(p - p' - k - k')\frac{d^3p'}{(2\pi)^3 2E'}\frac{d^3k}{(2\pi)^3 2\omega}\frac{d^3k'}{(2\pi)^3 2\omega'}$$

we can use that $d^3k = 2\omega\, \delta(k^2)\, d^4k$

and integrate d^4k with δ^4

$$d^3\Phi_3 = \frac{1}{(2\pi)^5}\frac{1}{4E'\omega'}\delta\left[(p-k'-p')^2\right] d^3p'\, d^3k'$$

let us define θ as the angle between $\vec{p}'$ and $\vec{k}'$

$$\Rightarrow (p - k' - p')^2 = m^2 - 2mE' - 2m\omega' + 2(k'p')$$

$$= m^2 - 2mE' - 2m\omega' + 2\omega' E'(1 - \cos\theta)$$

of internal consistency of the data over various momentum and angular regions lead to larger uncertainty and a preliminary result of $\rho = 0.747 \pm 0.005$.

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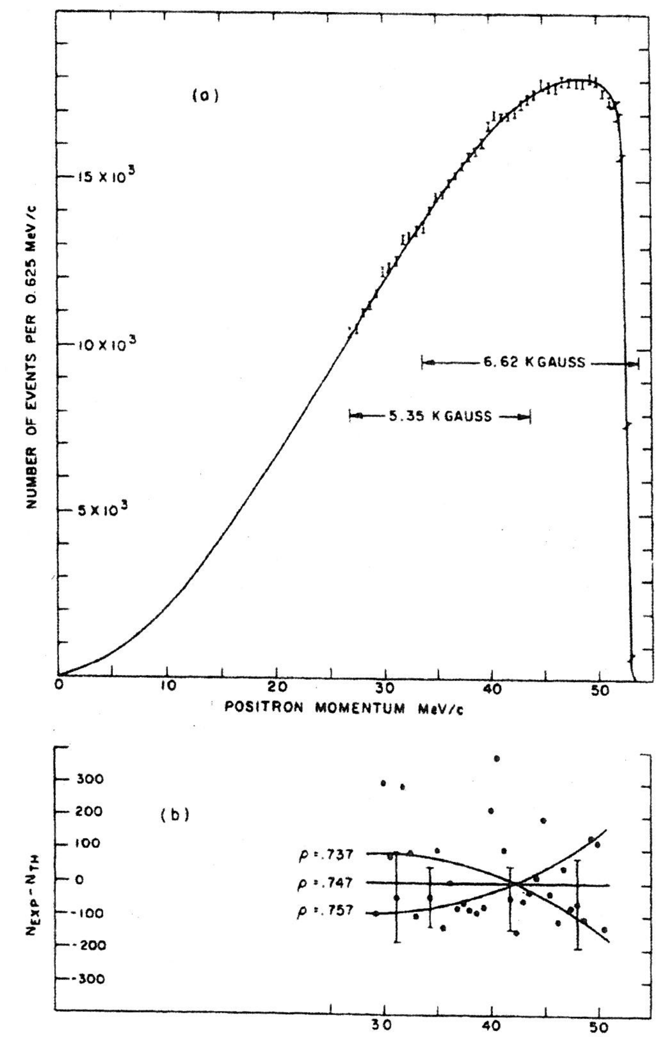


$\sim \dfrac{d\Gamma}{dE'}$

FIG. 4. (a) Experimental points for magnetic-field settings, normalized to the overlap region. The solid line is the theoretical spectrum for $\rho = 0.75$. The Michel spectrum,[4]

$$\rho(x)dx = \tfrac{1}{2}\{12x^2 - 12x^3 + \rho[(32/3)x^3 - 8x^2]\}dx,$$

where x is the positron momentum divided by its maximum value, has been corrected for internal radiation, bremsstrahlung, and ionization loss. (b) The deviation of experimental points from the best-fit theoretical curve for $\rho = 0.747$, showing typical experimental errors for four points. Curves for $\rho = 0.737$ and 0.757 are shown for comparison.

So all angular variables in $d^3p'\,d^3k'$ can be integrated except θ

$$d^3p'd^3k' = \omega'^2 E'^2 (4\pi)(2\pi)\, d\cos\theta\; d\omega' dE'$$

$$\Rightarrow d\Phi_3 = \frac{1}{(2\pi)^3}\,\frac{E'\omega'}{2}\,\delta\left[m^2 - 2mE' - 2m\omega' + 2\omega' E'(1-\cos\theta)\right]$$
$$d\omega' dE' d\cos\theta$$

$$= \frac{1}{(2\pi)^3}\,\frac{1}{4}\,\delta\left[\cos\theta - \frac{m^2 - 2mE' - 2m\omega'}{2\omega' E'}\right] d\omega' dE' d\cos\theta$$

we have to integrate in $dE'\,d\omega'$ making sure that

$$-1 \leq \frac{m^2 - 2mE' - 2m\omega'}{2\omega' E'} \leq 1 \Rightarrow \begin{cases} \frac{1}{2}m - E' \leq \omega' \leq \frac{m}{2} \\ 0 \leq E' \leq \frac{m}{2} \end{cases}$$

$$\text{So}\quad \frac{d\Gamma}{dE'} = \int_{\frac{1}{2}m - E'}^{m/2} d\omega' \,\frac{1}{(2\pi)^3}\,\frac{1}{4}\,\frac{1}{2m}\, 32 G_F^2 m^2 \omega'(m - 2\omega') d\omega'$$

$$= \frac{G_F^2}{12\pi^3}\, m^2 E'^2 \left(3 - \frac{4E'}{m}\right) \longrightarrow \text{Figure}$$

and the total decay width

$$\Gamma = \int_0^{m/2} dE' \frac{d\Gamma}{dE'} = \frac{G_F^2 m^5}{192\pi^3} \Rightarrow$$

