Discovery of ν's

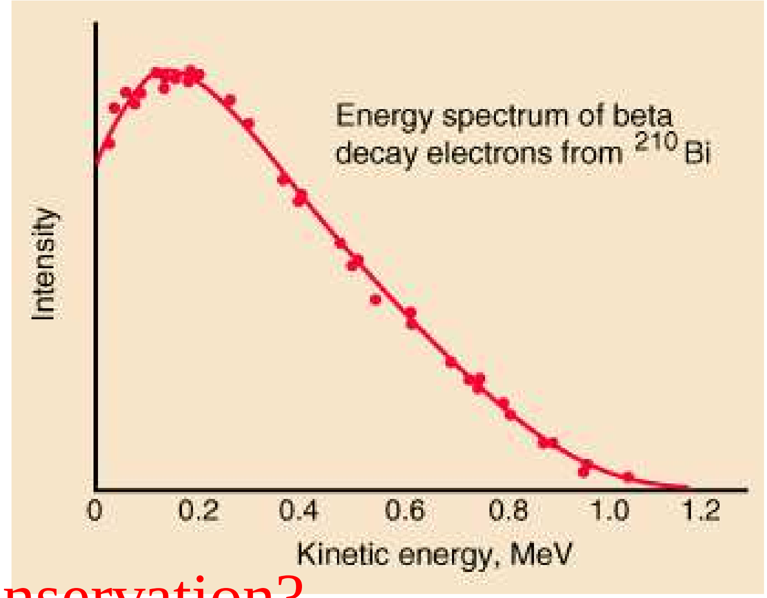
- At end of 1800's radioactivity was discovered and three types identified: α, β, γ

  β : an electron comes out of the radioactive nucleus.

- Energy conservation $\Rightarrow$ e^- should have had a fixed energy

$$(A,Z) \to (A,Z+1) + e^- \;\Rightarrow\; E_e = M(A,Z+1) - M(A,Z)$$

But 1914 James Chadwick showed that the electron energy spectrum is continuous



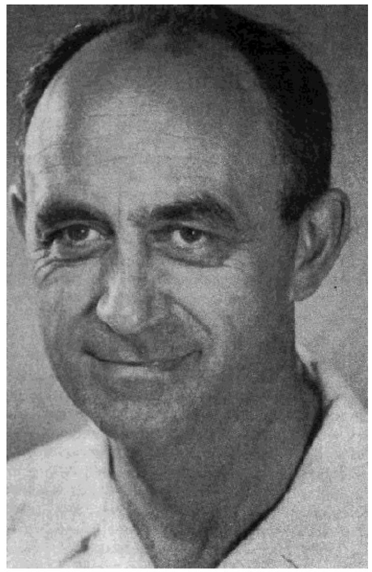
Do we throw away the energy conservation?

Discovery of ν's

• The idea of the neutrino came in 1930, when W. Pauli tried a desperate saving operation of "the energy conservation principle".

In his letter addressed to the "Liebe Radioaktive Damen und Herren" (Dear Radioactive Ladies and Gentlemen), the participants of a meeting in Tubingen. He put forward the hypothesis that a new particle exists as "constituent of nuclei", the "neutron" ν, able to explain the continuous spectrum of nuclear beta decay

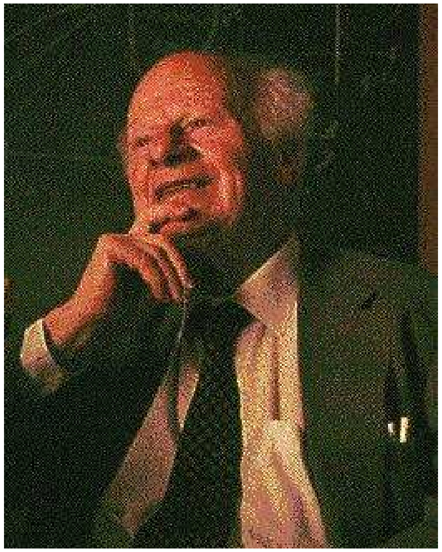
$$(A, Z) \to (A, Z+1) + e^- + \nu$$



• The ν is light (in Pauli's words: "the mass of the ν should be of the same order as the e mass"), neutral and has spin 1/2

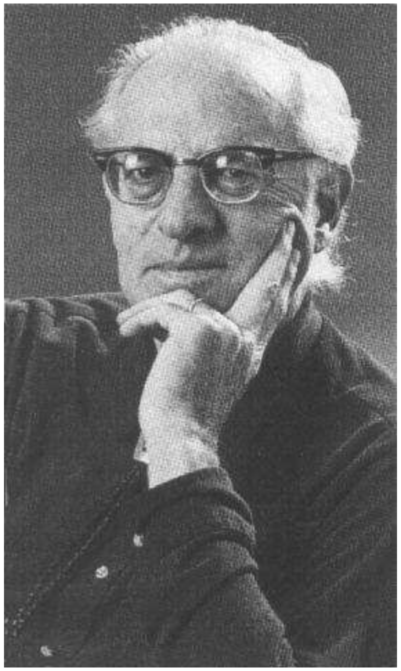
• In order to distinguish them from heavy neutrons, Fermi proposed to name them neutrinos.

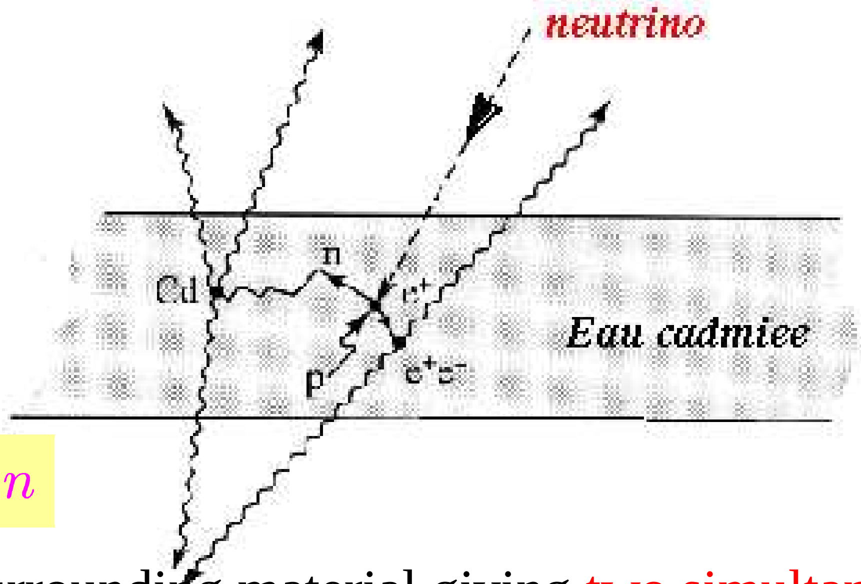
First Detection of ν's



In 1934, Hans Bethe and Rudolf Peierls showed that the cross section between ν and matter should be so small that a ν go through the Earth without deviation

In 1953 Frederick Reines and Clyde Cowan place a neutrino detector near a nuclear plant





400 litters of water and cadmium chloride.

$$\overline{\nu}_e + p \to e^+ + n$$

e^+ annihilates e^- of the surrounding material giving two simultaneous γ's.

neutron captured by a cadmium nucleus with emission of γ's some 15 msec after

The neutrino was there. Its tag was clearly visible

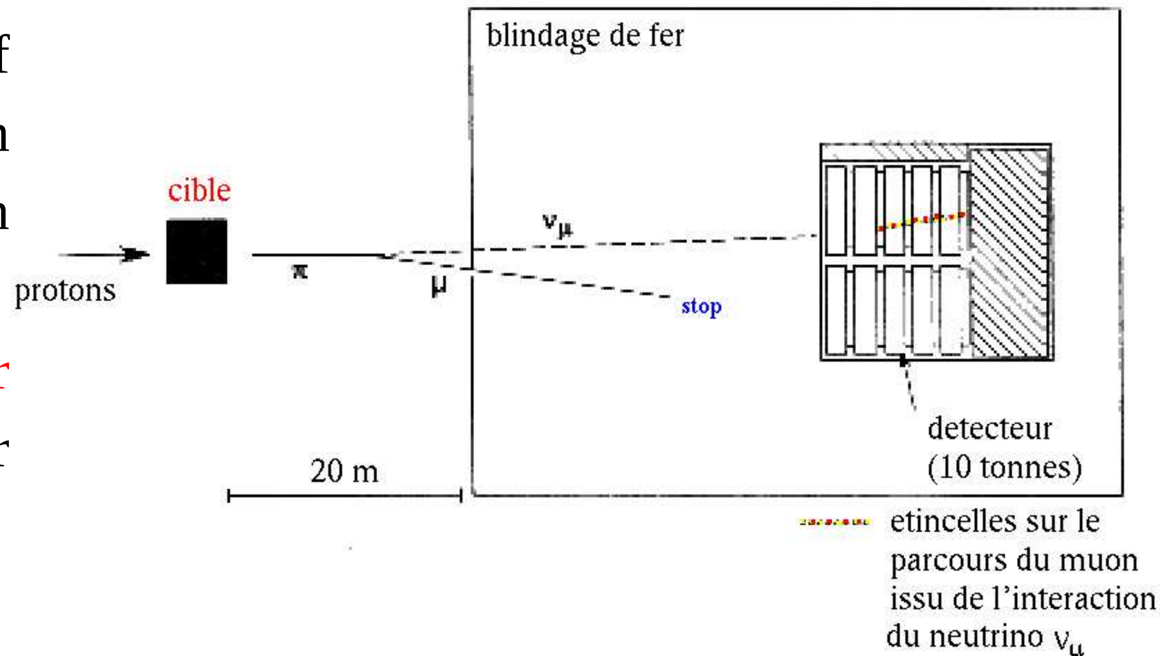
The Other Flavours

ν coming out of a nuclear reactor is $\overline{\nu}_e$ because it is emitted together with an e^-

Question: Is it different from the muon type neutrino ν_μ that could be associated to the muon? Or is this difference a theoretical arbitrary convention?

In 1959 M. Schwartz thought of producing an intense ν beam from π's decay (produced when a proton beam of GeV energy hits matter)

Schwartz, Lederman, Steinberger and Gaillard built a spark chamber (a 10 tons of neon gas) to detect ν_μ



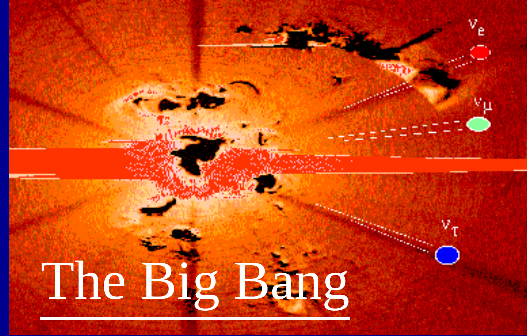
They observe 40 ν interactions: in 6 an e^- comes out and in 34 a μ^- comes out.

If $\nu_\mu \equiv \nu_e \Rightarrow$ equal numbers of μ^- and $e-$ $\Rightarrow$ Conclusion: ν_μ is a different particle

In 1977 Martin Perl discovers the particle tau $\equiv$ the third lepton family.

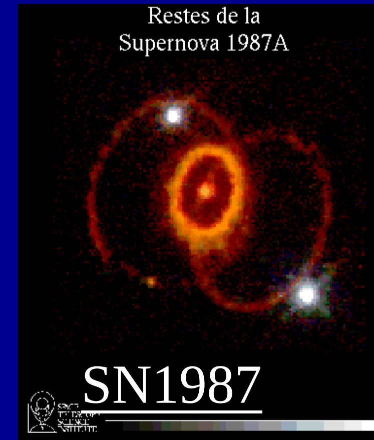
The ν_τ was observed by DONUT experiment at FNAL in 1998 (*officially* in Dec. 2000).

Sources of ν's



$\rho_\nu = 330/\text{cm}^3$

E_ν =0.0004 eV



$E_\nu \sim$ MeV

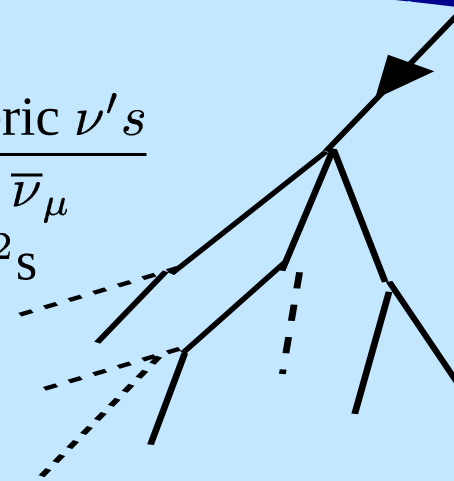
The Sun

ν_e

$\Phi_\nu^{Earth} = 6 \times 10^{10} \nu/\text{cm}^2\text{s}$

$E_\nu \sim$ 0.1–20 MeV

P



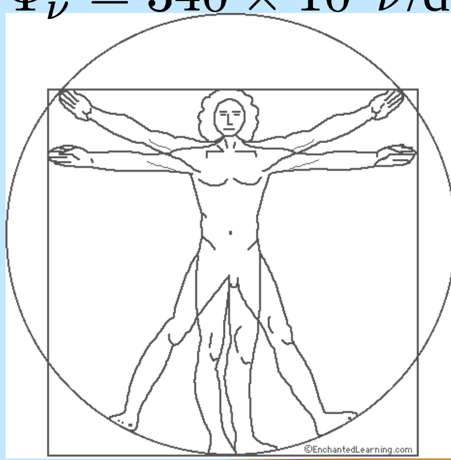
Atmospheric $\nu' s$

$\nu_e, \nu_\mu, \overline{\nu}_e, \overline{\nu}_\mu$

$\Phi_\nu \sim 1 \nu/\text{cm}^2\text{s}$

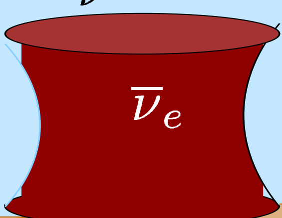
Human Body

$\Phi_\nu = 340 \times 10^6 \nu/\text{day}$



Nuclear Reactors

$E_\nu \sim$ few MeV



Earth's radioactivity

$\Phi_\nu \sim 6 \times 10^6 \nu/\text{cm}^2\text{s}$

Accelerators

$E_\nu \simeq$ 0.3–30 GeV

Fermilab
Discovering the Nature of Nature
CERN
～宇宙と物質の起源と構造を探る～
KEK
高エネルギー加速器研究機構

Concha Gonzalez-Garcia

ν in the SM

- The SM is a gauge theory based on the symmetry group

$$SU(3)_C \times SU(2)_L \times U(1)_Y \Rightarrow SU(3)_C \times U(1)_{EM}$$

- LEP tested this symmetry to 1% precission and the missing particles t, ν_τ

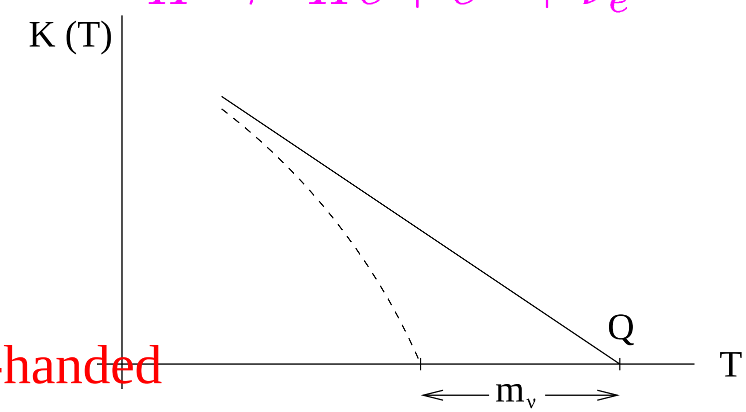
$(1,2)_{-\frac{1}{2}}$	$(3,2)_{\frac{1}{6}}$	$(1,1)_{-1}$	$(3,1)_{\frac{2}{3}}$	$(3,1)_{-\frac{1}{3}}$
$\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L$	$\begin{pmatrix}u^i\\ d^i\end{pmatrix}_L$	e_R	u^i_R	d^i_R
$\begin{pmatrix}\nu_\mu\\ \mu\end{pmatrix}_L$	$\begin{pmatrix}c^i\\ s^i\end{pmatrix}_L$	μ_R	c^i_R	s^i_R
$\begin{pmatrix}\nu_\tau\\ \tau\end{pmatrix}_L$	$\begin{pmatrix}t^i\\ b^i\end{pmatrix}_L$	τ_R	t^i_R	n^i_R

Notice there is no ν_R

$\Rightarrow$ Accidental global symmetry:

$B \times L_e \times L_\mu \times L_\tau$

$$^3H \rightarrow ^3He + e^- + \bar{\nu}_e$$



- When SM was invented upper bounds on m_ν

$m_{\nu_e} < 2.2$ eV

$m_{\nu_\mu} < 190$ KeV $\quad (\pi \rightarrow \mu\nu_\mu)$

$m_{\nu_\tau} < 18.2$ MeV $\quad (\tau \rightarrow n\pi\nu_\tau)$

- Neutrinos are conjured to be massless and left-handed

Neutrino Helicity

- The neutrino helicity was measured in 1957 in a experiment by Goldhaber et al.

- Using the electron capture reaction

$$e^- + {}^{152}Eu \to \nu + {}^{152}Sm^* \to {}^{152}Sm + \gamma$$

with $J({}^{152}Eu) = J({}^{152}Sm) = 0$ and $L(e^-) = 0$

- Angular momentum conservation $\Rightarrow$

$$\begin{cases} J_z(e^-) = J_z(\nu) + J_z(Sm^*) \\ = J_z(\nu) + J_z(\gamma) \\ \pm\frac{1}{2} = \pm\frac{1}{2} \quad \pm 1 \Rightarrow J_z(\nu) = -\frac{1}{2}J_z(\gamma) \end{cases}$$

- Nuclei are heavy $\Rightarrow \vec{p}({}^{152}Eu) \simeq \vec{p}({}^{152}Sm) \simeq \vec{p}({}^{152}Sm^*) = 0$

So momentum conservation $\Rightarrow \vec{p}(\nu) = -\vec{p}(\gamma) \Rightarrow$ ν helicity= γ helicity

- Goldhaber et al found γ had negative helicity $\Rightarrow$ ν has helicity -1

Thus so far ν was a particle with $m_\nu = 0$ and left handed.

