Theoretical Particle Physics: Assignment # 1

Due 02/26/08

1 Rewrite the standard model electroweak lagrangian for the 1st generation:

$$
\begin{aligned}
\mathcal{L} = & \tfrac{1}{4}W^i_{\mu\nu}W^{\mu\nu,i} - \tfrac{1}{4}B_{\mu\nu}B^{\mu\nu} \\
& + \overline{Q_L}\gamma^\mu\left(i\partial_\mu - g\tfrac{\sigma_k}{2}W^k_\mu - g'\tfrac{1}{6}B_\mu\right)Q_L \\
& + \overline{u_R}\gamma^\mu\left(i\partial_\mu - g'\tfrac{2}{3}B_\mu\right)u_R \\
& + \overline{d_R}\gamma^\mu\left(i\partial_\mu + g'\tfrac{1}{3}B_\mu\right)d_R \\
& + \overline{L_L}\gamma^\mu\left(i\partial_\mu - g\tfrac{\sigma_i}{2}W^i_\mu + g'\tfrac{1}{2}B_\mu\right)L_L + \overline{e_R}\gamma^\mu\left(i\partial_\mu + g'B_\mu\right)e_R \\
& + \left|\left(i\partial_\mu - g\tfrac{\sigma_i}{2}W^i_\mu - g'\tfrac{1}{2}B_\mu\right)\phi\right|^2 - \mu^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2 \\
& - \left(\lambda^u\overline{Q}_L(i\sigma_2)\phi^* u_R + \lambda^d\overline{Q}_L\phi d_R + \lambda^e\overline{L}_L\phi e_R + h.c.\right)
\end{aligned}
$$

after spontaneous symmetry breaking $\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}$ (in the unitary gauge) in terms of the physical states

$W^\pm_\mu = \frac{1}{\sqrt{2}}(W^1_\mu \mp iW^2_\mu)$ $\quad Z^\mu = \cos\theta_w W^3_\mu - \sin\theta_w B^\mu$ $\quad A^\mu = \sin\theta_w W^3_\mu + \cos\theta_w B^\mu$ $\quad h$
$e = e_L + e_R$ $\quad u = u_L + u_R$ $\quad d = d_L + d_R$ $\quad \nu$

In particular:

1.1 show that if the field A^μ corresponds to the photon then $\tan\theta_w = \frac{g'}{g}$ and $\frac{1}{e^2} = \frac{1}{g^2} + \frac{1}{g'^2}$

1.2 Obtain the expressions for the masses of all the particles and in particular show

$$M_W = \frac{1}{2}gv \qquad M_Z = \frac{1}{2\cos\theta_w}gv \qquad m_h^2 = 2\lambda v^2 \quad m_f = \frac{\lambda^f}{\sqrt{2}}v$$

1.3 If we write the terms of the coupling of the Z^0 to any of the fermions as

$$\mathcal{L}_{Z0,f} = -\frac{g_Z}{2}\bar{f}\gamma^\mu(g_V^f + \gamma^5 g_A^f)fZ_\mu \equiv -\frac{g_Z}{4}\bar{f}\gamma^\mu(R_f(1+\gamma^5) + L_f(1-\gamma^5)fZ_\mu \tag{1}$$

with $g_Z = g/\cos\theta_w$ obtain g_V^f, g_A^f, L_f and R_f for $f = u, d, e$ and ν.

2.1 Show that if the SM contained several Higgs multiplets with weak isospins T_i and hypercharges Y_i whose neutral states adquire a vev v_i, then the ρ parameter

$$\rho = \frac{M_W^2}{\cos^2\theta_w M_Z^2} = \frac{\sum_i v_i^2[T_i(T_i+1) - Y_i^2]}{2\sum_i Y_i^2 v_i^2} \tag{2}$$

(where we have used that in our notation $Q = Y + T_3$)

2.2 Assume that the SM contains a doublet with vev v_w and another scalar field with weak isospin T and hypercharge Y and vev v, show what bounds can be derived on the ratios of the vevs if we know that the ρ parameter has been meassured to be $0.01 \geq \rho - 1 \geq -0.03$. (For your amusement you can check Phys.Lett.B232:383,1989).