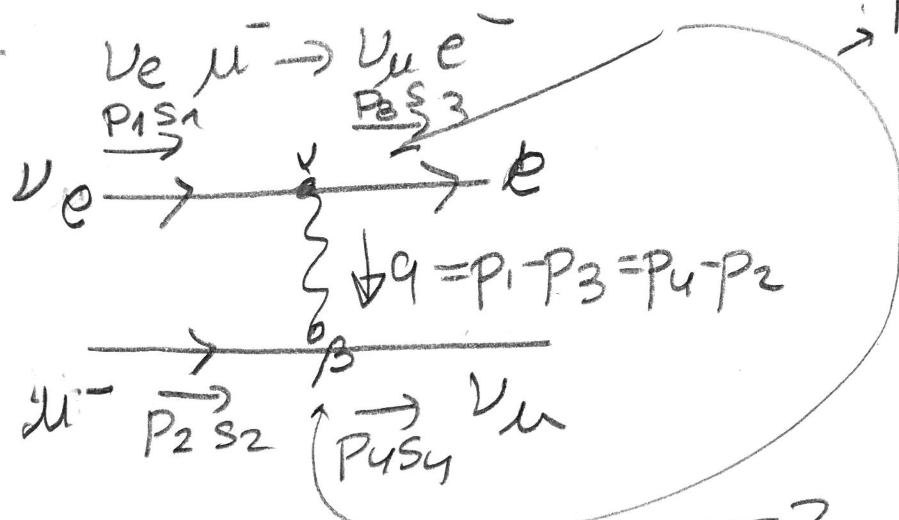
Using $\underset{\text{measured}}{\tau_\mu} = 2.2\times10^{-6}\,s$

$m_\mu = 105.7$ MeV

$\Rightarrow G_{F\,\mu dec} = 1.16632\times10^{-5}\ \text{GeV}^{-2}$

• $\nu_e \mu^- \to \nu_\mu e^-$ vs $\bar{\nu}_\mu \mu^- \to \bar{\nu}_e e^-$

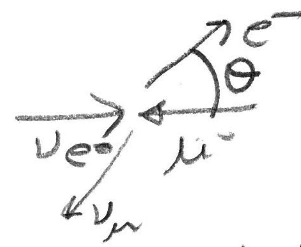
For $\nu_e \mu^- \to \nu_\mu e^-$



Notice vertex does not change generations (so ν_e with e, ν_μ " μ)

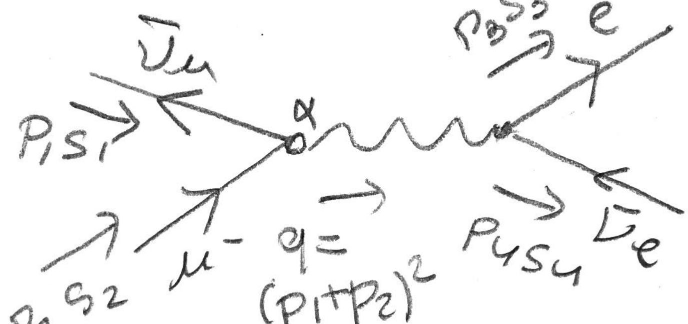
For $q^2 \ll M_W^2$ we get $\overline{|M|^2} = 16\, G_F^2\, s^2$

$$\Rightarrow \left.\frac{d\sigma}{d\Omega}\right|_{COM} = \frac{G_F^2}{4\pi^2}\, s$$



independent of θ

For $\bar{\nu}_\mu \mu^- \to \bar{\nu}_e e^-$



Same amplitude with

$p_1 \to -p_4$

$p_4 \to -p_1$

$\Rightarrow s = (p_1+p_2)^2 \to (p_2-p_4)^2 = t$

So in this case

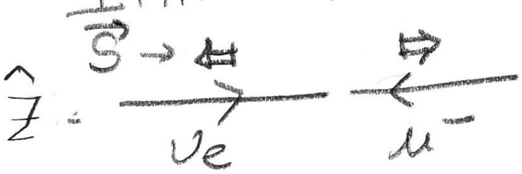
$$\left.\frac{d\sigma}{d\Omega}\right|_{COM} = \frac{G_F^2}{4\pi^2}\frac{t^2}{s} = \frac{G_F^2}{16\pi^2} s\,(1-\cos\theta)^2$$

so in this case $\frac{d\sigma}{d\Omega}(\theta=0)=0$

the difference can be understood from the V−A form of the vertex

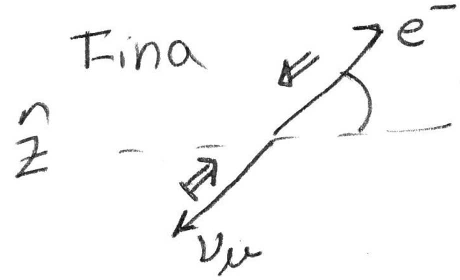
- In $\nu_e \mu^- \to e^- \nu_\mu$ neglecting masses all spinors are left-handed $\Rightarrow$ all 4 fermions have negative helicity