(because for massless fermions helicity$\equiv$ chirality...)

Dirac versus Majorana Neutrinos

- In the SM neutral bosons can be of two type:
  - Their own antiparticle such as γ, π^0 ...
  - Different from their antiparticle such as K^0, $\bar{K}^0$...

- In the SM ν are the only *neutral fermions*

$\Rightarrow$ OPEN QUESTION: are neutrino and antineutrino the same or different particles?

$\ast$ ANSWER 1: ν different from anti-ν $\Rightarrow$ ν is a *Dirac* particle (like e)

$\Rightarrow$ It is described by a *Dirac* field $\nu(x) = \sum_{s,\vec{p}} \left[a_s(\vec{p}) u_s(\vec{p}) e^{-ipx} + b_s^\dagger(\vec{p}) v_s(\vec{p}) e^{ipx} \right]$

$\Rightarrow$ And the charged conjugate neutrino field $\equiv$ the antineutrino field

$$\nu^C = \mathcal{C}\, \nu\, \mathcal{C}^{-1} = \eta_C^\star \sum_{s,\vec{p}} \left[b_s(\vec{p}) u_s(\vec{p}) e^{-ipx} + a_s^\dagger(\vec{p}) v_s(\vec{p}) e^{ipx} \right] = -\eta_C^\ast\, C\, \overline{\nu}^T$$

$(C = i\gamma^2\gamma^0)$

which contain two sets of creation–annihilation operators

$\Rightarrow$ These two fields can rewritten in terms of 4 chiral fields

ν_L , ν_R , $(\nu_L)^C$, $(\nu_R)^C$ with $\nu = \nu_L + \nu_R$ and $\nu^C = (\nu_L)^C + (\nu_R)^C$

Dirac versus Majorana Neutrinos

$\ast$ ANSWER 2: ν same as anti-ν $\Rightarrow$ ν is a *Majorana* particle : $\nu_M = \nu_M^C$

$$\Rightarrow \eta_C^\star \sum_{s,\vec{p}} \left[b_s(\vec{p}) u_s(\vec{p}) e^{-ipx} + a_s^\dagger(\vec{p}) v_s(\vec{p}) e^{ipx} \right] = \sum_{s,\vec{p}} \left[a_s(\vec{p}) u_s(\vec{p}) e^{-ipx} + b_s^\dagger(\vec{p}) v_s(\vec{p}) e^{ipx} \right]$$

$\Rightarrow$ So we can rewrite the field $\nu_M = \sum_{s,\vec{p}} \left[a_s(\vec{p}) u_s(\vec{p}) e^{-ipx} + \eta_C^\star a_s^\dagger(\vec{p}) v_s(\vec{p}) e^{ipx} \right]$

which contains only one set of creation–annihilation operators

$\Rightarrow$ A Majorana particle can be described with only 2 independent chiral fields:

ν_L and $(\nu_L)^C$ which verify $\nu_L = (\nu_R)^C$ $(\nu_L)^C = \nu_R$

- In the SM the interaction term for neutrinos

$$\mathcal{L}_{int} = \frac{i\,g}{\sqrt{2}} [(\bar{l}_\alpha \gamma_\mu P_L \nu_\alpha) W_\mu^- + (\bar{\nu}_\alpha \gamma_\mu P_L l_\alpha) W_\mu^+] + \frac{i\,g}{\sqrt{2}\cos\theta_W} (\bar{\nu}_\alpha \gamma_\mu P_L \nu_\alpha) Z_\mu$$

Only involves two chiral fields $P_L\,\nu = \nu_L$ and $\overline{\nu}\,P_R = \eta_C (\nu_L)^{C^T} C^\dagger$

$\Rightarrow$ Weak interaction cannot distinguish if neutrinos are Dirac or Majorana

The difference arises from *the mass term*

ν Mass Terms

- A fermion mass can be seen as at a Left-Right transition

$$m_f \overline{f_L} f_R + h.c. \quad \text{(this is not } SU(2)_L \text{ gauge invariant)}$$

- In the Standard Model mass comes from *spontaneous symmetry breaking* via Yukawa interaction of the left-handed doublet L_L with the right-handed singlet l_R:

$$\mathcal{L}_Y^{(\ell)} = -\frac{\sqrt{2}}{v}\overline{L_L} M^{(\ell)} \ell_R \phi + \text{h.c.} \quad \phi = \text{the scalar doublet}$$

- After spontaneous symmetry breaking

$$\phi \stackrel{SSB}{\rightarrow} \left\{ \begin{array}{c} 0 \\ \frac{v+H}{\sqrt{2}} \end{array} \right\} \Rightarrow \mathcal{L}_{\text{mass}}^{(\ell)} = -\bar{\ell}_L M^{(\ell)} \ell_R + \text{h.c.}$$

$M^{(\ell)}$= Dirac mass matrix for charged leptons

- ν's do not participate in QED or QCD and only ν_L is relevant for weak interactions $\Rightarrow$ there is no *dynamical* reason for introducing ν_R, so

How can we generate a mass for the neutrino?

ν Mass Terms: Dirac Mass

OPTION 1:

- One introduces ν_R which can couple to the lepton doublet by Yukawa interaction

$$\mathcal{L}_Y^{(\nu)} = -\frac{\sqrt{2}}{v}\overline{L_L}M^{(\nu)}\nu_R\widetilde{\phi} + \text{h.c.} \qquad (\widetilde{\phi} = i\tau_2\phi^*)$$

- Under spontaneous symmetry-breaking,

$$\mathcal{L}_Y^{(\nu)} \Rightarrow \mathcal{L}_{\text{mass}}^{(\text{Dirac})} = -\overline{\nu_L}M_D^\nu\nu_R + \text{h.c.} \equiv -\frac{1}{2}(\overline{\nu}_L M_D^\nu \nu_R + \overline{(\nu_R)^c}M_D^{\nu\,T}(\nu_L)^c) + \text{h.c.}$$

M_D^ν =Dirac mass for neutrinos

- $\mathcal{L}_{\text{mass}}^{(\text{Dirac})}$ involves the four chiral fields ν_L , ν_R , $(\nu_L)^C$, $(\nu_R)^C$

$\Rightarrow$ The eigenstates of M_D^ν are Dirac particles (same as quarks and charged leptons)

$\Rightarrow$ Total Lepton number is conserved *by construction* (not *accidentally*):

$$U(1)_L\,\nu = e^{i\alpha}\,\nu \quad \text{and} \quad U(1)_L\,\overline{\nu} = e^{-i\alpha}\,\overline{\nu}$$

$$U(1)_L\,\nu^C = e^{-i\alpha}\,\nu^C \quad \text{and} \quad U(1)_L\,\overline{\nu^C} = e^{i\alpha}\,\overline{\nu^C}$$

ν Mass Terms: Majorana Mass

OPTION 2:

• One does not introduce ν_R but uses that the field $(\nu_L)^c$ is right-handed, so that one can write a Lorentz-invariant mass term

$$\mathcal{L}_{\text{mass}}^{(\text{Maj})} = -\frac{1}{2}\overline{\nu_L} M_M^\nu \nu_L^c + \text{h.c.}$$

M_M^ν =Majorana mass for neutrinos

• But under any $U(1)$ symmetry

$$U(1)\,\nu^c = e^{-i\alpha}\,\nu^c \quad \text{and} \quad U(1)\,\overline{\nu} = e^{-i\alpha}\,\overline{\nu}$$

⇒ it can only appear for particles without electric charge

⇒ Total Lepton Number is not conserved

⇒ The eigenstates of M_M^ν are Majorana particles (verify $\nu_i^c = \nu_i$)

⇒ But $SU(2)_L$ gauge invariance is broken!!!

General $SU(2)_L$ invariant ν Mass Terms

OPTION 3:

- Introduce ν_{R_i} $(i = 1, m)$ and write all Lorentz and $SU(2)_L$ invariant mass term

$$\mathcal{L}_Y^{(\nu)} = -\frac{\sqrt{2}}{v}\overline{L_{L,j}} M^*_{D,ji} \nu_{R,i} \tilde{\phi} - \frac{1}{2}\overline{\nu_{R,j}} M_{N,ji} \nu^c_{R,i} + \text{h.c.}$$

- Under spontaneous symmetry-breaking

$$\begin{aligned}
\mathcal{L}^{(\nu)}_{\text{mass}} &= -\overline{\nu_{L,j}} M^*_{D,ji} \nu_{R,i} - \frac{1}{2}\overline{\nu_{R,j}} M_{N,ji} \nu^c_{R,i} + \text{h.c.} \\
&= -\frac{1}{2}(\overline{\nu_{R,j}} M^T_{D,ji} \nu_{L,i} + \overline{\nu^c_{L,j}} M_{D,ji} \nu^c_{R,i}) - \frac{1}{2}\overline{\nu_{R,j}} M_{N,ji} \nu^c_{R,i} + \text{h.c.} \\
&\equiv -\frac{1}{2}\overline{\vec{\nu}^c} M^\nu \vec{\nu} + \text{h.c.}
\end{aligned}$$

with $\vec{\nu} = \begin{pmatrix} \nu_{L,k} \\ \nu^c_{R,l} \end{pmatrix}$ and $M^\nu = \begin{pmatrix} 0 & M_D \\ M_D^T & M_N \end{pmatrix}$

- In general if $M_N \neq 0 \Rightarrow$ 3+m Majorana neutrino states (verify $\nu_i^c = \nu_i$)
  how many are light depends on hierarchy between M_D and M_N

$\Rightarrow$ Total Lepton Number is not conserved

The See-Saw Mechanism

- A particular realization of OPTION 3: Assume $M_N \gg m_D \Rightarrow$
  - m Heavy ν's of mass $m_{\nu_H} \sim M_N$
  - 3 light neutrinos ν's of mass $m_{\nu_l} = m_D M_N^{-1} m_D^T$
  - The heavier ν_H the lighter $\nu_l \Rightarrow$ See-Saw Mechanism
  - Arises in many extensions of the SM: SO(10) GUTS, Left-right...

ν Mass from Non-Renormalizable Operator

If SM is an effective low energy theory, for $E \ll \Lambda_{\rm NP}$

– The same particle content as the SM and same pattern of symmetry breaking

– But there can be non-renormalizable (dim>4) operators

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \sum_n \frac{1}{\Lambda_{\rm NP}^{n-4}} \mathcal{O}_n$$

First NP effect $\Rightarrow$ dim=5 operator

There is only one!

$$\mathcal{O}_5 = \frac{Z^\nu_{ij}}{\Lambda_{\rm NP}} \left(\overline{L_{L,i}} \tilde{\phi}\right) \left(\tilde{\phi}^T L^C_{L,j}\right)$$

which after symmetry breaking

induces a ν Majorana mass

$$(M_\nu)_{ij} = Z^\nu_{ij} \frac{v^2}{\Lambda_{\rm NP}}$$

$\mathcal{O}_5$ breaks total lepton and lepton flavour numbers

Implications:

– It is natural that ν mass is the first evidence of NP

– Naturally $m_\nu \ll$ other fermions masses $\sim \lambda^f v$ if $\Lambda_{NP} >> v$

– see-saw with heavy neutrinos integrated out is a particular example of this

Lepton Mixing

• Charged current and mass for 3 charged leptons ℓ_i and N neutrinos ν_j. In weak basis

$$\mathcal{L}_{CC}+\mathcal{L}_M = -\frac{g}{\sqrt{2}}\sum_{i=1}^{3}\overline{\ell^W_{L,i}}\gamma^\mu \nu^W_i W^+_\mu - \sum_{i,j=1}^{3}\overline{\ell^W_{L,i}} M_{\ell\, ij}\,\ell^W_{R,j} - \frac{1}{2}\sum_{i,j=1}^{N}\overline{\nu^{c\,W}_i} M_{\nu\, ij}\,\nu^W_j + \text{h.c.}$$

• Changing to mass basis by rotations

$$\ell^W_{L,i} = V^\ell_{L\,ij}\,\ell_{L,j} \qquad \ell^W_{R,i} = V^\ell_{R\,ij}\,\ell_{R,j} \qquad \nu^W_i = V^\nu_{ij}\,\nu_j$$

$$V^{\ell\,\dagger}_L M_\ell V^\ell_R = \text{diag}(m_e, m_\mu, m_\tau) \quad V^{\nu\,\dagger} M^\dagger_\nu M_\nu V^\nu = \text{diag}(m^2_1, m^2_2, m^2_3, \dots, m^2_N)$$

$V^\ell_{L,R} \equiv$ Unitary 3×3 matrices and $V^\nu \equiv$ Unitary $N\times N$ matrix.

• The charged current in the mass basis

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}}\overline{\ell^i_L}\,\gamma^\mu\, U^{ij}_{\text{LEP}}\,\nu_j\, W^+_\mu$$

$U_{\text{LEP}} \equiv 3\times N$ matrix

$$U^{ij}_{\text{LEP}} = \sum_{k=1}^{3} {V^{\ell}_L}^{\dagger\, ik}\, V^{\nu\, kj}$$

Lepton Mixing

- For example for 3 Dirac ν's : 3 Mixing angles + 1 Dirac Phase

$$U_{\rm LEP} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{-i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{21} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- For 3 Majorana ν's : 3 Mixing angles + 1 Dirac Phase + 2 Majorana Phases

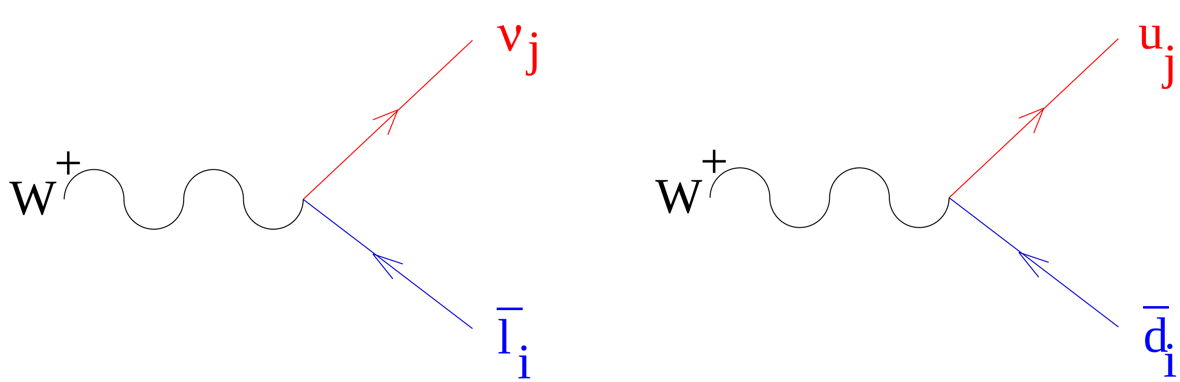
$$U_{\rm LEP} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{-i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{21} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\phi_2} & 0 \\ 0 & 0 & e^{i\phi_3} \end{pmatrix}$$

Effects of ν Mass

- Neutrino masses can have kinematic effects

- Also if neutrinos have a mass the charged current interactions of leptons are not diagonal (same as quarks)

$$\frac{g}{\sqrt{2}} W^+_\mu \sum_{ij} \left(U^{ij}_{LEP}\, \overline{\ell^i}\, \gamma^\mu\, L\, \nu^j + U^{ij}_{CKM}\, \overline{U^i}\, \gamma^\mu\, L\, D^j \right) + h.c.$$



- SM gauge invariance *does not imply* $U(1)_{L_e} \times U(1)_{L_\mu} \times U(1)_{L_\tau}$ symmetry

- Total lepton number $U(1)_L = U(1)_{Le+L_\mu+L_\tau}$ can be or cannot be still a symmetry depending on whether neutrinos are Dirac or Majorana

Neutrino Mass Scale: Tritium β Decay

- Fermi proposed a kinematic search of ν_e mass from beta spectra in ^3H beta decay

$$^3H \rightarrow ^3He + e + \overline{\nu}_e$$

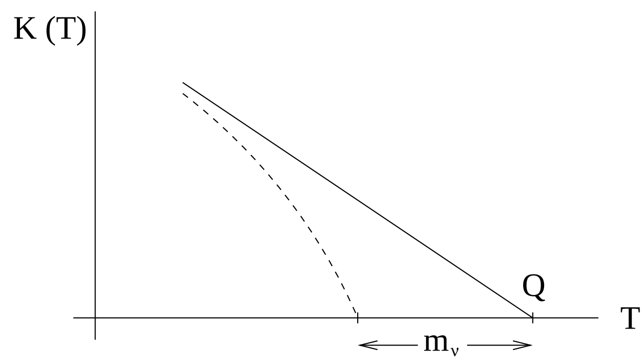
- For "allowed" nuclear transitions, the electron spectrum is given by phase space alone

$$K(T) \equiv \sqrt{\frac{dN}{dT}\frac{1}{C p\, E\, F(E)}} \propto \sqrt{(Q-T)\sqrt{(Q-T)^2 - m_\nu^2}}$$

$T = E_e - m_e$, Q= maximum kinetic energy, (for ^3H beta decay $Q = 18.6$ KeV)

- $m_\nu \neq 0 \Rightarrow$ distortion from the straight-line at the end point of the spectrum

$$m_\nu = 0 \;\Rightarrow\; T_{\max} = Q$$
$$m_\nu \neq 0 \;\Rightarrow\; T_{\max} = Q - m_\nu$$



– At present only a bound: $m_{\nu_e}^{eff} \equiv \sqrt{\sum m_j^2 |U_{ej}|^2} < 2.2$ eV (at 95 % CL)
(Mainz & Troisk experiments)

– Katrin proposed to improve present sensitivity to $m_{eff}^{\beta} \sim 0.3\,\text{eV}$

