Midterm practice problems

1. A very high-energy cosmic ray neutron can collide with the cosmic microwave background (CMBR) photons to become a Δ resonance of mass $M_\Delta = 1.232$ GeV. Assuming that all CMBR photons have the same energy, $E = kT$, where $k = 1.38 \times 10^{-23}$ J K^{-1} is the Boltzman constant and $T = 2.725$ K
   (a) Determine the minimum neutron energy for which the Δ can to be produced
   (b) Knowing that the lifetime of the neutron is approximately 900 s, from how far could that neutron with that minimum energy come (you can approximate the neutron mass by 1 GeV)?

2. A photon accelerator is made from electrons of energy E_0 "bouncinig" off a laser photon of energy E_l with $E_0 \gg E_l \gg m_e$ (so you can set $m_e = 0$ from the begining).
   (a) Find the energy of the outgoing photon as a function of the angle θ between the initial and final photon directions and the initial energies E_0 and E_l.
   (b) For what angle θ does the photon emerges with most energy?

3. During almost 2 decades the accelerator LEP at CERN in Switzerland (http://www.cern.th) collided beams of electrons and positrons with the aim of studying in detail our understanding of the particle interactions and to test the Standard Model. In the first phase (called LEPI) the energy of the beams was tunned to produce a large amount of Z^0 bosons at rest to obtain a precise determination of its mass and its decay properties. Answer the following questions
   (a) We know now the mass of the Z^0 What was the energy of the colliding beams to produce the Z^0's at rest?
   (b) Z^0 quickly decays, about 6% of the times in $\tau^+\tau^-$. How far do the τ travel before they also decay? (look for τ mass and lifetime in PDG
   (c) Subsequently τ^+ decays into $\pi^+ + \bar{\nu}_\tau$ and τ^- decays into $\mu^- + \nu_\tau + \bar{\nu}_\mu$. Give the expression (analytical) of the energy of the π^+ (in the LAB frame) as a function of its angle θ relative to the τ^+ direction.

4. In the scattering $A + B \to C + D$ we define the Mandelstam variables

$$s = (p_A + p_B)^2 = (p_C + p_D)^2 \qquad t = (p_A - p_C)^2 = (p_B - p_D)^2$$
$$u = (p_A - p_D)^2 = (p_B - p_C)^2$$

   where p_i is the four-momentum of particle i.
   a) Show that they verify

$$u + s + t = m_A^2 + m_B^2 + m_C^2 + m_D^2$$

   b) Show that in the COM system and for $s \gg m^2_{A,B,C,D}$

$$t \simeq -\frac{s}{2}(1 - \cos\theta)$$
$$u \simeq -\frac{s}{2}(1 + \cos\theta)$$

   where θ is the scattering angle in the COM

5. The spin, parity and charge conjugation quantum numbers for some mesons are:

	s	P	C		s	P	C
π^0	0	-1	+1	ω	1	-1	-1
$\pi^\pm, K^\pm$	0	-1		b_1^0	1	+1	-1
ρ^0	1	-1	-1	a_1^0	1	1	1

The ϕ is a neutral meson with $s = 1$ with which decays as

$$\phi \to \rho_0 \, \pi_0$$

via strong interactions.
(a) What can you say about the orbital angular momentum of the $\rho_0 \, \pi_0$
(b) What is the P and C of the ϕ according to this decay (justify the answer)?
(c) What is the spin and orbital angular momentum of $q\bar{q}$ constituents of the ϕ

6. Explain the quantum numbers of the ω, a_1^0 and b_1^0 mesons given above in terms of the quantum number of its constituents

7. The η is a pseudoscalar meson. Explain why the decay $\eta \to \pi^+\pi^-$ is forbidden by em and strong interactions while the decay $\eta \to \gamma\gamma$ is possible.

8. Using the spinors in the chiral representation show that
(a) $[u^+(\vec{p})]^P \equiv \gamma^0 u^+(-\vec{p}) = A u^-(\vec{p})$
(b) $[u^-(\vec{p})]^P \equiv \gamma^0 u^-(-\vec{p}) = B u^+(\vec{p})$
(c) $[u^+(\vec{p})]^C \equiv i\gamma^2\gamma^0 \overline{u^+(\vec{p})}^T = D v^+(\vec{p})$
(d) $[u^-(\vec{p})]^C \equiv i\gamma^2\gamma^0 \overline{u^-(\vec{p})}^T = E v^-(\vec{p})$

where A, B, D, E are just phases. Reason what these relations mean.

9. Draw the Feynman diagrams and write the Feynman amplitudes in QED for
(a) $e^-e^- \to e^-e^-$
(b) $e^-\gamma \to e^-\gamma$
(c) $e^+\gamma \to e^+\gamma$
(d) $e^-e^- \to \mu^-\mu^-$