Initial



$\Rightarrow \vec{J}_{in}=0 \quad J_z^{int}=0$

Final



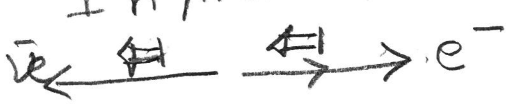
$\Rightarrow \vec{J}_{fin}=0$ for any θ

- In $\bar{\nu}_\mu \mu^- \to e^- \bar{\nu}_e$ for $\bar{\nu}_e$ and $\bar{\nu}_\mu$ left handed $\Rightarrow$ positive helicity

Initial

$\Rightarrow J_z^{inital} = +1$

In final state for $\theta=0$ (e^- same direction as $\bar{\nu}_\mu$)



$\Rightarrow J_z^{final} = -1 \Rightarrow$ no possible ang. momentum conservation

Similarly one can obtain for nucleus β decay

(a bit more complicated because $m_p \simeq m_n$ so we must keep the masses and p, n are inside nucleus). Ie for

$$^{14}O \longrightarrow {}^{14}N + e^+ + \nu_e$$

8p 7p

6n 7n

(So basic process is $p \to n\, e^+ \nu_e$ which can only happen in nucleus so $E_p > E_n$ despite $m_p < m_n$

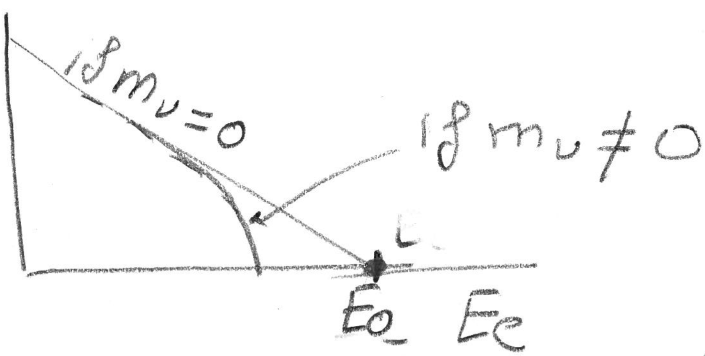
One gets

$$\frac{dn}{dE_e} = \frac{G_F^2}{\pi^3} E_e \underbrace{|\vec{p}_e|}_{=\sqrt{E_e^2 - m_e^2}} (E_0 - E_e)^2$$ with $E_0 = M_O - M_N$

Notice that if $m_\nu = 0$ $E_0 = E_e + E_\nu \Rightarrow E_e = E_0 - E_\nu \leq E_0 - m_\nu$

measuring E_e we can reconstruct

$$\sqrt{\frac{dN}{dE_e}\frac{1}{E_0}\frac{1}{|\vec{p}_e|}}$$



this is the Kurie plot and this end of the energy spectrum of the e^- in β decay is the most sensitive laboratory test of m_ν. We want E_0 to be as low as possible. Best case is tritium $^3H \to {}^3He + e^- + \bar{\nu}_e$

Best limit from Katrin experiment $\Rightarrow m_\nu \lesssim 0.45\,\mathrm{eV}$

From data for ^{16}O decay comparing with prediction we find

$$G_F\Big|_{\substack{\text{nuclear}\\ \beta\text{ decay}}} = 1.136 \times 10^{-5}\,\mathrm{GeV}^{-2}$$

and from μ decay we got

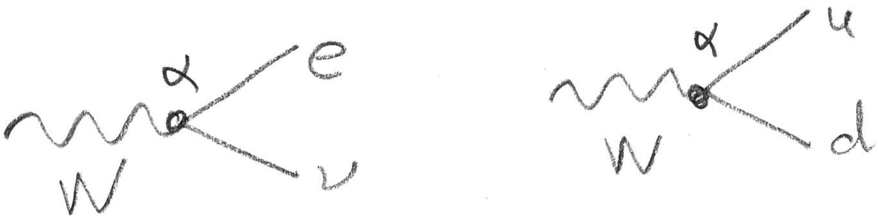
$$G_F\Big|_{\mu\,\text{decay}} = 1.166 \times 10^{-5}\,\mathrm{GeV}^{-2}$$

$$\Rightarrow \frac{G_F\big|_{\beta\,\text{dec}}}{G_F\big|_{\mu\,\text{dec}}} \simeq 0.975$$

So the coupling of weak charged interactions of quarks and leptons are "almost" the same. In fact the couplings are the same and this small difference can be explained as we will see in sec 4.