Neutrino Mass Scale: Other Channels

Muon neutrino mass

- From the two body decay at rest

$$\pi^- \to \mu^- + \overline{\nu}_\mu$$

- Energy momentum conservation:

$$m_\pi = \sqrt{p_\mu^2 + m_\mu^2} + \sqrt{p_\mu^2 + m_\nu^2}$$
$$m_\nu^2 = m_\pi^2 + m_\mu^2 - 2 + m_\mu \sqrt{p^2 + m_\pi^2}$$

- Measurement of p_μ plus the precise knowledge of m_π and $m_\mu \Rightarrow m_\nu$
- The present experimental result bound:

$$m_{\nu_\mu}^{eff} \equiv \sum m_j |U_{\mu j}|^2 < 190 \text{ KeV}$$

Tau neutrino mass

- The τ is much heavier $m_\tau = 1.776$ GeV

$\Rightarrow$ Large phase space $\Rightarrow$ difficult precision for m_ν

- The best precision is obtained from hadronic final states

$$\tau \to n\pi + \nu_\tau \quad \text{with } n \geq 3$$

- Lep I experiments obtain:

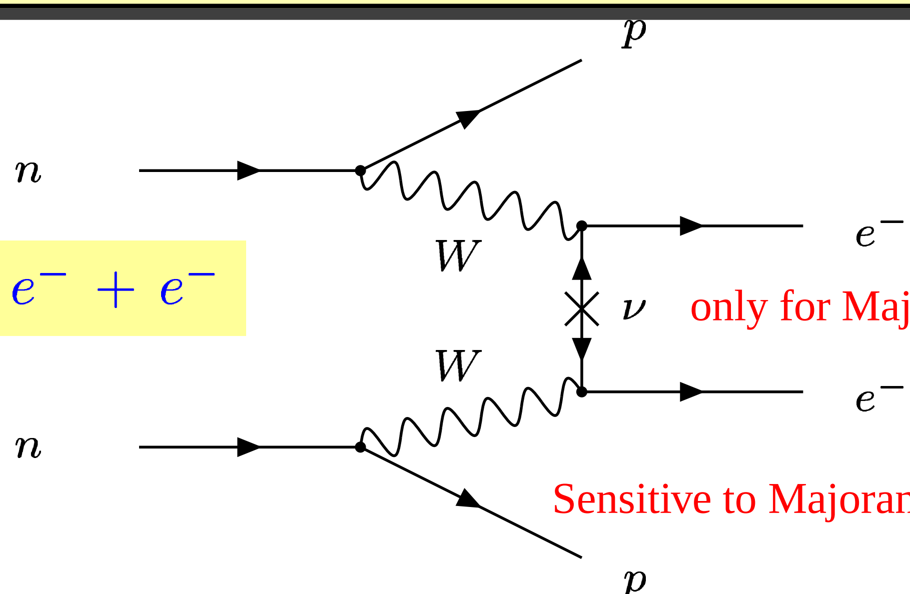
$$m_{\nu_\tau}^{eff} \equiv \sum m_j |U_{\tau j}|^2 < 18.2 \text{ MeV}$$

$\Rightarrow$ If mixing angles U_{ej} are not negligible

Best kinematic limit on Neutrino Mass Scale comes from Tritium Beta Decay

Neutrino Mass Scale: ν-less Double-β Decay

$$(A,Z) \to (A,Z+2) + e^- + e^-$$



only for Majorana ν's

Sensitive to Majorana CP phases

- Amplitude involves the product of two leptonic currents: $\left[\overline{e}\gamma^\mu L\nu\right]\left[\overline{e}\gamma^\mu L\nu\right]$
  - If ν Dirac $\Rightarrow$ ν *annihilates* a *neutrino* and *creates* an *antineutrino*
    $\Rightarrow$ no same state $\Rightarrow$ Amplitude = 0
  - If ν Majorana $\Rightarrow$ $\nu = \nu^c$ *annihilates and creates* a *neutrino=antineutrino*
    $\Rightarrow$ same state $\Rightarrow$ Amplitude $\propto$ $\overbrace{\nu\,(\nu^c})^T \neq 0$
- Amplitude of ν-less-$\beta\beta$ decay is proportional to $|\langle m_{ee}\rangle| = |\sum U_{ej}^2 m_j|$
  - Present bound: $|\langle m_{ee}\rangle| < 0.35$ eV +theor. uncert. < 1.05 eV (90% CL)
  - Several proposed experiments to reach $|\langle m_{ee}\rangle| \sim 10^{-2}$ eV

Concha Gonzalez-Garcia

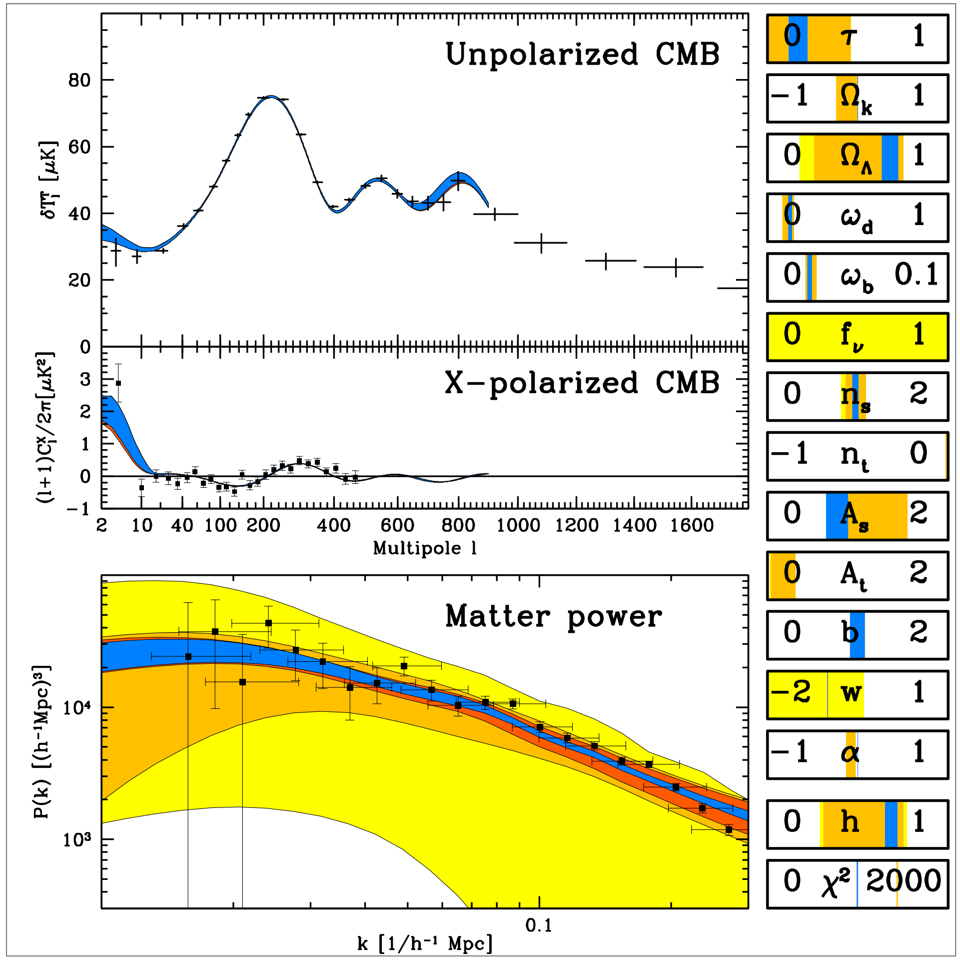
Neutrino Mass Scale in Cosmology

$\sum m_{\nu_i}$ has effects on:

Cosmic Microwave Background Temperature Fluctuations

Most recent from WMAP

Large scale structure:
– 2° Field Galaxy Redshift Survey (2dFGRS)
–Sloan Digital Sky Survey (SDSS)



Tegmark *et. al* astro-ph/0310723

Problem: 13 parameters to be determined!!

$\Rightarrow$ limit on $\sum m_{\nu_i}$ depends on
prior and data used to constraint
other 12 parameters

$\sum m_{\nu_i} \leq 0.7 \;-\; 2.1$ eV at 95 % CL

Effects of ν Mass: Flavour Transitions

- Flavour ($\equiv$ Interaction) basis (production and detection): ν_e, ν_μ and ν_τ
- Mass basis (free propagation in space-time): ν_1, ν_2 and ν_3 $\ldots$
- In general interaction eigenstates $\neq$ propagation eigenstates

$$U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \ldots \end{pmatrix} = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

$\Rightarrow$ Flavour is not conserved during propagation $\Rightarrow$ ν can be detected with different (or same) flavour than produced

- The probability $P_{\alpha\beta}$ of producing neutrino with flavour α and detecting with flavour β has to depend on:
  - Misalignment between interaction and propagation states ($\equiv U$)
  - Difference between propagation eigenvalues
  - Propagation distance

Vacuum Oscillations

- A state mixture of 2 neutrino species $|\nu_e\rangle$ and $|\nu_X\rangle$ or equivalently of $|\nu_1\rangle$ and $|\nu_2\rangle$

$$|\Phi(x)\rangle = \Phi_e(x)|\nu_e\rangle + \Phi_X(x)|\nu_X\rangle = \Phi_1(x)|\nu_1\rangle + \Phi_2(x)|\nu_2\rangle$$

where $|\nu_e\rangle = \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle$ and $|\nu_X\rangle = -\sin\theta|\nu_1\rangle + \cos\theta|\nu_2\rangle$

$$\left[U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\right]$$

- Evolution of $\Phi(x) =$ is given by the Dirac Equations. Calling $\Phi_i(x) = \nu_i(x)\phi_i$

In the relativistic limit $\sqrt{E^2 - m_i^2} \simeq E - \frac{m_i^2}{2E}$

[Under some approximations spinorial part factorizes out. Check Baltz and Wesener PRD37 (1988) 3364]

$$-i\frac{\partial}{\partial x}\begin{pmatrix} \nu_1(x) \\ \nu_2(x) \end{pmatrix} = \left[E - \frac{m_1^2+m_2^2}{4E} - \begin{pmatrix} -\frac{\Delta m^2}{4E} & 0 \\ 0 & \frac{\Delta m^2}{4E} \end{pmatrix}\right]\begin{pmatrix} \nu_1(x) \\ \nu_2(x) \end{pmatrix}$$

where $\Delta m^2 = m_2^2 - m_1^2$.

- Defining $L_0 = \frac{4\pi E}{\Delta m^2} = \frac{\pi}{1.27} \frac{E\,(\mathrm{GeV})}{\Delta m^2\;(\mathrm{eV}^2)}$ Km,

$$-i\frac{\partial}{\partial x}\begin{pmatrix}\nu_1(x)\\ \nu_2(x)\end{pmatrix} = \left[E - \frac{m_1^2+m_2^2}{4E} - \begin{pmatrix}-\frac{\pi}{L_0} & 0\\ 0 & \frac{\pi}{L_0}\end{pmatrix}\right]\begin{pmatrix}\nu_1(x)\\ \nu_2(x)\end{pmatrix}$$

- After a distance L

$$|\Phi(L)\rangle = e^{-i\alpha L}\left(e^{-i\pi L/L_0}\nu_1(0)|\nu_1\rangle + e^{i\pi L/L_0}\nu_2(0)|\nu_2\rangle\right)$$

- If ν was produced with flavour $\alpha \Rightarrow \nu_\alpha(0) = 1$ and $\nu_\beta(0) = 0$

it can be detected with flavour β with probability

$$P_{\alpha\beta} = |\langle\nu_\beta|\Phi(L)\rangle|^2 = \delta_{\alpha\beta} + 2(\tfrac{1}{2} - \delta_{\alpha\beta})\sin^2 2\theta\,\sin^2\frac{\pi L}{L_0}$$

- $P_{\alpha\neq\beta} \equiv P_{osc}$ the probability of flavour $\beta \neq \alpha$ to *appear* after a distance L
- $P_{\alpha\alpha} = 1 - P_{osc}$ is the probability of flavour α to *survive* after a distance L

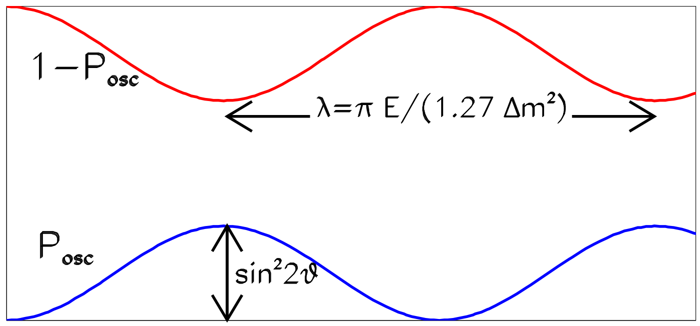
Concha Gonzalez-Garcia

$$P_{\alpha\beta} = |\langle \nu_\beta | \Phi(L) \rangle|^2 = \delta_{\alpha\beta} + 2(\tfrac{1}{2} - \delta_{\alpha\beta}) \sin^2 2\theta \sin^2 \frac{\pi L}{L_0}$$

- The probabilities oscillate in L and $1/E$

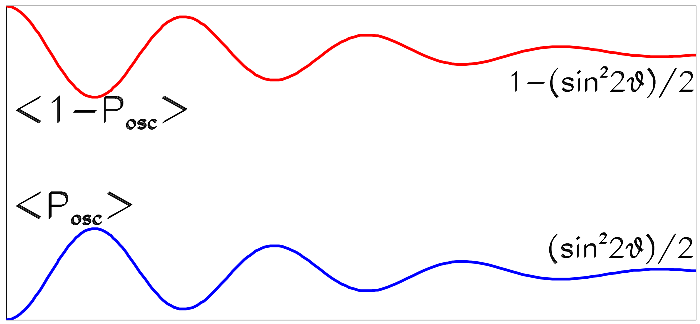
$$P_{osc} = \sin^2(2\theta)\,\sin^2\left(\frac{\Delta m^2 L}{E}\right) \quad \text{Appear}$$

$$P_{\alpha\alpha} = 1 - P_{osc} \quad \text{Disappear}$$



- In real experiments
  neutrinos are not monochromatic

$$\Rightarrow \langle P_{\alpha\beta} \rangle = \int dE_\nu \frac{d\Phi}{dE_\nu} \sigma_{CC}(E_\nu) P_{\alpha\beta}(E_\nu)$$



- Maximal sensitivity for $\Delta m^2 \sim E/L$

  – $\Delta m^2 \ll E/L \;\Rightarrow$ No time to oscillate

  $\Rightarrow \langle \sin^2(1.27 \Delta m^2 L/E) \rangle \simeq 0 \rightarrow \langle P_{osc} \rangle \simeq 0$

  – $\Delta m^2 \gg E/L \;\Rightarrow$ Averaged oscillations

  $\Rightarrow \langle \sin^2(1.27 \Delta m^2 L/E) \rangle \simeq \frac{1}{2} \rightarrow \langle P_{osc} \rangle \simeq \frac{1}{2}\sin^2(2\theta)$

Vacuum Oscillations

- The oscillation probability for N neutrinos:

$$P_{\alpha\beta} = \delta_{\alpha\beta} - 4\sum_{j\neq i}^{n} \mathrm{Re}[U^\star_{\alpha i} U_{\beta i} U_{\alpha j} U^\star_{\beta j}] \sin^2\left(\frac{\Delta_{ij}}{2}\right) + 2\sum_{j\neq i} \mathrm{Im}[U^\star_{\alpha i} U_{\beta i} U_{\alpha j} U^\star_{\beta j}] \sin\left(\Delta_{ij}\right)$$

$$\frac{\Delta_{ij}}{2} = \frac{(E_i - E_j)L}{2} = 1.27\frac{(m_i^2 - m_j^2)}{\mathrm{eV}^2}\frac{L/E}{\mathrm{Km/GeV}}$$

– The first term $\delta_{\alpha\beta} - 4\sum_{j\neq i}^{n} \mathrm{Re}[U^\star_{\alpha i} U_{\beta i} U_{\alpha j} U^\star_{\beta j}] \sin^2\left(\frac{\Delta_{ij}}{2}\right)$ equal for $\overline{\nu}$ $(U \to U^*)$

$\to$ conserves CP

– The last piece $2\sum_{j\neq i} \mathrm{Im}[U^\star_{\alpha i} U_{\beta i} U_{\alpha j} U^\star_{\beta j}] \sin\left(\Delta_{ij}\right)$ opposite sign for $\overline{\nu}$

$\to$ violates CP

- $P_{\alpha\beta}$ depends on Theoretical Parameters
  - $\Delta m_{ij}^2 = m_i^2 - m_j^2$ The mass differences
  - $U_{\alpha j}$ The mixing angles (and Dirac phases)

  and on Two *Experimental* Parameters:
  - E The neutrino energy
  - L Distance ν source to detector

- No information on mass scale nor Majorana phases

To allow observation of neutrino oscillations:

