Theoretical Particle Physics: Assignment # 3

Due 03/27/08

1 Show that only for a massless fermion chirality eigenstates are helicity eigenstates.

2.1 Show that the mass matrix M in a Majorana mass term

$$-\mathcal{L}_{Maj} = -\frac{1}{2}\overline{\nu_{L,i}} M_{ij} (\nu_{L,j})^c + h.c \tag{1}$$

must be symmetric $M_{ij} = M_{ji}$.

2.2 Show that the Dirac mass term:

$$-\mathcal{L}_{Dir} = \overline{\nu_{L,i}} M^*_{Dij} \nu_{R,j} + h.c. \tag{2}$$

can be written as:

$$\begin{aligned} -\mathcal{L}_{Dir} &= \frac{1}{2}\left[\overline{\nu_{L,i}}(M_D^*)_{ij}\nu_{R,j} + \overline{(\nu_{R,i})^c}(M_D^\dagger)_{ij}(\nu_{L,j})^c\right] + h.c. = \\ & \frac{1}{2}\left[\overline{\nu_{R,i}}(M_D^T)_{ij}\nu_{L,j} + \overline{(\nu_{L,i})^c}(M_D)_{ij}(\nu_{R,j})^c\right] + h.c. \end{aligned} \tag{3}$$

3 Show that for $M_R >> M_D$ the see-saw mass matrix

$$M = \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix}$$

where M_D is a $3 \times m$ matrix and M_R is a $m \times m$ matrix, can be diagonalized to order ϵ^3 [where $\epsilon = \mathcal{O}(M_D/M_R)$] by a U matrix such that

$$U^T M U = \begin{pmatrix} m_{light} & 0 \\ 0 & m_{heavy} \end{pmatrix} \tag{4}$$

where

$m_{light} = -V_1^T M_D M_R^{-1} M_D^T V_1$ is 3×3 real and diagonal

$m_{heavy} = V_2^T \left[M_R + \frac{1}{2}(M_R^*)^{-1} M_D^\dagger M_D + \frac{1}{2} M_D^T M_D^* (M_R^*)^{-1}\right] V_2$ is m × m real and diagonal

Hint: Write $U = (\exp(iR))V$ with $R = \begin{pmatrix} 0 & S \\ S^\dagger & 0 \end{pmatrix}$ and $V = \begin{pmatrix} V_1 & 0 \\ 0 & V_2 \end{pmatrix}$. Assume that R is order ϵ and expand U to the require order in R, impose Eq.(4) and from the requirement that the off-diagonal sub-blocks vanish, find R.