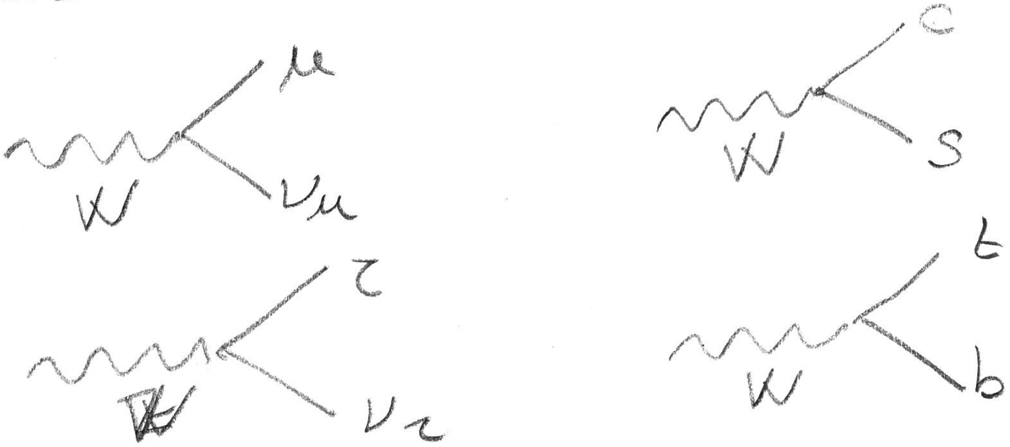
4 Fermionic mixing

We have written the vertices for the 1st generation



all $\dfrac{+ig_W}{\sqrt{2}} \; \gamma^\alpha \dfrac{(1-\gamma_5)}{2}$

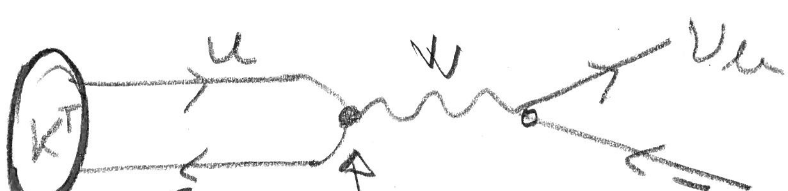
and we can make replicas for the other generations



Doing this we assume that the W does not couple to fermions of different generations.

However we observe the decay $K^+ (\equiv u\bar{s}) \longrightarrow \mu^+ \nu_\mu$

⇒



vertex with u (≡ 1st generation
and s (≡ 2nd "

To understand this we need to clarify how we assign flavour $\overset{= \text{generation}}{\text{to}}$ the fermions

We have two observables to define what we call

u vs c vs t since they have the same $Q = 2/3$

or d vs s vs b $\quad Q = 1/3$

Let us call $\begin{pmatrix} u' \\ d' \end{pmatrix} \begin{pmatrix} c' \\ s' \end{pmatrix} \begin{pmatrix} t' \\ b' \end{pmatrix}$ the quark generations

which do not mix in the weak charge vertex (so you can put a prime in the vertices in previous page).

Let us call u, c, t to the quarks with $Q = 2/3$ and mass ordered as $m_u < m_c < m_t$

and d, s, b to the quarks with $Q = -1/2$ a

$m_d < m_s < m_b$

We can always chose $u = u'$, $c = c'$, $t = t'$

But in general $\quad$ unitary 3×3 matrix

$$\begin{pmatrix} d \\ s \\ b \end{pmatrix} = U_{3\times3} \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}$$

Let us consider 2 generations. In this case

$$U_{2\times2} = \begin{pmatrix}\cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c\end{pmatrix}$$

Cabbibo angle

So $d = \cos\theta_c\, d' + \sin\theta_c\, s'$

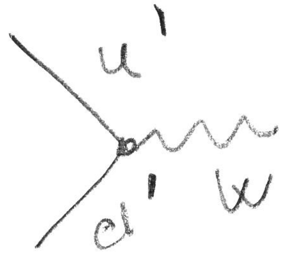
$s = -\sin\theta_c\, d' + \cos\theta_c\, s'$