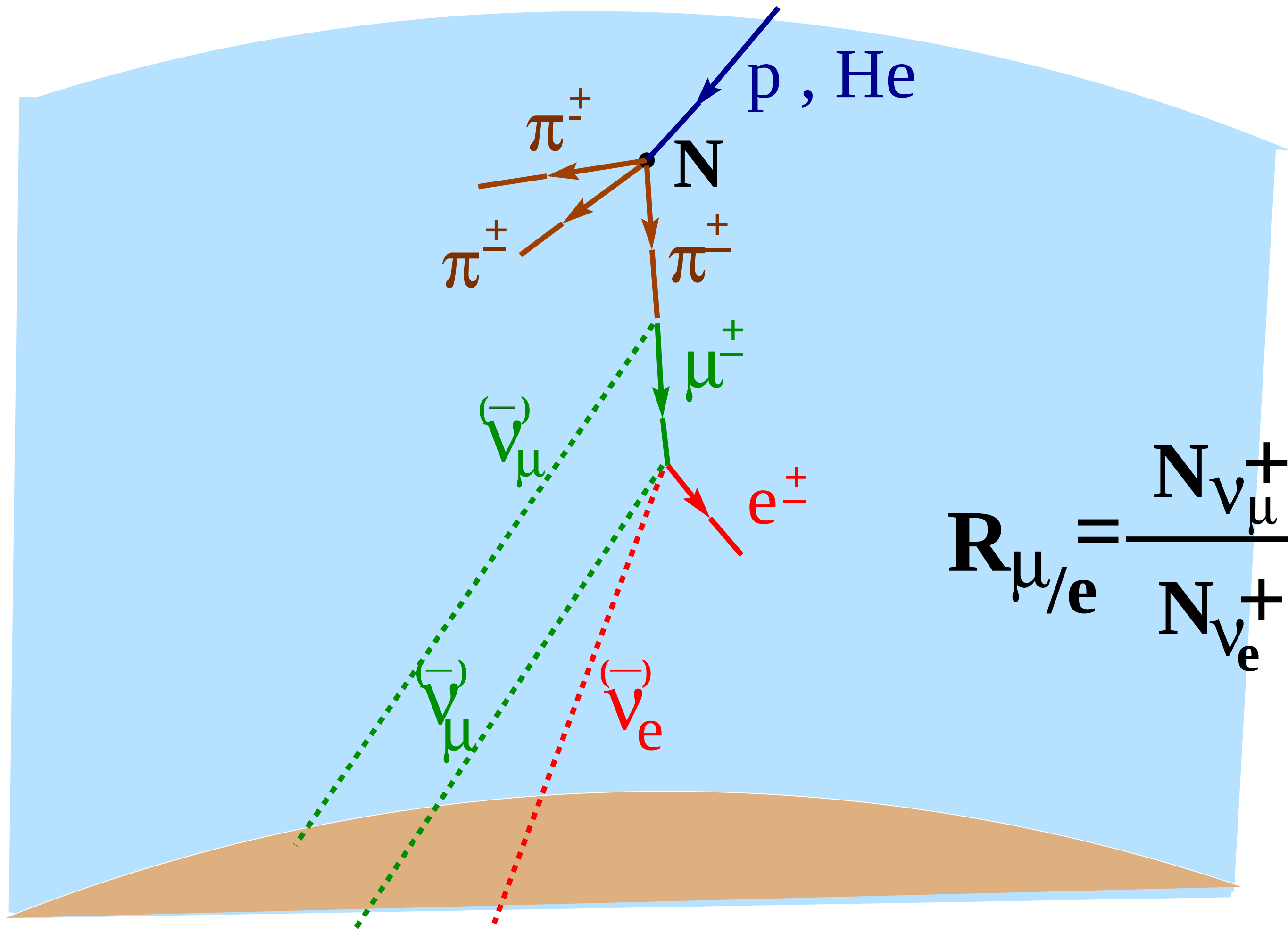
– Nature has to be good: $\theta \not\ll 0$

– Need the right set up ($\equiv$right $\langle \frac{L}{E} \rangle$) for Δm^2

Source	E (GeV)	L (Km)	Δm^2 (eV^2)
Solar	10^{-3}	10^7	10^{-10}
Atmospheric	0.1–10^2	10–10^3	10^{-1}–10^{-4}
Reactor	10^{-3}	SBL: 0.1–1	10^{-2}–10^{-3}
		LBL: 10–10^2	10^{-4}–10^{-5}
Accelerator	10	SBL: 0.1	$\gtrsim 0.01$
		LBL: 10^2– 10^3	10^{-2}–10^{-3}

Atmospheric Neutrinos

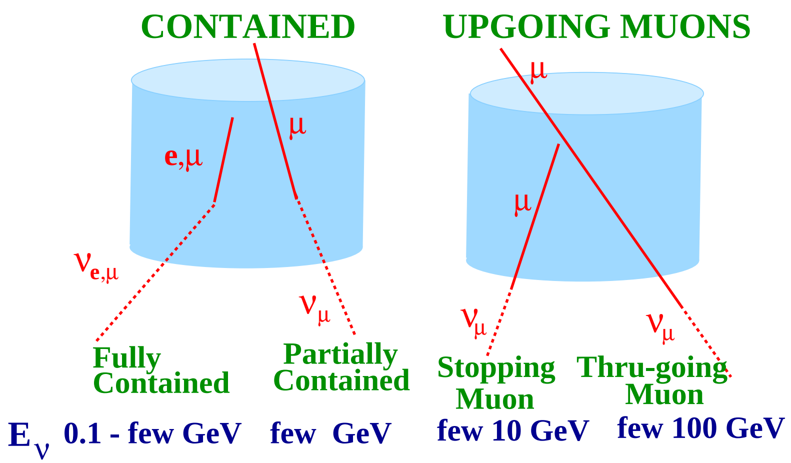
Atmospheric $\nu_{e,\mu}$ are produced by the interaction of cosmic rays (p, He ...) with the atmosphere



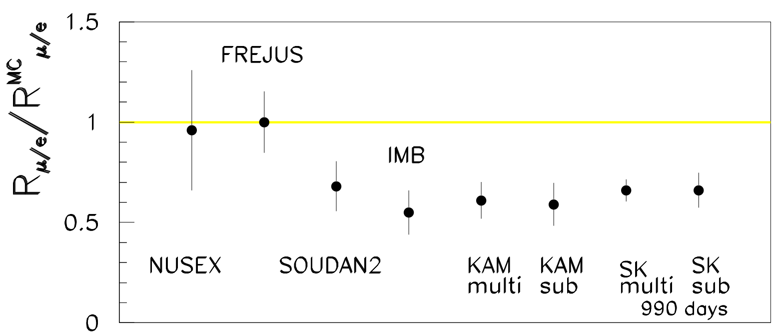
$$R_{\mu/e} = \frac{N_{\nu_\mu} + N_{\bar{\nu}_\mu}}{N_{\nu_e} + N_{\bar{\nu}_e}} \sim 2$$

Atmospheric Neutrinos: Data

EVENT CLASSIFICATION

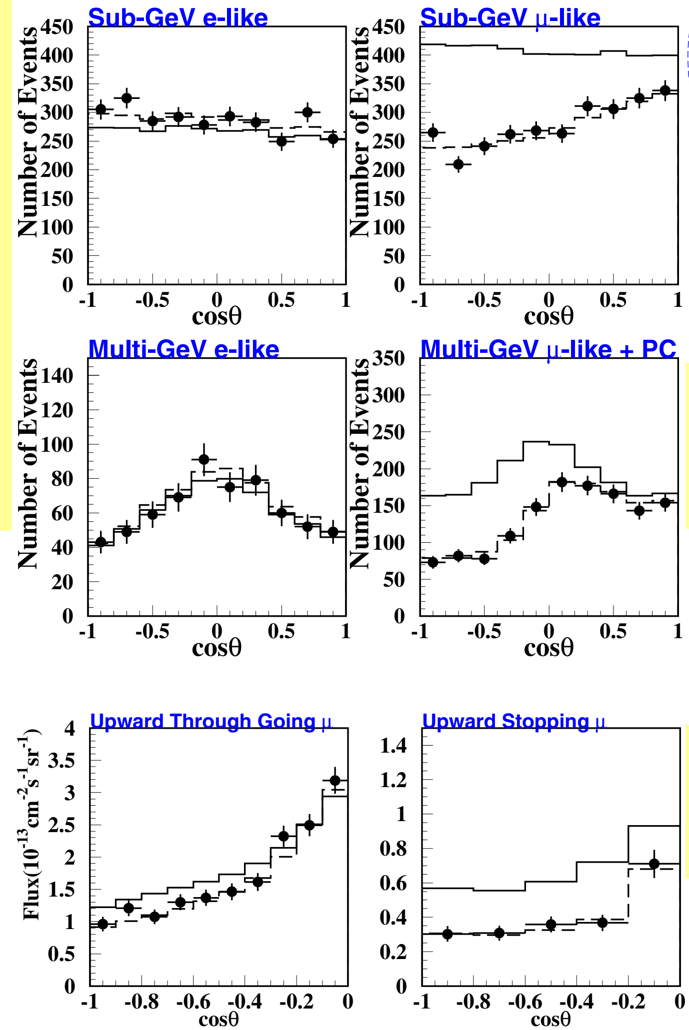


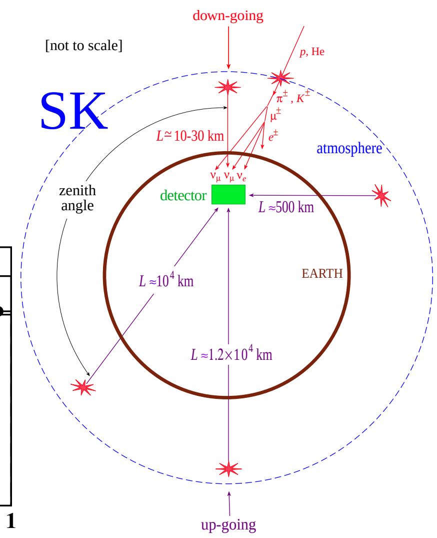
- Total Rates for Contained Events



- Angular Distribution at SK

ν_e in agreement with SM



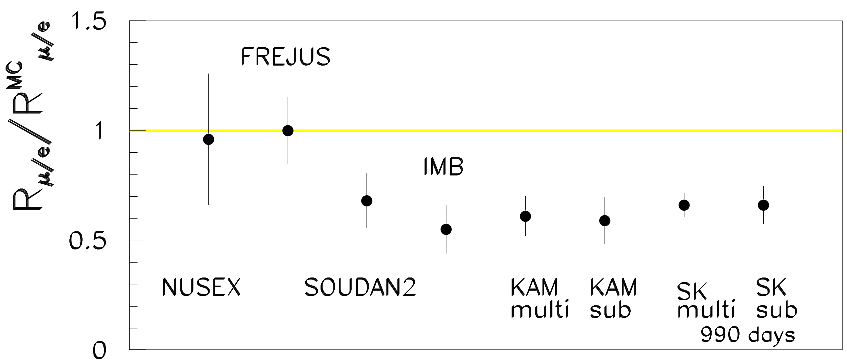


→ Deficit grows with L

→ Decreases with E

Atmospheric ν Oscillations: Parameter Estimate

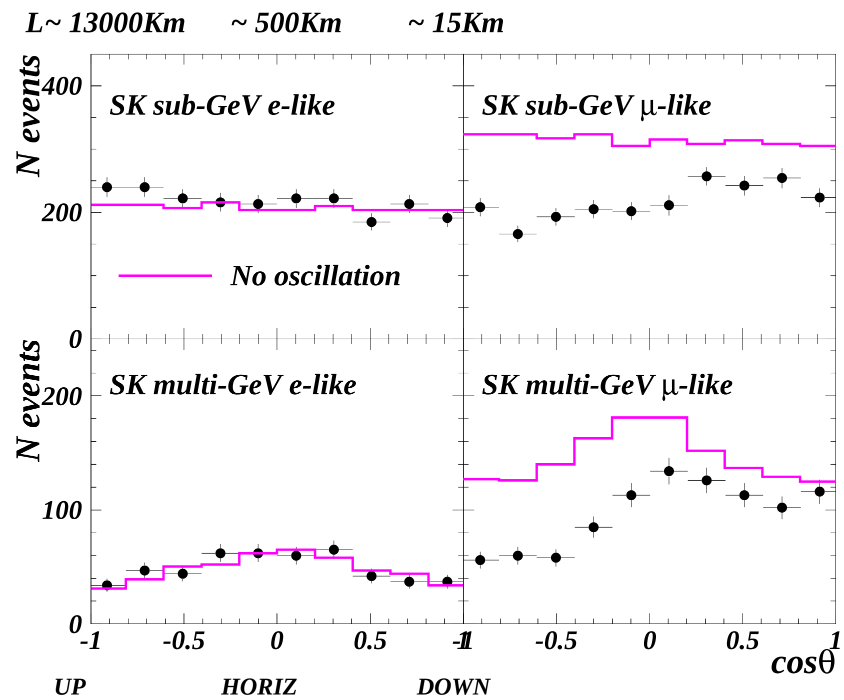
- From Total Contained Event Rates:



$$\langle P_{\mu\mu} \rangle = 1 - \sin^2 2\theta \sin^2 \frac{\Delta m^2 L}{2E}$$
$$\sim 0.5 - 0.7$$

$$\Rightarrow \sin^2 2\theta \gtrsim 0.6$$

- From Angular Distribution:

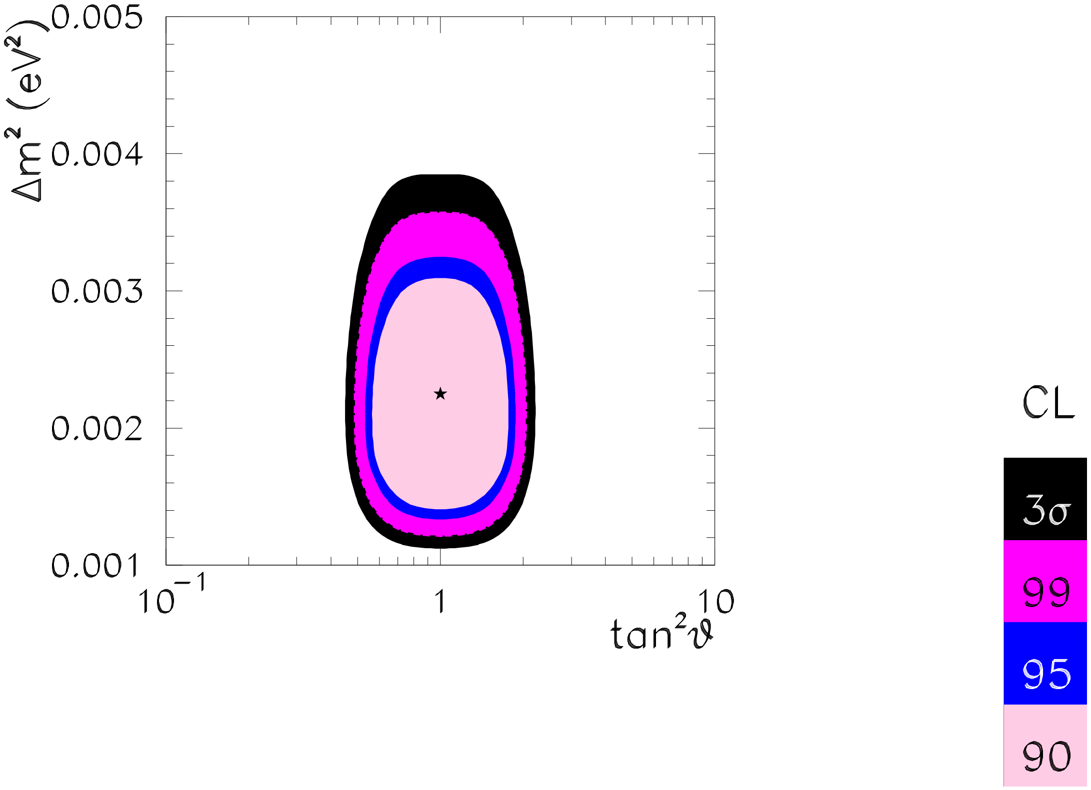


For $E \sim 1$ GeV deficit at $L \sim 10^2 - 10^4$ Km

$$\frac{\Delta m^2(\text{eV}^2) L(\text{km})}{2E(\text{GeV})} \sim 1$$

$$\Rightarrow \Delta m^2 \sim 10^{-4} - 10^{-2} \text{eV}^2$$

Atmospheric ν Oscillation Solution: $\nu_\mu \to \nu_\tau$



Best fit:

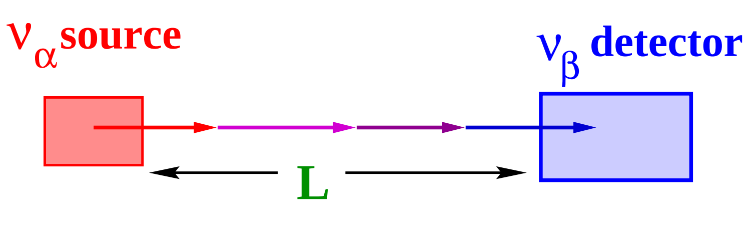
$\Delta m^2 = 2.2 \times 10^{-3}$ eV^2

$\tan^2\theta = 1$

ν Oscillations: Lab Searches at Short Distance

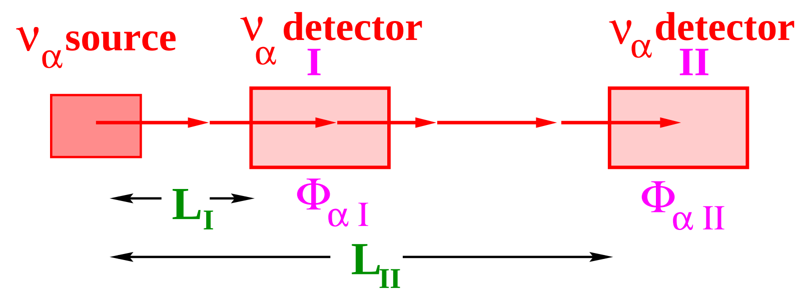
- In laboratory experiments ν source: Accelerator or Nuclear Reactor

Appearance Experiment



Searches for β diff α

Disappearance Experiment



Compares $\Phi_{\alpha\, I}$ and $\Phi_{\alpha\, II}$ to look for loss

Experiment	$\langle \frac{E/MeV}{L/m} \rangle$		α	β
CCFR	100	FNAL	ν_μ, ν_e	ν_τ
E531	25	FNAL	ν_μ, ν_e	ν_τ
Nomad	13	CERN	ν_μ, ν_e	ν_τ
Chorus	13	CERN	ν_μ, ν_e	ν_τ
E776	2.5	BNL	ν_μ	ν_e
Karmen2	2.5	Rutherford	$\bar{\nu}_\mu$	$\bar{\nu}_e$
LSND	3	Los Alamos	$\bar{\nu}_\mu$	$\bar{\nu}_e$

Experiment	$\langle \frac{E/MeV}{L/m} \rangle$		α
CDHSW	1.4	CERN	ν_μ
BugeyIII	0.05	Reactor	$\bar{\nu}_e$
Chooz	0.005	Reactor	$\bar{\nu}_e$

