Theoretical Particle Physics: Assignment # 1
Due 02/26/08

1 Rewrite the standard model electroweak lagrangian for the 1st generation:
\[
\mathcal{L} = \frac{1}{4} W^i_{\mu\nu} W^{i\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \\
+ Q_L \gamma^\mu \left( i \partial_\mu - g_S^\mu W^k_{\mu} - g' B^1_{\mu} \right) Q_L \\
+ \bar{u} R \gamma^\mu \left( i \partial_\mu - g' B^2_{\mu} \right) u_R \\
+ \bar{d} R \gamma^\mu \left( i \partial_\mu + g' B^2_{\mu} \right) d_R \\
+ \bar{L} L \gamma^\mu \left( i \partial_\mu - g' B^1_{\mu} \right) L + e R \gamma^\mu (i \partial_\mu + g' B^2_{\mu}) e_R \\
+ \left( i \partial_\mu - g' B^1_{\mu} \right) \phi^\dagger (L - \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2) \\
- (\lambda^u Q_L (i \sigma_2) \phi^u u_R + \lambda^d Q_L \phi^d R + \lambda^e L \phi e_R + h.c.)
\]

after spontaneous symmetry breaking $\phi = \frac{1}{\sqrt{2}} \left( \begin{array}{c} 0 \\ v + h \end{array} \right)$ (in the unitary gauge) in terms of the physical states

\[
W_{\mu}^\pm = \frac{1}{\sqrt{2}} (W_{\mu}^1 \mp i W_{\mu}^2) \quad Z^\mu = \cos \theta_w W_{\mu}^3 - \sin \theta_w B^\mu \quad A^\mu = \sin \theta_w W_{\mu}^3 + \cos \theta_w B^\mu \quad h = e_L + e_R \quad u = u_L + u_R \quad d = d_L + d_R \quad \nu
\]

In particular:

1.1 show that if the field $A^\mu$ corresponds to the photon then $\tan \theta_w = \frac{\sqrt{2}}{g}$ and $\frac{1}{\tan \theta_w} = \frac{1}{g^2} + \frac{1}{g^2}$

1.2 Obtain the expressions for the masses of all the particles and in particular show

\[
M_W = \frac{1}{2} g v \quad M_Z = \frac{1}{2 \cos \theta_w} g v \quad m_h^2 = 2 \lambda v^2 \quad m_f = \frac{\sqrt{2}}{\lambda} v
\]

1.3 If we write the terms of the coupling of the $Z^0$ to any of the fermions as

\[
\mathcal{L}_{Z^0f} = - \frac{g_Z}{2} \bar{f} \gamma^\mu (g_V^f + \gamma^5 g_A^f) f Z_\mu \equiv - \frac{g_Z}{4} \bar{f} \gamma^\mu (R_f (1 + \gamma^5) + L_f (1 - \gamma^5)) f Z_\mu
\]

with $g_Z = g / \cos \theta_w$ obtain $g_V^f$, $g_A^f$, $L_f$ and $R_f$ for $f = u, d, e$ and $\nu$.

2.1 Show that if the SM contained several Higgs multiplets with weak isospins $T_i$ and hypercharges $Y_i$ whose neutral states acquire a vev $v_i$, then the $\rho$ parameter

\[
\rho = \frac{M_W^2}{\cos^2 \theta_w M_Z^2} = \frac{\sum_i v_i^2 [T_i (T_i + 1) - Y_i^2]}{2 \sum_i Y_i^2 v_i^2}
\]

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(where we have used that in our notation $Q = Y + T_3$)

2.2 Assume that the SM contains a doublet with vev $v_w$ and another scalar field with weak isospin $T$ and hypercharge $Y$ and vev $v$, show what bounds can be derived on the ratios of the vevs if we know that the $\rho$ parameter has been measured to be $0.01 \geq \rho - 1 \geq -0.03$. (For your amusement you can check Phys.Lett.B232:383,1989).