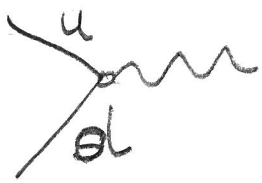
non relativistic mass eigenstate

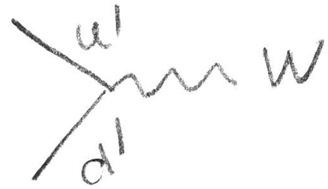
So $K^+ = u\bar{s} \neq u'\bar{s}'$

$\pi^+ = u\bar{d} \neq u'\bar{d}'$

So 

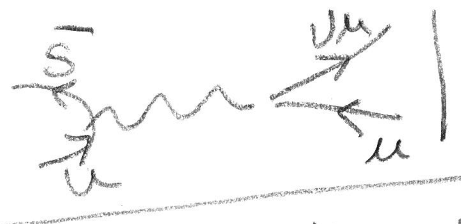
$\equiv -\sin c$ 

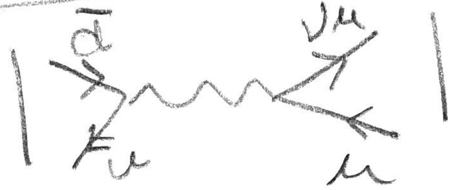


$\equiv \cos\theta_c$ 

So comparing

$$\frac{\Gamma(K^+\to\mu^+\nu_\mu)}{\Gamma(\pi^+\to\mu^+\nu_\mu)} \propto$$

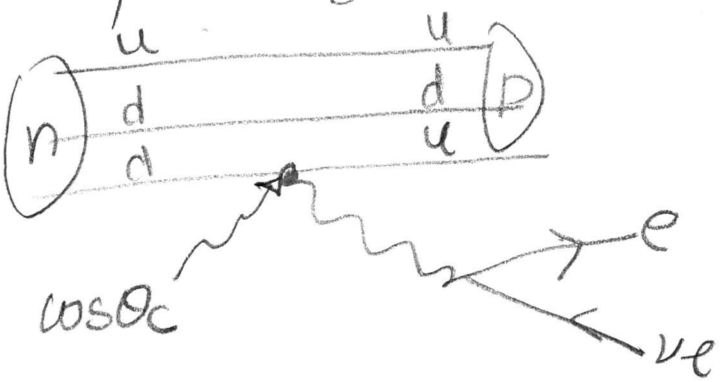
$\Big|$  $\Big|^2$

$\Big|$  $\Big|^2$

$$\sim \frac{\sin^2\theta_c}{\cos^2\theta_c}$$

From data on this ratio $\sin\theta_c = 0.12$ ($\theta_c \simeq 13^\circ$)

So in β-decay



$$\Rightarrow G_F^{\beta\,\mathrm{dec}} = \cos\theta_c \, G_F^{\mu\text{-dec}}$$
$$\cos\theta_c = \sqrt{1-0.22^2} \simeq 0.975$$

For 3 generations $U_{3\times3} \equiv V_{CKM}$ so

$$\mathcal{L}^{\text{quarks}}_{\text{charge weak}} = -\frac{g_W}{\sqrt{2}} \sum_{ij} V_{ij}\, \bar{u}_i \, \gamma^\alpha \frac{(1-\gamma_5)}{2} d_j W^+_\alpha + \text{h.c.}$$

mass states

V_{CKM} is a 3×3 unitary matrix

- a complex 3×3 matrix has ~~9~~ 18 parameters
  9 real (modulus) and 9 phases
- unitarity $\Rightarrow \sum_j |U_{ij}|^2 = 1$ for $i=1,2,3 \Rightarrow$ 3 real conditions
  $\sum_i U_{ij} U^*_{jk} = 0$ for $j \neq k \Rightarrow$ 3 complex conditions

$\Rightarrow$ unitary 3×3 matrix has 3 real parameters
and $9-3=6$ phases

of those 6 phases 5 can be just absorbed in the wave function of the quark $\Rightarrow$ 1 physical phase

We can parametrize the matrix as $S_{ij} \equiv \sin\theta_{ij}$, $C_{ij} \equiv \cos\theta_{ij}$

$$V_{CKM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & C_{23} & S_{23} \\ 0 & -S_{23} & C_{23} \end{pmatrix} \begin{pmatrix} C_{13} & 0 & S_{13}e^{-i\delta} \\ 0 & & \\ S_{13}e^{i\delta} & 0 & C_{13} \end{pmatrix} \begin{pmatrix} C_{12} & S_{12} & 0 \\ -S_{12} & C_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

and from the experiments we find

$$\left.\begin{array}{l} \theta_{12} \sim \theta_C \sim 13^\circ \\ \theta_{22} \sim 2.3^\circ \\ \theta_{13} \sim 0.2^\circ \end{array}\right\} \Rightarrow \text{hierarchical mixing}$$

$$\delta \sim 68^\circ$$

How about leptons? In principle we can do the same. But if ν's are massless $m_{\nu_1} = m_{\nu_2} = m_{\nu_3} = 0$ so we can always define $\nu_e = \nu'_e$, $\nu_\mu = \nu'_\mu$, $\nu_\tau = \nu'_\tau$

So $U_{LEP} = I_{3\times 3}$ by default

$\Rightarrow$ lepton flavour are not mixed

But from oscillation experiments we know $m_{\nu_1} \neq m_{\nu_2} \neq m_{\nu_3} \neq 0$ so

So in the mass basis

$$\mathcal{L}^{\text{leptons}}_{\text{chargeweak}} = -\frac{g_W}{\sqrt{2}} \sum_{ij} U^{LEP}_{ij} \, \bar{\nu}_i \gamma^\alpha \frac{(1-\gamma_5)}{2} \, l^-_j W^+_\alpha$$

$+ h.c$

and we have measured

$\theta^{LEP}_{12} \sim 30^\circ$

$\theta^{LEP}_{23} \sim 45\%$ $\Rightarrow U^{LEP}$ is very different from V_{CKM}

$\theta^{LEP}_{13} \sim 8\%$ Why??