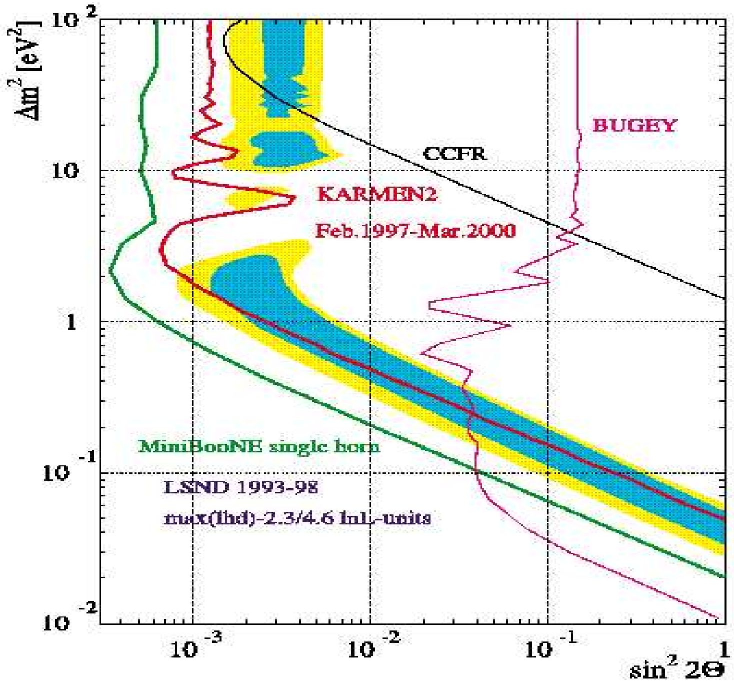
LSND

- The only short distance signal for oscillation: $L = 30$ m with $\langle E_\nu \rangle \sim 30$ MeV
- Used the proton beam of Los Alamos $p + Target \to \pi^+ + X$

$$\pi^+ \to \nu_\mu \, \mu^+$$

$$\mu^+ \to e^+ \, \nu_e \, \overline{\nu_\mu}$$

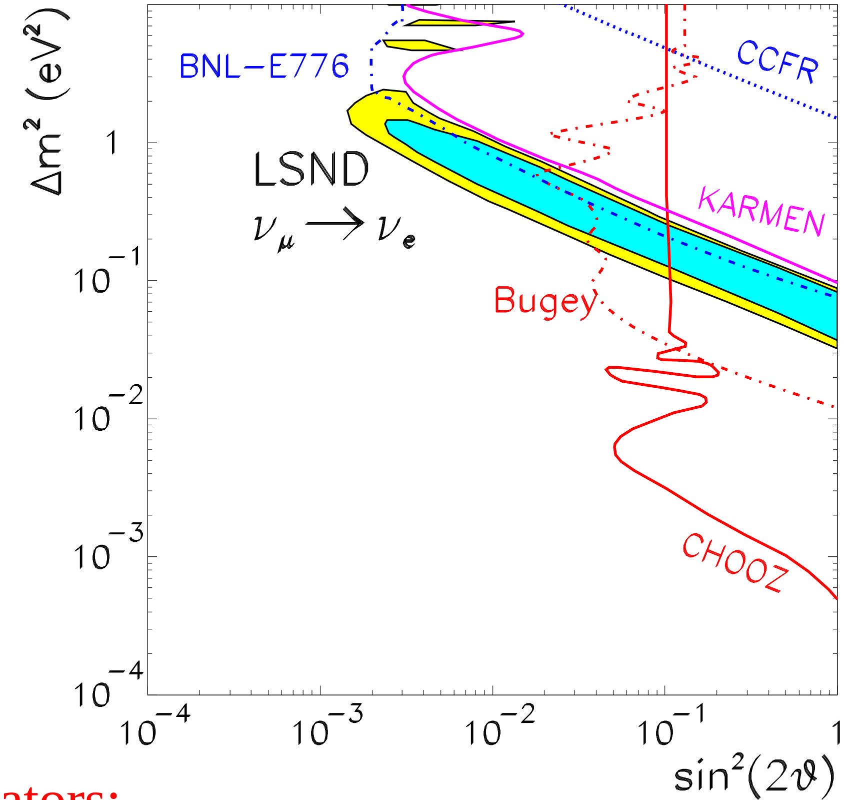
- observed $\overline{\nu_\mu} \to \overline{\nu_e}$ with probability $\langle P_{e\mu} \rangle = (0.26 \pm 0.07 \pm 0.05)\%$



- $Karmen$ which searched for the same signal and did not observe oscillations.
- $MiniBoone$ in Fermilab is running to solve this.

Summary of $\nu_\mu \to \nu_e$ at Short Baseline

- Reactor disappearance experiments
  - $\Rightarrow$ Lower E and longer L
  - $\Rightarrow$ are more sensitive to lower Δm^2



- Accelerator appearance experiments
  - $\Rightarrow$ higher E shorter L more precision
  - $\Rightarrow$ better limits on mixing

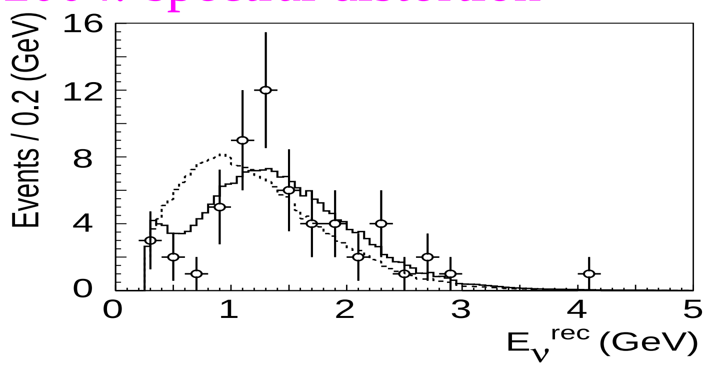
- To reach small $\Delta m^2 \gtrsim 10^{-4}$ eV^2
  - $\Rightarrow$ very large L and intermediate E
  - $\Rightarrow$ Long Baseline Experiments at Accelerators:

- To reach smaller $\Delta m^2 \gtrsim 10^{-5}$ eV^2 $\Rightarrow$ Long Baseline Experiments at Reactors

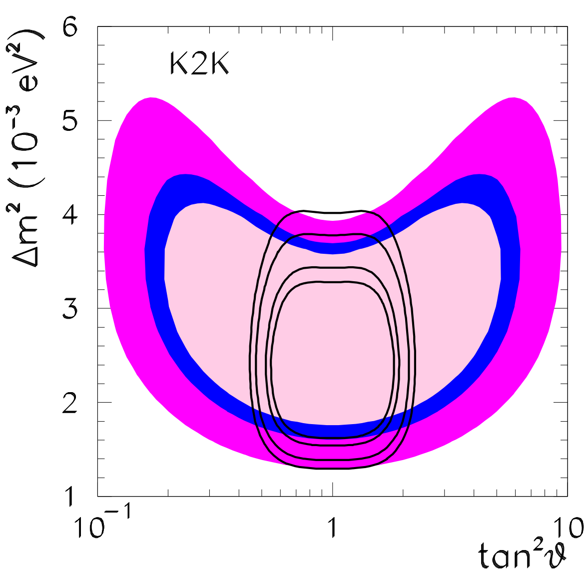
ATM Test at Long Baseline Experiments

K2K	ν_μ at KEK	SK	L=250 km
MINOS	ν_μ at Fermilab	Soundan	L=735 km
Opera/Icarus	ν_μ at CERN	Gran Sasso	L=740 km

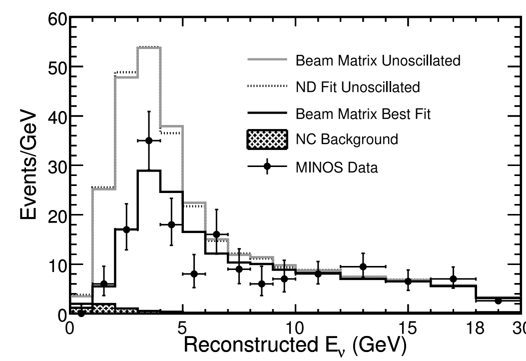
K2K 2004: spectral distortion



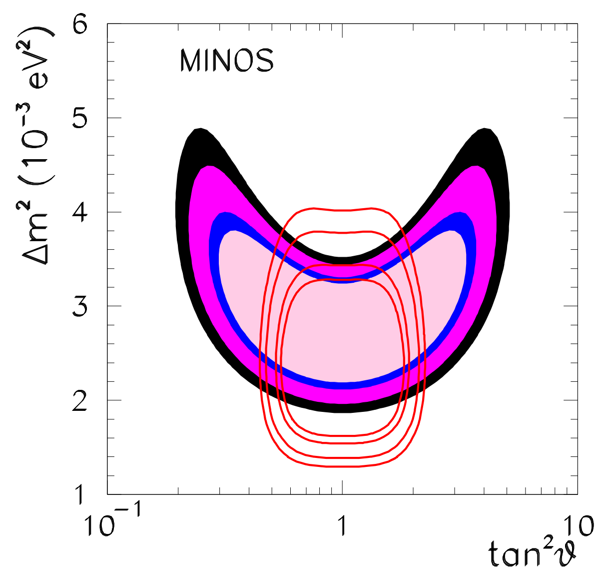
Confirmation of ATM oscillations



MINOS 2006: spectral distortion



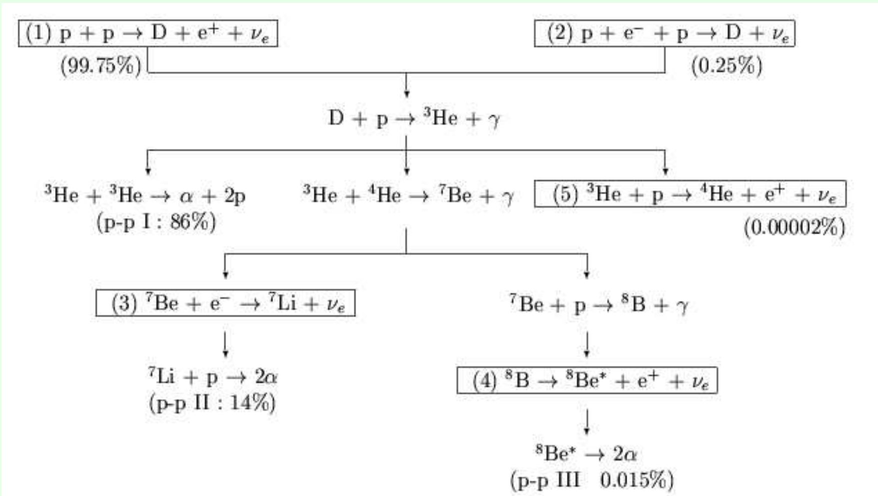
Confirmation of ATM oscillations



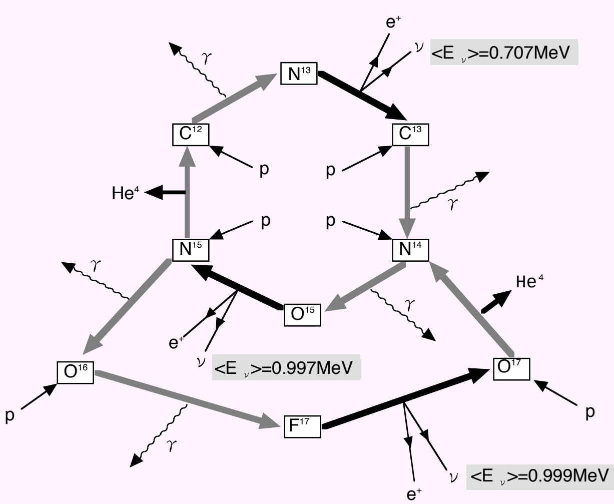
Solar Neutrinos: Fluxes

Concha Gonzalez-Garcia

pp chain



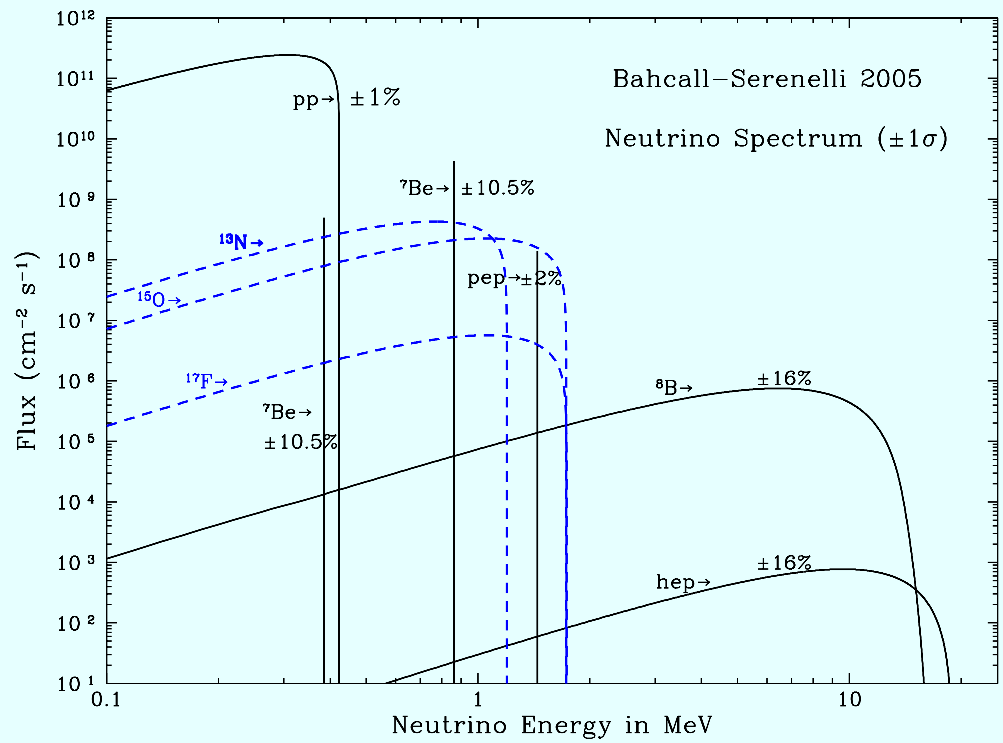
CNO cycle



- Sun shines by :

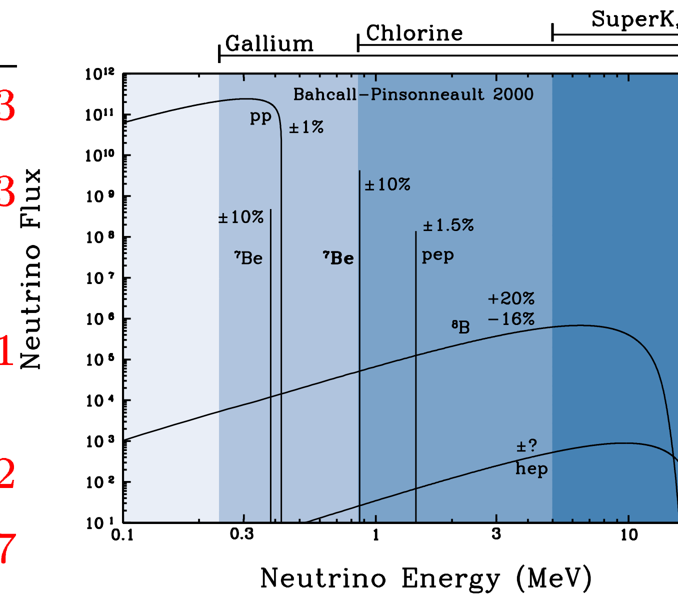
$$4\,p \;\rightarrow\; {}^4He \;+\; 2\,e^+ \;+\; 2\,\nu_e \;+\; \gamma$$

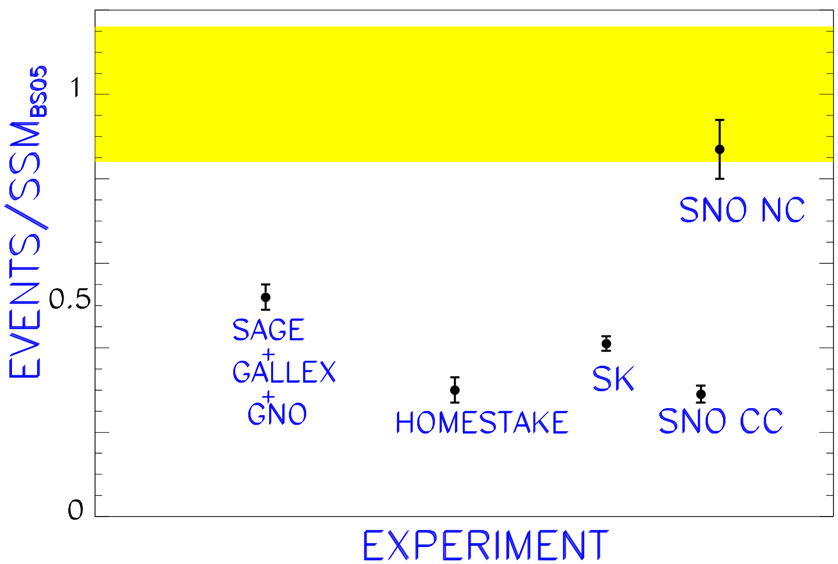
Solar Standard Model Fluxes



Solar Neutrinos: Data

	Experiment	Detection	Flavour	E_{th} (MeV)	$\frac{\text{Data}}{\text{BS05}}$
radio-chemical	Homestake	$^{37}\text{Cl}(\nu, e^-)^{37}\text{Ar}$	ν_e	$E_\nu > 0.81$	0.30 ± 0.03
	Sage + Gallex+GNO	$^{71}\text{Ga}(\nu, e^-)^{71}\text{Ge}$	ν_e	$E_\nu > 0.23$	0.52 ± 0.03
real time	Kam $\Rightarrow$ SK	ES $\nu_x e^- \to \nu_x e^-$	$\nu_e, \nu_{\mu/\tau}$ $\left(\frac{\sigma_{\mu\tau}}{\sigma_e} \simeq \frac{1}{6}\right)$	$E_e > 5$	0.41 ± 0.01
	SNO	CC $\nu_e d \to p p e^-$	ν_e	$T_e > 5$	0.29 ± 0.02
		NC $\nu_x d \to \nu_x d$	$\nu_e, \nu_{\mu/\tau}$	$T_\gamma > 5$	0.87 ± 0.07
		ES $\nu_x e^- \to \nu_x e^-$	$\nu_e, \nu_{\mu/\tau}$	$T_e > 5$	0.41 ± 0.05