δ_{LEP} not ~~k~~nown yet

5 CP violation

In homeworks we saw that

$$\mathcal{L} = c\,\overline{\Psi}_a\,\gamma^\mu(1-\gamma_5)\Psi_b V_\mu + h.c.$$

violates both $\mathcal{C}$ and $\mathcal{P}$ but it would conserve $\mathcal{CP}$

To verify experimentally if $\mathcal{CP}$ is conserved in weak charge interactions one studies the system of neutral kaons $K^0 \equiv d\bar{s}$ $\quad \overline{K^0} = s\bar{d}$

they are pseudo-scalar mesons $\Rightarrow \underset{\text{spin}}{S=0}$ and $P=-1$

and how they decay on pions $\pi^+ = (u\bar{d})$ $\pi^- = (d\bar{u})$ $\pi^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$

which are also pseudo scalar mesons

So under $\mathcal{P}$

wave func

$$\phi_{K^0} \longrightarrow -\phi_{K^0}$$
$$\phi_{\overline{K^0}} \longrightarrow -\phi_{\overline{K^0}}$$

under $\mathcal{C}$

$$\phi_{K^0} \longrightarrow \phi_{\overline{K^0}}$$
$$\phi_{\overline{K^0}} \longrightarrow \phi_{K^0}$$

So if we make the combination

$$\phi_{K_1} \equiv \frac{1}{\sqrt{2}}(\phi_{K^0} + \phi_{\overline{K^0}}) \xrightarrow{\mathcal{CP}} -\phi_{K_1}$$

$$\phi_{K_2} = \frac{1}{\sqrt{2}}(\phi_{K^0} - \phi_{\overline{K^0}}) \longrightarrow \phi_{K_2}$$

So K_1 and K_2 are eigenstates of $\mathcal{CP}$

For pions

$$\phi_{\pi^+\pi} \xrightarrow{\mathcal{P}} (-)^2 (-)^\ell \phi_{\pi^+\pi} \overset{\ell=0}{=} \phi_{\pi^+\pi^-}$$

$$\phi_{\pi^+\pi^-} \xrightarrow{\mathcal{C}} (-)^\ell \phi_{\pi^+\pi} = \phi_{\pi^+\pi^-}$$

$$\Rightarrow \phi_{\pi^+\pi^-} \xrightarrow{\mathcal{CP}} \phi_{\pi^+\pi^-}$$

$$\phi_{\pi^+\pi^-\pi^0} \xrightarrow{\mathcal{P}} (-)^3 (-)^\ell \phi_{\pi\pi^-\pi^0} \overset{\ell=0}{=} -\phi_{\pi^+\pi\pi^0}$$

$$\xrightarrow{\mathcal{C}} \phi_{\pi^+\pi^-\pi^0}$$

$$\text{so } \phi_{\pi^+\pi^-\pi^0} \xrightarrow{\mathcal{CP}} -\phi_{\pi^+\pi^-\pi^0}$$

So if $\mathcal{CP}$ is conserved in weak decays

K_1 should decay only in $\pi^+\pi^-\pi^0$
" " " " $\pi^+\pi^-$
K_2 "

and because of phase space $\tau_1 > \tau_2$

So if we start with a beam of K_0 and put a detector at d with $c\tau_1 \ll d \ll c\tau_2$ at that distance all the K_2 component of K_0 will have decayed and only K_1 is left and we should only observe decay in 3 π's.

But the experiment found about 1 per mill of times a decay ~~in~~ in $\pi^+\pi^-$.

So the Long lived state was not purely K_1 but it had some small component of K_2

$$|K_{LONG}\rangle \simeq |K_1\rangle + \epsilon |K_2\rangle \quad \text{with } \epsilon \simeq 2\times10^{-3}$$

$\Rightarrow$ CP is violated in weak charge interactions

Also it is observed that

$$\Gamma(K_{LONG} \to e^+ \pi^- \nu_e) \neq \Gamma(K_{LONG} \to e^- \pi^+ \bar{\nu}_e)$$

by about 3 per mill.

Notice that CP allows to define in absolute terms what we call e^+ vs e^- (e^+ is the one produced 3‰ times more in K_{LONG} decay)

We say that CP violation allows for particle-antiparticle asymmetry. CP violation is one of the conditions that Sakharov found that were required to generate the matter-antimatter in the Universe

(the other two are B violation and departure of thermal equilibrium) where in $\mathcal{L}_{\text{charge weak}}$ is encoded the possibility of CP violation.

For quarks

$$\mathcal{L} = -\frac{g_W}{\sqrt{2}} \Big\{ \sum_{ij} V_{ij}^{CKM} \, \bar{u}_i \gamma^\alpha \frac{(1-\gamma_5)}{2} d_j W_\alpha^+ + V_{ij}^{CKM*} \, \bar{d}_j \gamma^\alpha \frac{(1-\gamma_5)}{2} u_i W_\alpha \Big\}$$

under $\mathcal{C}$ and $\mathcal{P}$

$$\mathcal{P} (\bar{u}_i \gamma^\alpha (1-\gamma_5) d_j) \mathcal{P}^{-1} = \bar{u}_i \gamma^\beta (1+\gamma_5) d_j \, \underset{\begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}}{P_\beta^\alpha}$$

and $\mathcal{C} (\bar{u}_i \gamma^\beta (1+\gamma_5) d_j) \mathcal{C}^{-1} = -\bar{d}_j \gamma^\beta (1-\gamma_5) u_i$