All experiments measuring mostly ν_e observed a deficit

Deficit is energy dependent

Deficit disappears in NC

Solar Neutrinos: Flavour Conversion Evidence

SK and SNO measure $\Phi_{^8\mathrm{B}}$ in different reactions

$$\begin{array}{lll} \mathrm{ES}\ \ \nu_x e^- \to \nu_x e^- & \Phi_{^8\mathrm{B}}^{\mathrm{SK,ES}} & = (2.35 \pm 0.08) \times 10^6\ \mathrm{cm}^{-2}\mathrm{s}^{-1} \\ \mathrm{CC}\ \ \nu_e d \to p p e^- & \Phi_{^8\mathrm{B}}^{\mathrm{SNO,CC}} & = (1.68 \pm 0.1) \times 10^6\ \mathrm{cm}^{-2}\mathrm{s}^{-1} \\ \mathrm{NC}\ \ \nu_x d \to \nu_x d & \Phi_{^8\mathrm{B}}^{\mathrm{SNO,NC}} & = (4.94 \pm 0.42) \times 10^6\ \mathrm{cm}^{-2}\mathrm{s}^{-1} \end{array}$$

* In the SSM with SM interaction all results should be equal

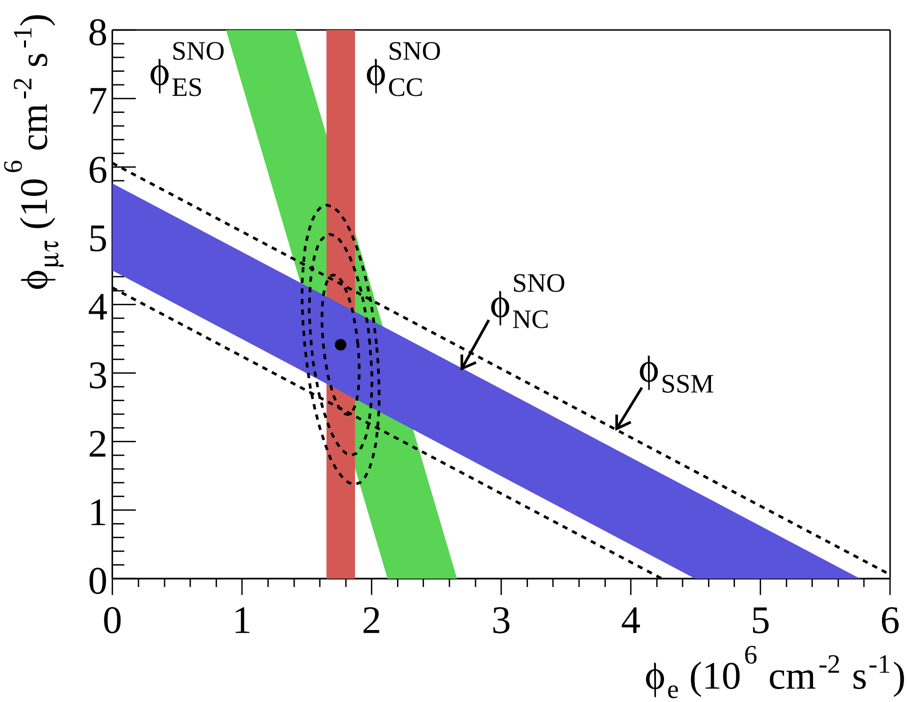
$$\Phi_{^8\mathrm{B}}^{\mathrm{ES,SK}} = \Phi_{^8\mathrm{B}}^{\mathrm{CC,SNO}} \Rightarrow 3.2\sigma \text{ out}$$

$$\Phi_{^8\mathrm{B}}^{\mathrm{NC,SNO}} = \Phi_{^8\mathrm{B}}^{\mathrm{CC,SNO}} \Rightarrow 7\sigma \text{ out}$$

* If flavour conversion everything fits perfectly:

$$\Phi^{\mathrm{CC}} = \Phi_e$$
$$\Phi^{\mathrm{ES}} = \Phi_e + r\,\Phi_{\mu\tau}$$
$$\Phi^{\mathrm{NC}} = \Phi_e + \Phi_{\mu\tau}$$

$$\left(r = \frac{\sigma_{\mathrm{ES}}(\nu_e)}{\sigma_{\mathrm{ES}}(\nu_\mu)} \simeq \frac{1}{6} \right)$$



Neutrinos in Matter:Effective Potentials

- In SM the characteristic ν-p interaction cross section

$$\sigma \sim \frac{G_F^2 E^2}{\pi} \sim 10^{-43}\text{cm}^2 \quad \text{at } \text{E}_\nu \sim \text{MeV}$$

- So if a beam of $\Phi_\nu \sim 10^{10} \nu' s$ was aimed at the Earth only 1 would be deflected

  so it seems that for neutrinos *matter does not matter*

- But that cross section is for *inelastic* scattering

  Does not contain forward elastic *coherent* scattering

- In *coherent* interactions $\Rightarrow$ ν and medium remain unchanged

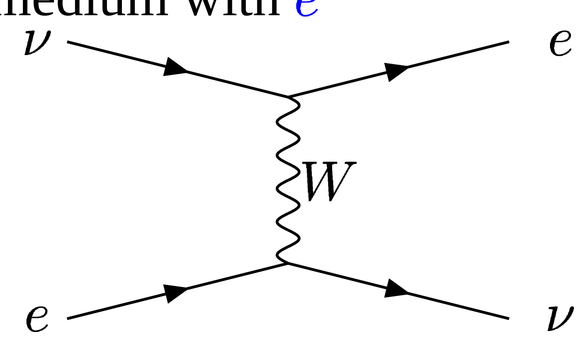
  Interference of scattered and unscattered ν waves

Neutrinos in Matter:Effective Potentials

- Coherence $\Rightarrow$ decoupling of ν evolution equation from equations of the medium.

- The effect of the medium is described by an effective potential depending on density and composition of matter

- For example for ν_e in medium with e^-



$$V_{CC} = \sqrt{2} G_F N_e$$

$N_e \equiv$ electron number density

- The effective potential has opposite sign for neutrinos y antineutrinos

- Other potentials for ν_e $(\overline{\nu}_e)$ due to different particles in medium

medium	V_C	V_N
e^+ and e^-	$\pm\sqrt{2}G_F(N_e - N_{\bar{e}})$	$\mp\frac{G_F}{\sqrt{2}}(N_e - N_{\bar{e}})(1-4\sin^2\theta_W)$
p and $\bar{p}$	0	$\pm\frac{G_F}{\sqrt{2}}(N_p - N_{\bar{p}})(1-4\sin^2\theta_W)$
n and $\bar{n}$	0	$\mp\frac{G_F}{\sqrt{2}}(N_n - N_{\bar{n}})$
Neutral $(N_e = N_p)$	$\pm\sqrt{2}G_F N_e$	$\mp\frac{G_F}{\sqrt{2}}N_n$

Neutrinos in Matter: Evolution Equation

Evolution Eq. for $|\nu\rangle = \nu_1|\nu_1\rangle + \nu_2|\nu_2\rangle \equiv \nu_e|\nu_e\rangle + \nu_X|\nu_X\rangle$ $(X = \mu, \tau, \text{sterile})$

(a) In vacuum in the mass basis:

$$-i\frac{\partial}{\partial x}\begin{pmatrix}\nu_1(x)\\ \nu_2(x)\end{pmatrix} = \left[E - \frac{m_1^2+m_2^2}{4E} - \begin{pmatrix}-\frac{\Delta m^2}{4E} & 0\\ 0 & \frac{\Delta m^2}{4E}\end{pmatrix}\right]\begin{pmatrix}\nu_1(x)\\ \nu_2(x)\end{pmatrix}$$

(b) In vacuum in the weak basis

$$-i\frac{\partial}{\partial x}\begin{pmatrix}\nu_e\\ \nu_X\end{pmatrix} = \left[E - \frac{m_1^2+m_2^2}{4E} - \begin{pmatrix}-\frac{\Delta m^2}{4E}\cos 2\theta & \frac{\Delta m^2}{4E}\sin 2\theta\\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta\end{pmatrix}\right]\begin{pmatrix}\nu_e\\ \nu_X\end{pmatrix}$$

(c) In matter (e, p, n) in weak basis

$$-i\frac{\partial}{\partial x}\begin{pmatrix}\nu_e\\ \nu_X\end{pmatrix} = \left[E - V_X - \frac{m_1^2+m_2^2}{2E} - \begin{pmatrix}V_e - V_X - \frac{\Delta m^2}{4E}\cos 2\theta & \frac{\Delta m^2}{4E}\sin 2\theta\\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta\end{pmatrix}\right]\begin{pmatrix}\nu_e\\ \nu_X\end{pmatrix}$$

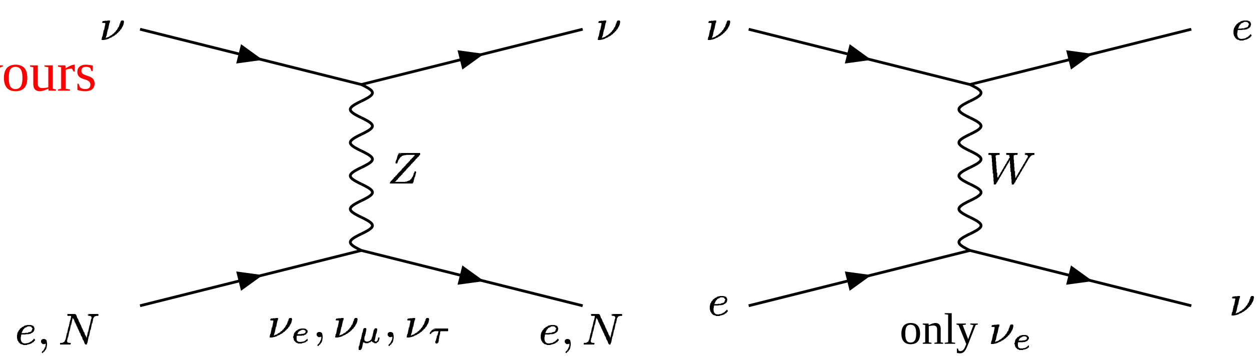
(c)$\neq$ (b) because different flavours

have different interactions

For example $X = \mu, \tau$:

$$V_{CC} = V_e - V_X = \sqrt{2}G_F N_e$$

(opposite sign for $\overline{\nu}$)

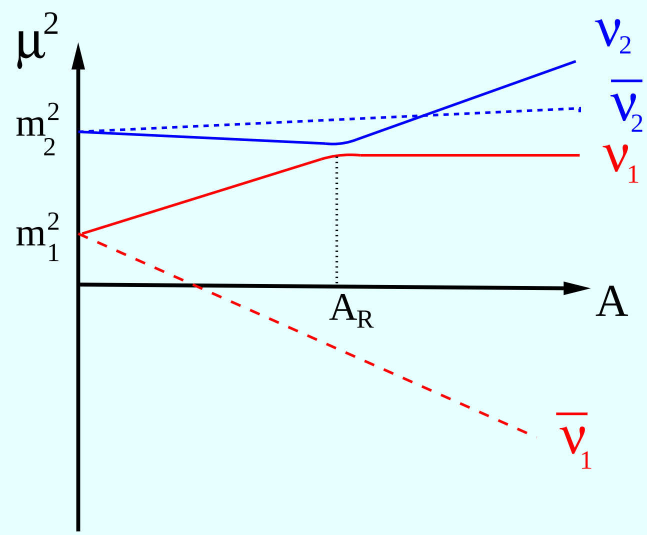


⇒ Effective masses and mixing are different than in vacuum

⇒ If matter density varies along ν trajectory the effective masses and mixing vary too

The effective masses: $(A = 2E(V_e - V_X))$

$$\mu_{1,2}(x) = \frac{m_1^2 + m_2^2}{2} + E(V_e + V_X)$$
$$\pm \frac{1}{2}\sqrt{(\Delta m^2 \cos 2\theta - A)^2 + (\Delta m^2 \sin 2\theta)^2}$$

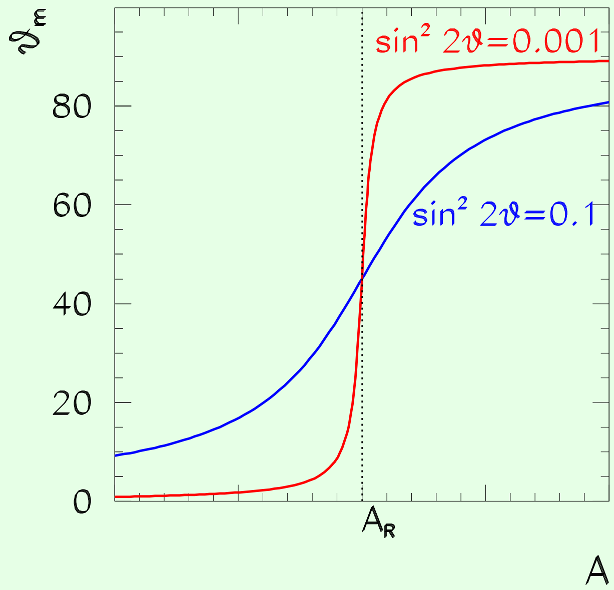


At *resonant* potential: $A_R = \Delta m^2 \cos 2\theta$

Minimum $\Delta\mu^2 = \mu_2^2 - \mu_1^2$

The mixing angle in matter

$$\tan 2\theta_m = \frac{\Delta m^2 \sin 2\theta}{\Delta m^2 \cos 2\theta - A}$$



* At $A = 0$ (vacuum) ⇒ $\theta_m = \theta$
* At $A = A_R$ ⇒ $\theta_m = \frac{\pi}{2}$
* At $A >> A_R$ ⇒ $\theta_m = \frac{\pi}{2} - \theta$

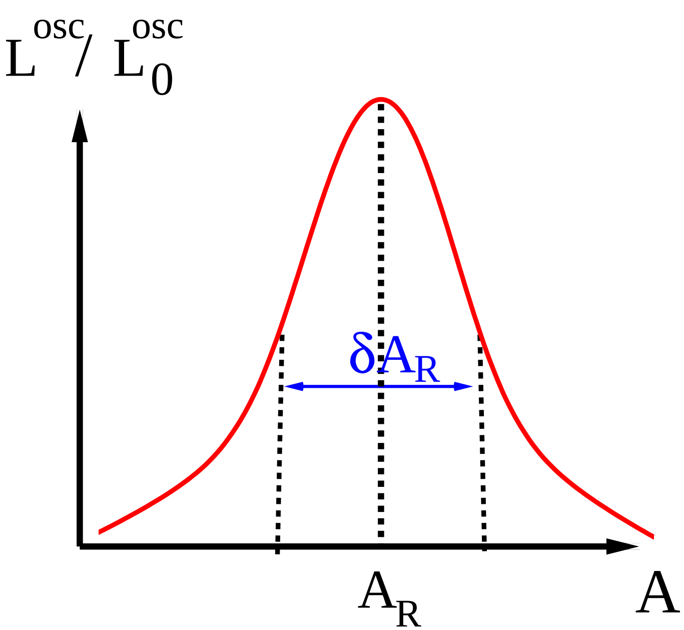
The oscillation length in vacuum

$$L_0^{osc} = \frac{4\pi E}{\Delta m^2}$$

The oscillation length in matter

$$L^{osc} = \frac{L_0^{osc}}{\sqrt{(\Delta m^2 \cos 2\theta - A)^2 + (\Delta m^2 \sin 2\theta)^2}} \equiv \frac{4\pi E}{\Delta \mu^2}$$

L^{osc} presents a resonant behaviour



At the resonant point

$$L_R^{osc} = \frac{L_0^{osc}}{\sin 2\theta}$$

The width of the resonance in potential:

$$\delta V_R = \frac{\Delta m^2 \sin 2\theta}{E}$$

The width of the resonance in distance:

$$\delta r_R = \frac{\delta V_R}{|\frac{dV}{dr}|_R}$$

- In terms of the mass eigenstates in matter: $$\begin{pmatrix} \nu_e \\ \nu_X \end{pmatrix} = U[\theta_m(x)] \begin{pmatrix} \nu_1^m(x) \\ \nu_2^m(x) \end{pmatrix}$$

- For constant potential θ_m and μ_i are constant along ν evolution

  $\Rightarrow$ the evolution is determined by masses and mixing in matter as

$$P_{osc} = \sin^2(2\theta_m)\, \sin^2\left(\frac{\Delta\mu^2 L}{2E}\right)$$

- For varying potential: $$\begin{pmatrix} \dot\nu_e \\ \dot\nu_X \end{pmatrix} = \dot U[\theta_m(x)] \begin{pmatrix} \nu_1^m(x) \\ \nu_2^m(x) \end{pmatrix} + U[\theta_m(x)] \begin{pmatrix} \dot\nu_1^m(x) \\ \dot\nu_2^m(x) \end{pmatrix}$$

$\Rightarrow$ the evolution equation in flavour basis (removing diagonal part)

$$i \begin{pmatrix} \dot\nu_e \\ \dot\nu_X \end{pmatrix} = \frac{1}{2E} \begin{pmatrix} A - \frac{\Delta m^2}{2}\cos 2\theta & \frac{\Delta m^2}{2}\sin 2\theta \\ \frac{\Delta m^2}{2}\sin 2\theta & \frac{\Delta m^2}{2}\cos 2\theta \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_X \end{pmatrix}$$

$\Rightarrow$ the evolution equation in instantaneous mass basis

$$i \begin{pmatrix} \dot\nu_1^m \\ \dot\nu_2^m \end{pmatrix} = \frac{1}{2E} U^\dagger(\theta_m) \begin{pmatrix} A - \frac{\Delta m^2}{2}\cos 2\theta & \frac{\Delta m^2}{2}\sin 2\theta \\ \frac{\Delta m^2}{2}\sin 2\theta & \frac{\Delta m^2}{2}\cos 2\theta \end{pmatrix} U(\theta_m) \begin{pmatrix} \nu_1^m \\ \nu_2^m \end{pmatrix} - i\, U^\dagger \dot U(\theta_m) \begin{pmatrix} \nu_1^m \\ \nu_2^m \end{pmatrix}$$

$$\Rightarrow \quad i \begin{pmatrix} \dot\nu_1^m \\ \dot\nu_2^m \end{pmatrix} = \frac{1}{4E} \begin{pmatrix} -\Delta\mu^2(x) & -4\, i\, E\, \dot\theta_m(x) \\ 4\, i\, E\, \dot\theta_m(x) & \Delta\mu^2(x) \end{pmatrix} \begin{pmatrix} \nu_1^m \\ \nu_2^m \end{pmatrix}$$

- The evolution equation in instantaneous mass basis

$$i \begin{pmatrix} \dot{\nu}_1^m \\ \dot{\nu}_2^m \end{pmatrix} = \frac{1}{4E} \begin{pmatrix} -\Delta\mu^2(x) & -4\,i\,E\,\dot{\theta}_m(x) \\ 4\,i\,E\,\dot{\theta}_m(x) & \Delta\mu^2(x) \end{pmatrix} \begin{pmatrix} \nu_1^m \\ \nu_2^m \end{pmatrix}$$

$\Rightarrow$ It is not diagonal $\Rightarrow$ Instantaneous mass eigenstates $\neq$ eigenstates of evolution

$\Rightarrow$ Transitions $\nu_1^m \to \nu_2^m$ can occur $\equiv$ *Non adiabaticity*

- For $\Delta\mu^2(x) \gg 4\,E\,\dot{\theta}_m(x)$ $\left[\frac{1}{V}\frac{dV}{dx}\Big|_R \ll \frac{\Delta m^2}{2E}\frac{\sin^2 2\theta}{\cos 2\theta} \right]$ $\equiv$ Slowly varying matter potent

$\Rightarrow$ ν_i^m behave approximately as *evolution eigenstates*

$\Rightarrow$ ν_i^m do not mix in the evolution This is the *adiabatic* transition approximation

The adiabaticity condition

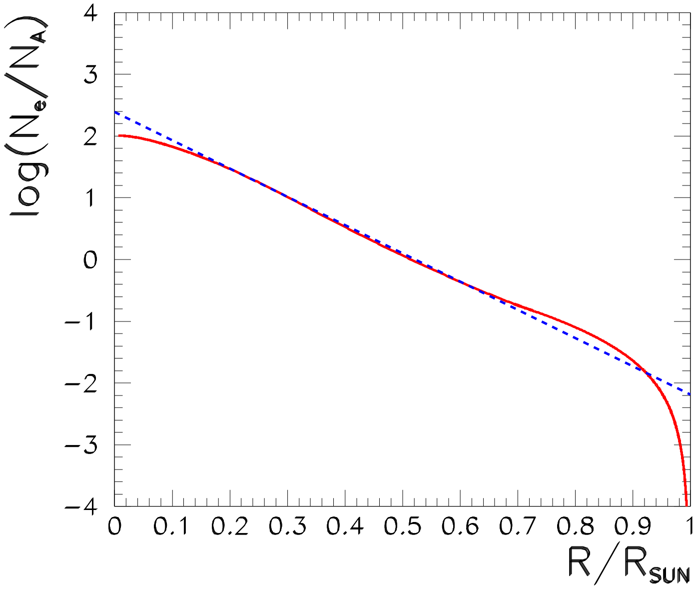
$$\frac{1}{V}\frac{dV}{dx}\Big|_R \ll \frac{\Delta m^2}{2E}\frac{\sin^2 2\theta}{\cos 2\theta} \equiv 2\pi\,\delta r_R \gg L_R^{osc}$$

$\Rightarrow$ Many oscillations take place in the resonant region

Neutrinos in The Sun : MSW Effect

- Solar neutrinos are ν_e produced in the core ($R \lesssim 0.3 R_\odot$) of the Sun

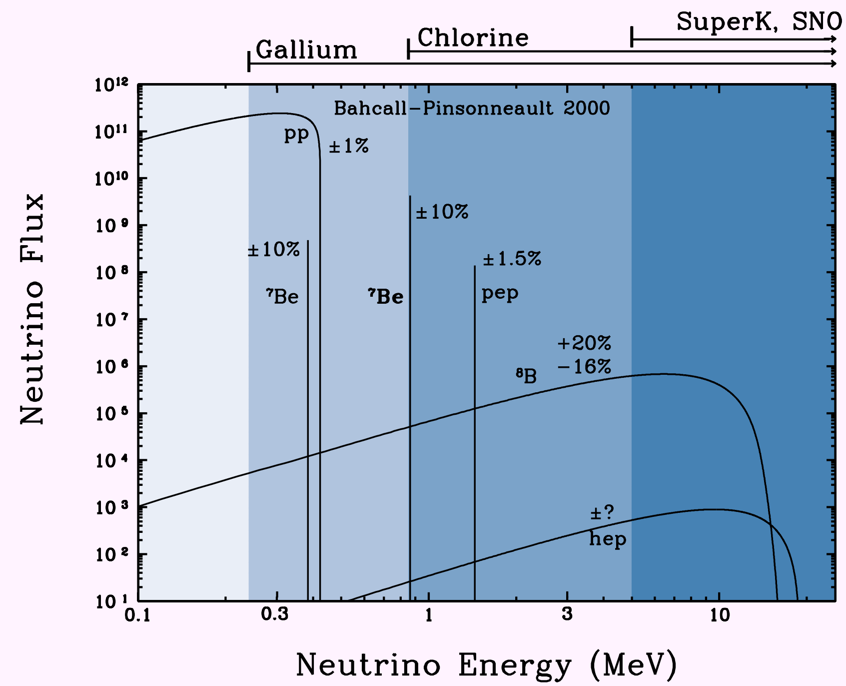
The solar matter density



$$V_{CC} = \sqrt{2} G_F N_e \sim 10^{-14} \frac{N_e}{N_A} \text{ eV}$$

At core: $V_{CC,0} \sim 10^{-14}$–10^{-12} eV

The energy spectrum of solar $\nu_e' s$



$E_\nu \sim 0.1$–10 MeV

- For $\nu_e \leftrightarrow \nu_{\mu(\tau)}$, in vacuum $\nu_e = \cos\theta\, \nu_1 + \sin\theta\, \nu_2$
- For 10^{-9} eV$^2 \lesssim \Delta m^2 \lesssim 10^{-4}$ eV^2 $\Rightarrow$ $2E_\nu V_{CC,0} > \Delta m^2 \cos 2\theta$

$\Rightarrow$ ν can cross resonance condition in its way out of the Sun

For $\theta \ll \frac{\pi}{4}$: In vacuum $\nu_e = \cos\theta\,\nu_1 + \sin\theta\,\nu_2$ is mostly ν_1

In Sun core $\nu_e = \cos\theta_{m,0}\,\nu_1 + \sin\theta_{m,0}\,\nu_2$ is mostly ν_2

If $\frac{(\Delta m^2/\text{eV}^2)\sin^2 2\theta}{(E/\text{MeV})\cos 2\theta} \gg 3\times 10^{-9}$

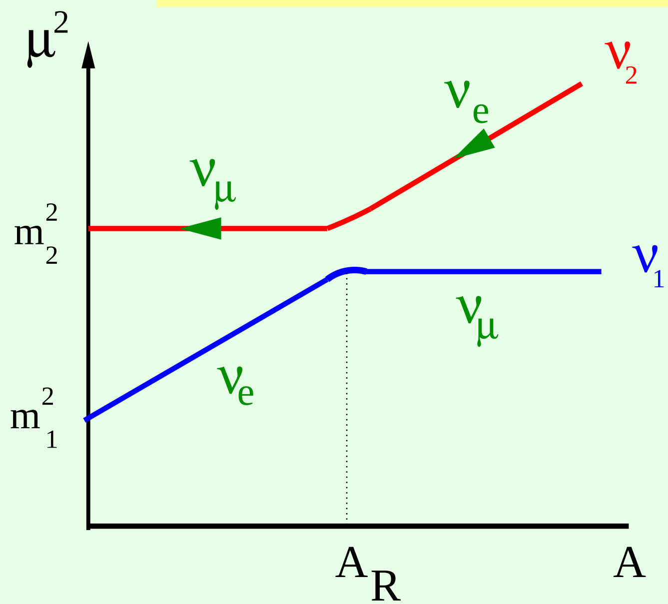
$\Rightarrow$ Adiabatic transition

$*$ ν is mostly ν_2 before and after resonance

$*$ $\theta_m \downarrow$ *dramatically* at resonance

$\Rightarrow$ ν_e component $\downarrow \Rightarrow P_{ee}\downarrow$

This is the MSW effect



$$P_{ee} = \frac{1}{2}\left[1 + \cos 2\theta_{m,0}\cos 2\theta\right]$$

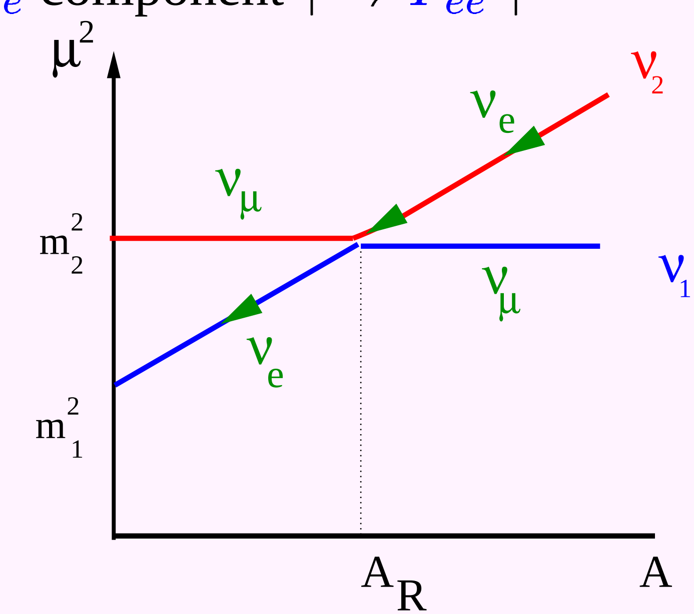
If $\frac{(\Delta m^2/\text{eV}^2)\sin^2 2\theta}{(E/\text{MeV})\cos 2\theta} \lesssim 3\times 10^{-9}$

$\Rightarrow$ Non-Adiabatic transition

$*$ ν is mostly ν_2 till the resonance

$*$ At resonance the state can jump into ν_1 (with probability P_{LZ})

$\Rightarrow$ ν_e component $\uparrow \Rightarrow P_{ee}\uparrow$

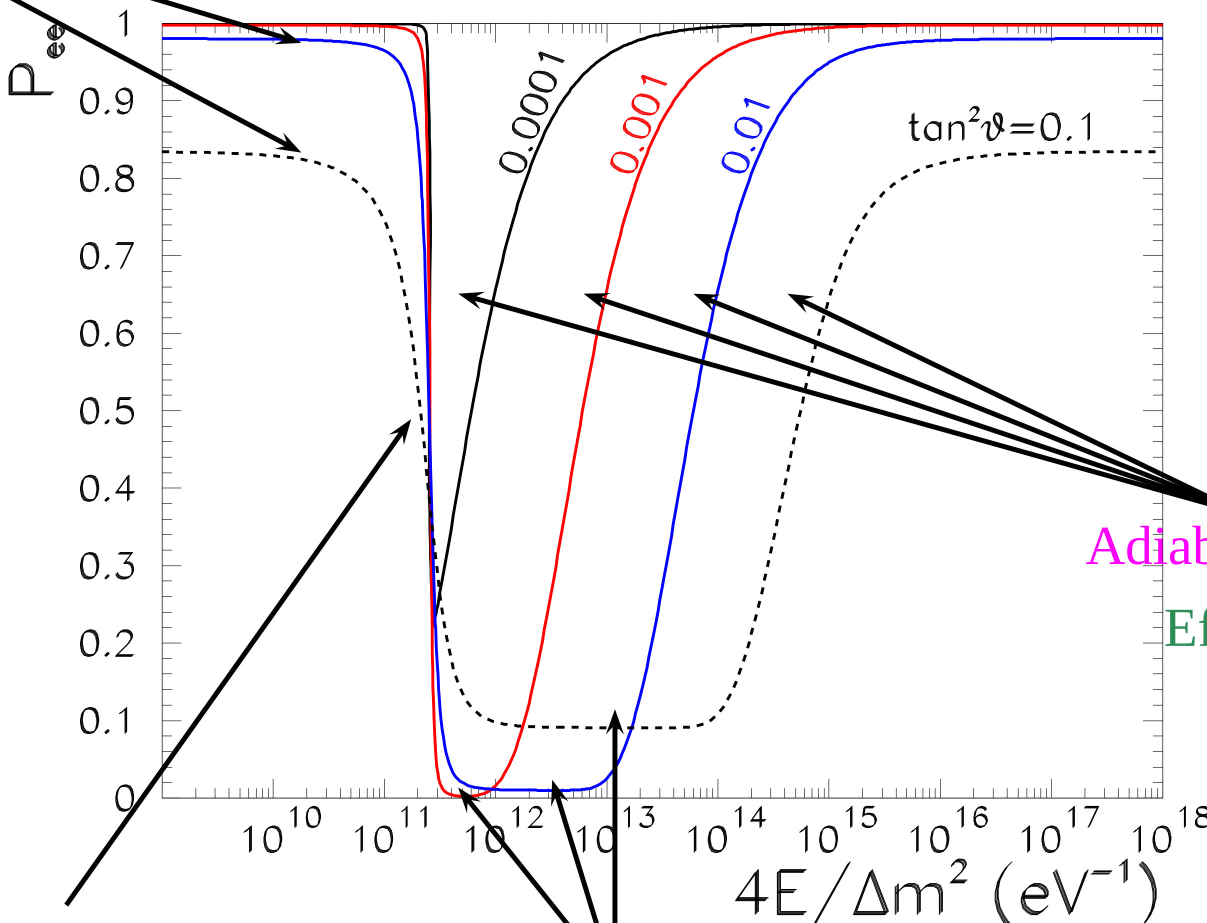


$$P_{ee} = \frac{1}{2}\left[1 + (1-2P_{LZ})\cos 2\theta_{m,0}\cos 2\theta\right]$$

Concha Gonzalez-Garcia

Neutrinos in The Sun : MSW Effect

ν does not cross resonance: $P_{ee} = 1 - \frac{1}{2}\sin^2 2\theta > \frac{1}{2}$