$$P W_\alpha^+ \mathcal{P}^{-1} = P_\alpha^\gamma W_\gamma^+ \qquad \mathcal{C} W_\alpha^+ \mathcal{C}^{-1} = -W_\alpha$$

$$\mathcal{P} W_\alpha P^{-1} = P_\alpha^\gamma W_\alpha \qquad \mathcal{C} W_\alpha \mathcal{C}^{-1} = -W_\alpha^+$$

$$\Rightarrow \mathcal{C}\mathcal{P} (\bar{u}_i \gamma^\alpha (1-\gamma_5) d_j W_\alpha^+) (\mathcal{C}\mathcal{P})^{-1}$$

$$= \bar{d}_j \gamma^\beta (1-\gamma_5) u_i W_\gamma \underbrace{P_\beta^\alpha P_\alpha^\gamma}_{\delta^\gamma_\beta}$$

$$= \bar{d}_j \gamma^\alpha (1-\gamma_5) u_i W_\alpha$$

So

$$\Rightarrow \mathcal{CP}\,\mathcal{L}\,(\mathcal{CP})^{-1} = -\frac{g_W}{\sqrt{2}}\left\{\sum_{ij} V_{ij}^{CKM}\,\bar{d}_j\gamma^\alpha\frac{(1-\gamma_5)}{2}u_i W_\alpha + V_{ij}^{CKM*}\,\bar{u}_i\gamma^\alpha\frac{(1-\gamma_5)}{2}d_j W_\alpha^+\right\}$$

$$= \mathcal{L} \Leftrightarrow V_{ij}^{CKM} = V_{ij}^{CKM*}$$

So $\cancel{CP}$ is possible in the SM because V^{CKM} can have physical complex phase which can be (and it is) non-zero.

For $m_\nu = 0$ the phase in V_{CKM} is the only source of $\cancel{CP}$ in the SM.

Now we know that $m_\nu \neq 0$ and there is also a $U_{LEP} \neq I_{3\times3}$ and U_{LEP} can also contain a complex phase $\Rightarrow \cancel{CP}$ also with the leptons $\Rightarrow$ new mechanism to generate the matter-antimatter asymmetry.

There is a huge experimental program to determine the $\cancel{CP}$ for the leptons

6 Weak neutral currents

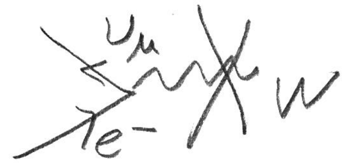
So far we have discussed weak interactions in which there is a change of electric charge ±1 between the fermions in the vertex. ≡ weak charge current.

In 1973 some processes including ν's were observed

$$\nu_\mu e^- \to \nu_\mu e^-$$

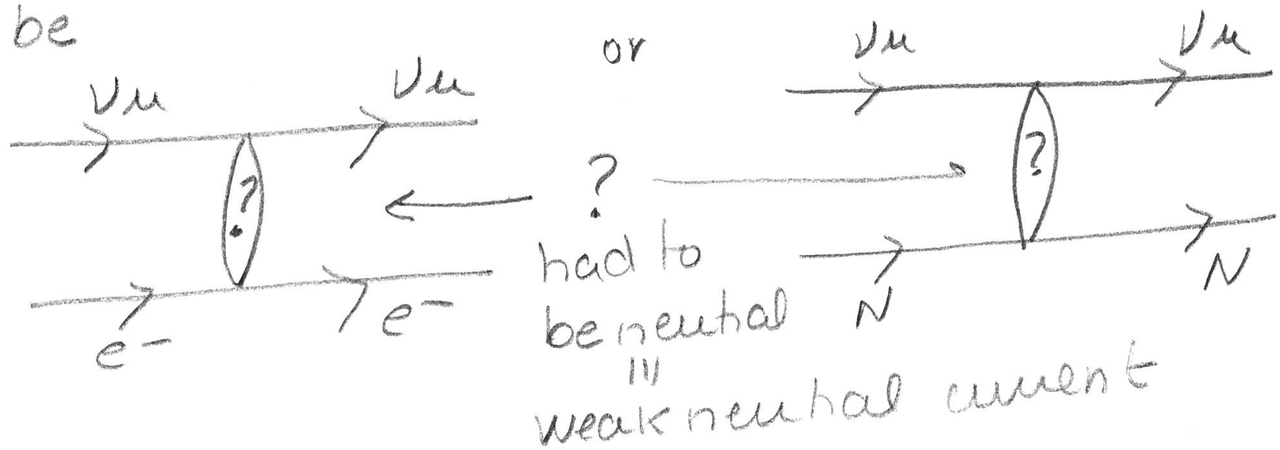
$$\nu_\mu \underset{p, or\, n}{\underset{\equiv}{N}} \to \nu_\mu N$$

Since ν's do not have electric charge nor colour these processes had to be due to weak int.
But weak interactions of leptons do not change flavour (in limit $m_\nu \to 0$) so



Furthermore in $\nu_\mu N \to \nu_\mu N$ there is no charge lepton

So the amplitudes for these processes had to be



the observed cross sections and many other similar could explain with pure neutral current

amplitudes

same G_F as μ dec

constant $\simeq 1$

$$\mathcal{M}(\nu_\mu \underset{\substack{|||\\ e^- \text{ or } q}}{f} \longrightarrow \nu_\mu f) = -\frac{8G_F}{\sqrt{2}}\rho \left[\bar{u}_{\nu_\mu}\gamma^\alpha \frac{(1-\gamma_5)}{2} u_{\nu_\mu}\right] \times \left[\bar{u}_f \gamma_\alpha \left(C_L^f \frac{(1-\gamma_5)}{2} + C_R^f \frac{(1+\gamma_5)}{2}\right) u_f\right]$$

with

f	C_L^f	C_R^f	Q_f
ν	$1/2$	0	0
e	-0.27	0.23	-1
u	0.35	-0.15	$2/3$
d	-0.43	0.07	$-1/3$

$$\Longrightarrow \quad C_R^f = -Q_f X \qquad C_L^{\nu, u} = \frac{1}{2} - Q_f X \qquad C_R^{e, d} = -\frac{1}{2} - Q_f X$$

with $X \simeq 0.22 - 0.23$

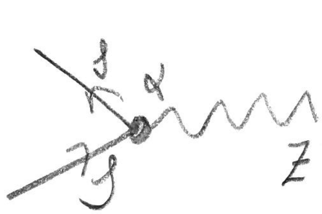
These amplitudes can be understood in terms of an effective interaction with

- interaction is mediated by a neutral vector $\equiv Z^\mu$
- Z has a mass M_Z

So the effective Lagrangian

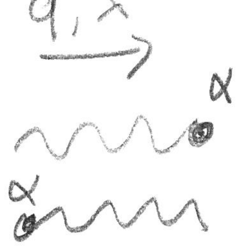
$$\mathcal{L}^{NC} = - \sum_{f = u, d, \nu, e} \underbrace{g_Z}_{\text{coupling constant}} \bar{\Psi}_f \gamma^\alpha \Big(C_L^f \underset{\frac{1-\gamma_5}{2}}{P_L} + C_R^f \underset{\frac{1+\gamma_5}{2}}{P_R} \Big) \Psi_f \, Z_\alpha$$

. So the FR for this interaction



$$-i g_Z \gamma^\alpha \left[C_L^f \frac{(1-\gamma_5)}{2} + C_R^f \frac{(1+\gamma_5)}{2} \right]$$

- Z^μ external lines



Z incoming $\quad \epsilon_\lambda^\alpha(q)$

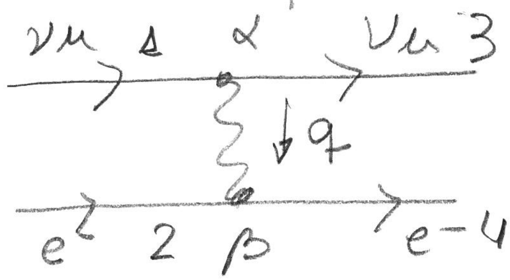
Z outgoing $\quad \epsilon_\lambda^{\alpha *}(q)$

since $M_Z \neq 0$
$\lambda = \pm 1, 0$

- Z propagator

$$-i \, \frac{g_{\alpha\beta} - \frac{q_\alpha q_\beta}{M_Z^2}}{q^2 - M_Z^2}$$

For example



$$\mathcal{M} = g_Z^2 \left[\bar{u}^3_{\nu_\mu} \gamma^\alpha c_L^\nu \frac{(1-\gamma_5)}{2} u^1_{\nu_\mu} \right] \frac{g_{\alpha\beta} - \frac{q_\alpha q_\beta}{M_Z^2}}{q^2 - M_Z^2} \left[\bar{u}^4_e \gamma^\beta \left(c_L^e \frac{(1-\gamma_5)}{2} + c_R^e \frac{(1+\gamma_5)}{2} \right) u^2_e \right]$$

$$\xrightarrow{q^2 \ll M_Z^2} -\frac{g_Z^2}{M_Z^2} \left[\gamma^\alpha \qquad \right] \left[\gamma_\alpha \qquad \right]$$

$$\Rightarrow \frac{8 G_F}{\sqrt{2}} \rho = \frac{g_Z^2}{M_Z^2}$$ and in CC $\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8 M_W^2}$

$$\Rightarrow \frac{g_W^2}{M_W^2} \rho = \frac{g_Z^2}{M_Z^2}$$ and since $\rho \simeq 1$

$$\frac{M_W^2}{M_Z^2} \sim \frac{g_W^2}{g_Z^2}$$ $\Rightarrow$ the masses and couplings of the weak CC and NC come in the same ratio (?)

So there is no generation mixing in NC weak interaction ≡ GIM mechanism
in Glashow, Iliopoulos and Maiami

Notice that for this to happen we need to consider full generations (1 up and one down quark per generation).

But when NC were discovered we only have observed 3 quarks (u, d, s). So the observation of no flavour changing NC would not be easily explained.

GIM realized that they could explain it if the postulated the existence of a 4th quark with $Q = Q_u = +\frac{2}{3}$.

In 1974 the J/ψ (≡ meson made of these 4th "charm" quark and antiquark $c\bar{c}$) was discovered!