Adiabacity breaking

Effect of P_{LZ}

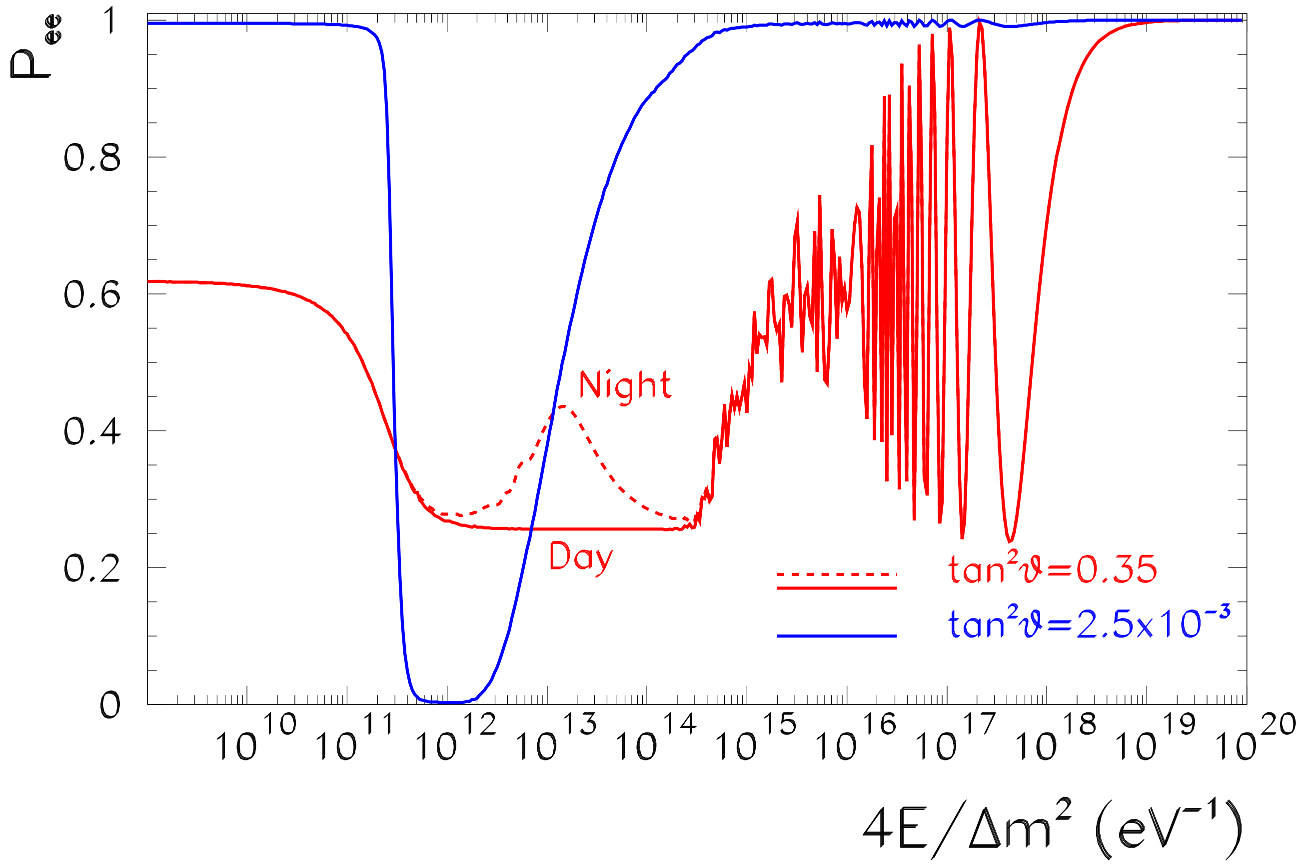
ν crosses resonance

MSW effect

Adiabatic MSW transition $P_{ee} = \sin\theta < \frac{1}{2}$

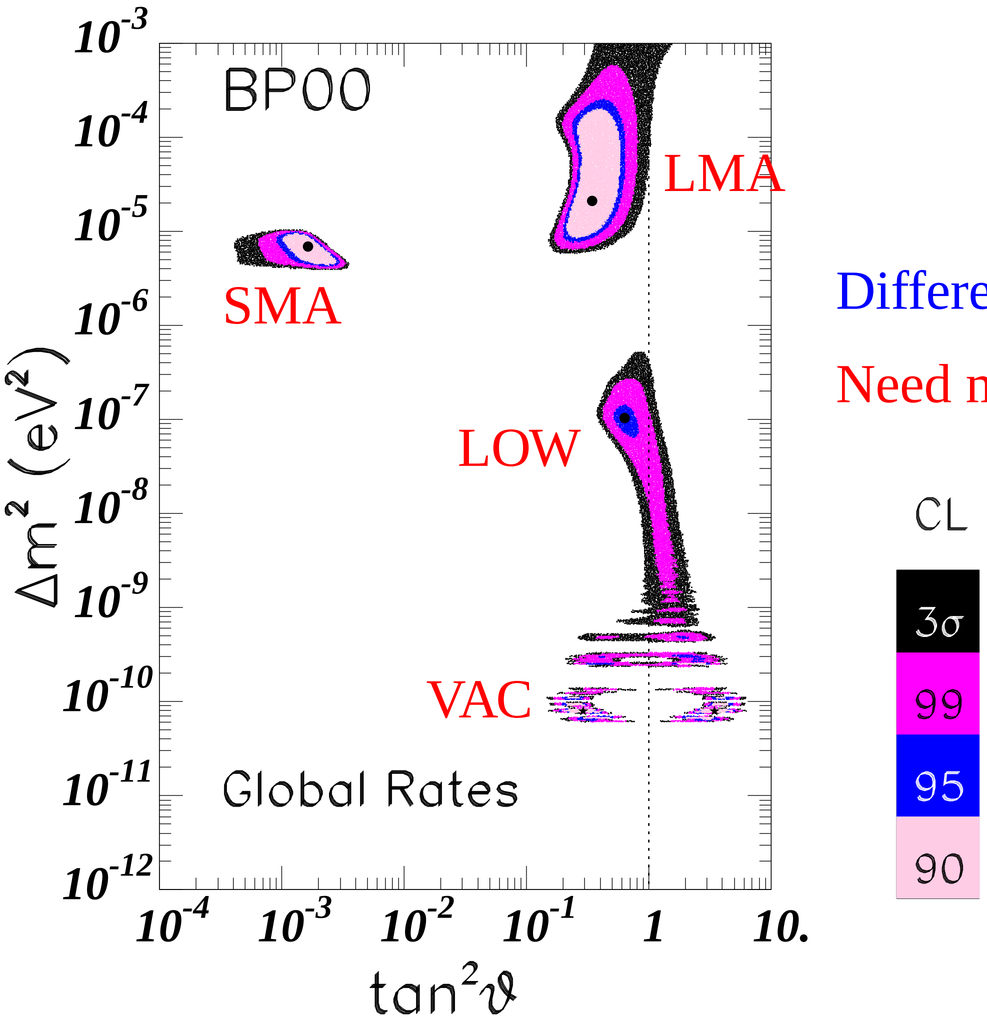
Neutrinos from The Sun : The Full Story

$$A(\nu_e \to \nu_e) = A_{Sun}(\nu_e \to \nu_1) \times A_{\text{vac}}(\nu_1 \to \nu_1) \times A_{Earth}(\nu_1 \to \nu_e)$$
$$+A_{Sun}(\nu_e \to \nu_2) \times A_{\text{vac}}(\nu_2 \to \nu_2) \times A_{Earth}(\nu_2 \to \nu_e)$$



Solar Neutrinos: Oscillation Solutions

Allowed regions by Fit to Total Rates: Cl, Ga, SK and SNO CC



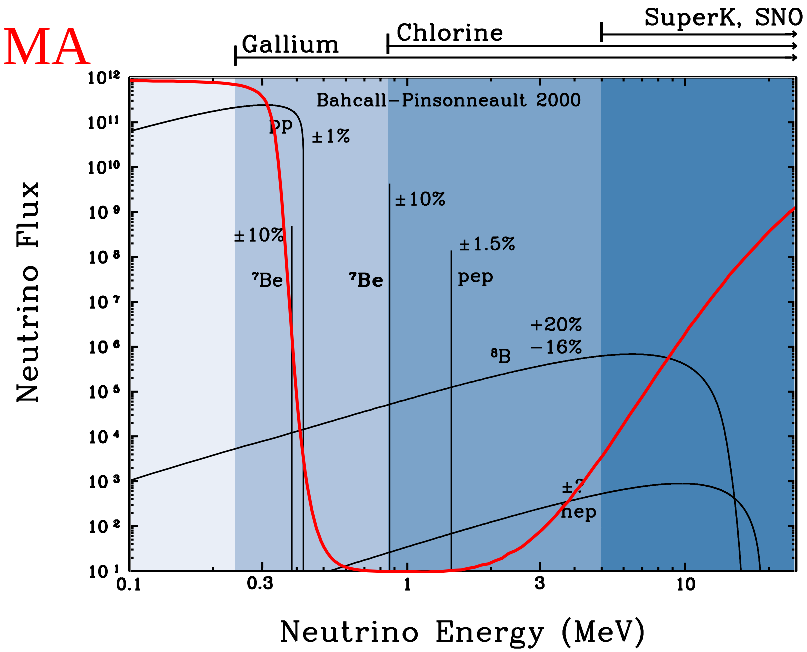
Different regimes can explain the Total Rates

Need more observables to discriminate

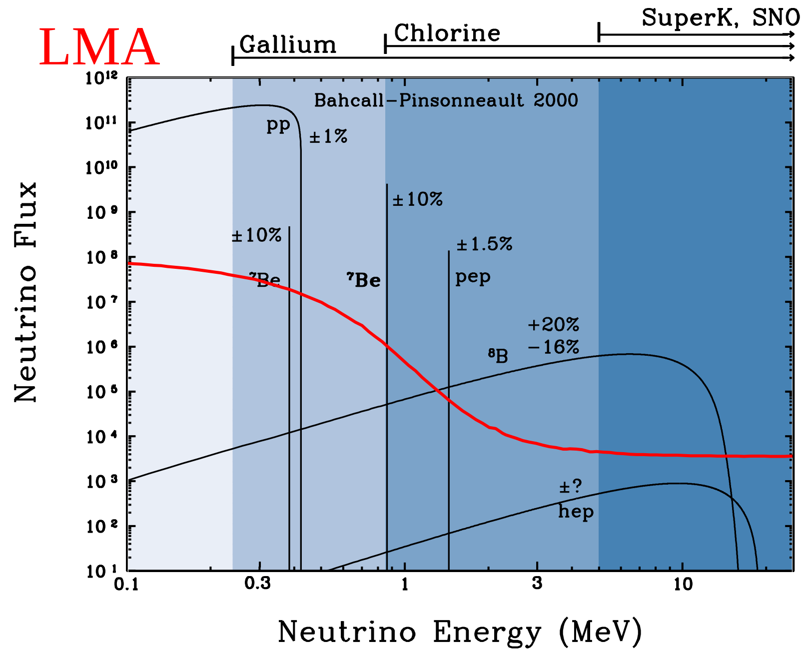
Concha Gonzalez-Garcia

Energy Dependence of P_{ee} for Different Solutions

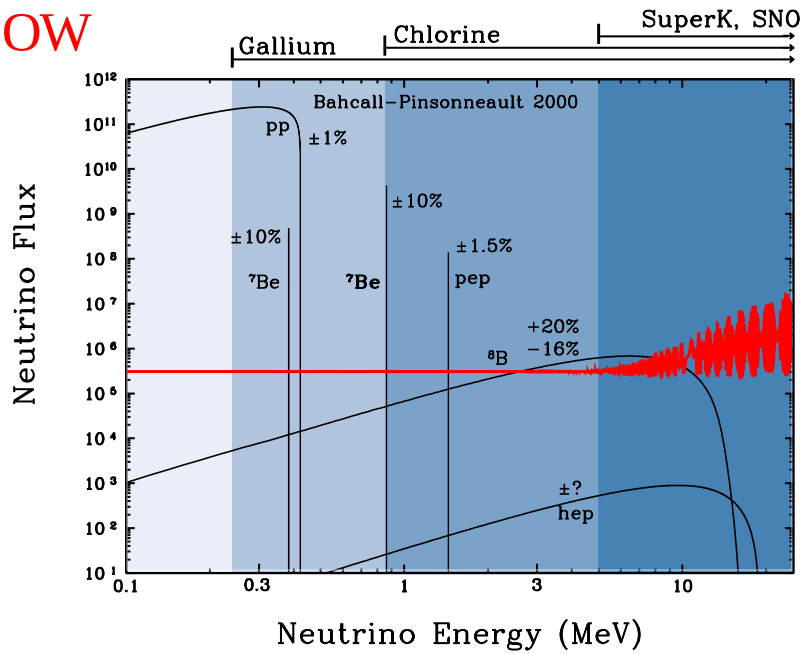
SMA



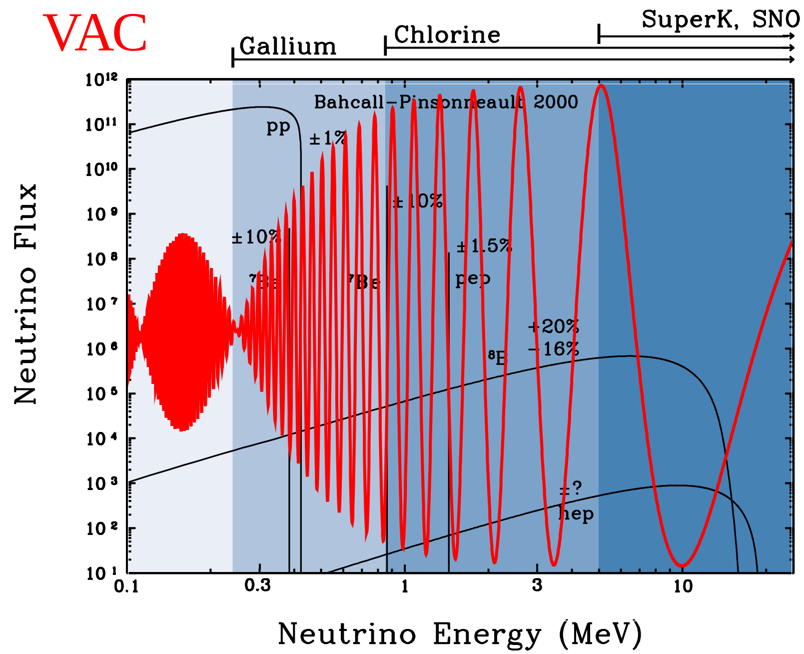
LMA



LOW



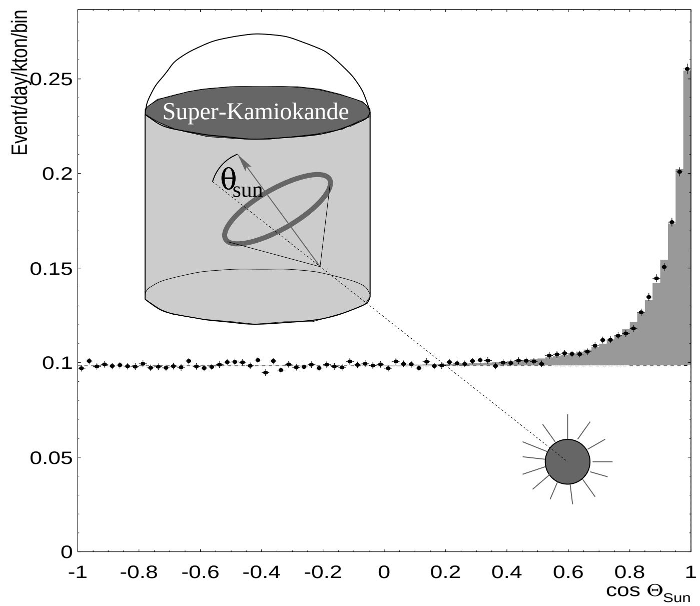
VAC



- Real Time experiments can also give information on Energy and Direction of $\nu' s$ and can search for Energy and Time variations of the effect

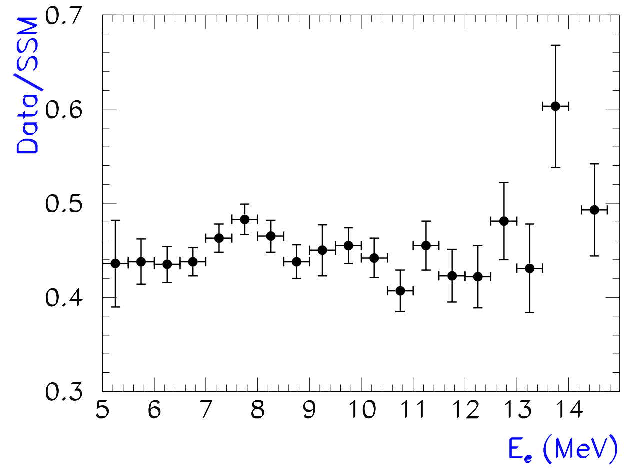
- From SK (Confirmed by SNO)

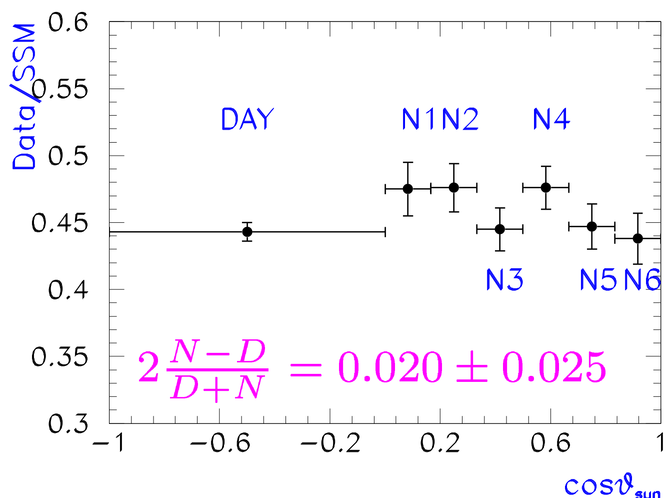
ν's come from the SUN



No Energy Distorsion

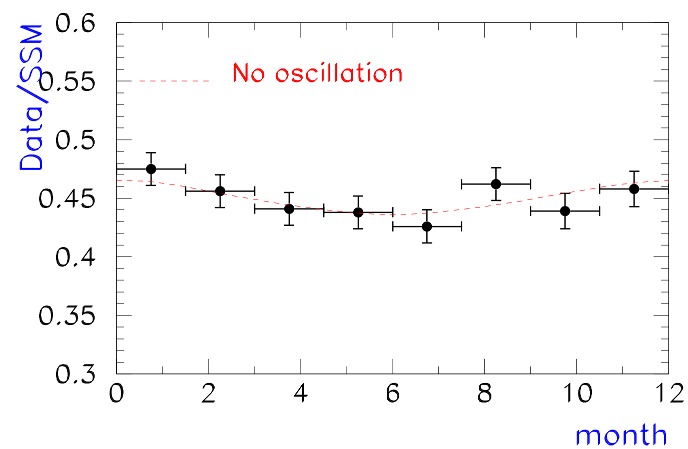
Deficit indep $E_\nu \gtrsim 5$ MeV





No Earth Matter Effect:

Small Day-Night Asymmetry

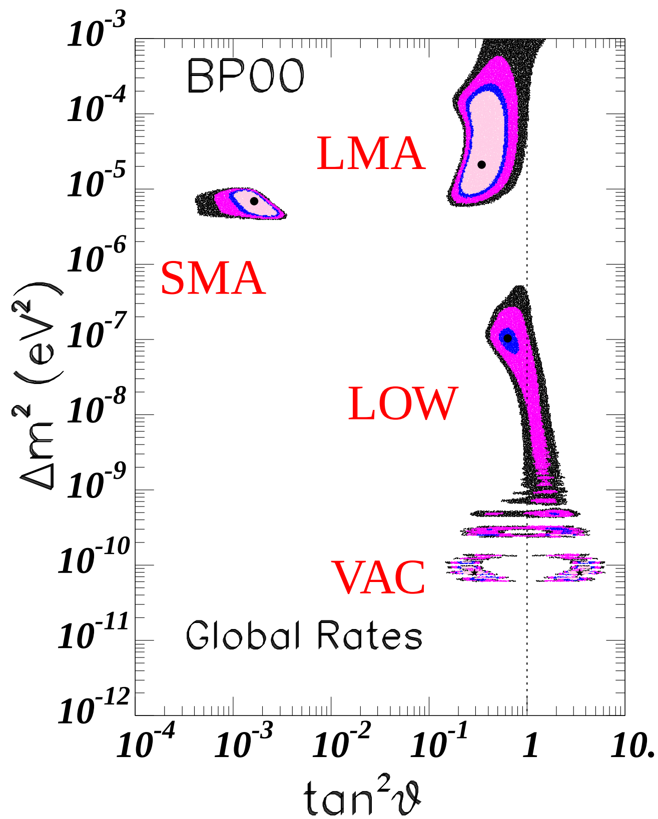


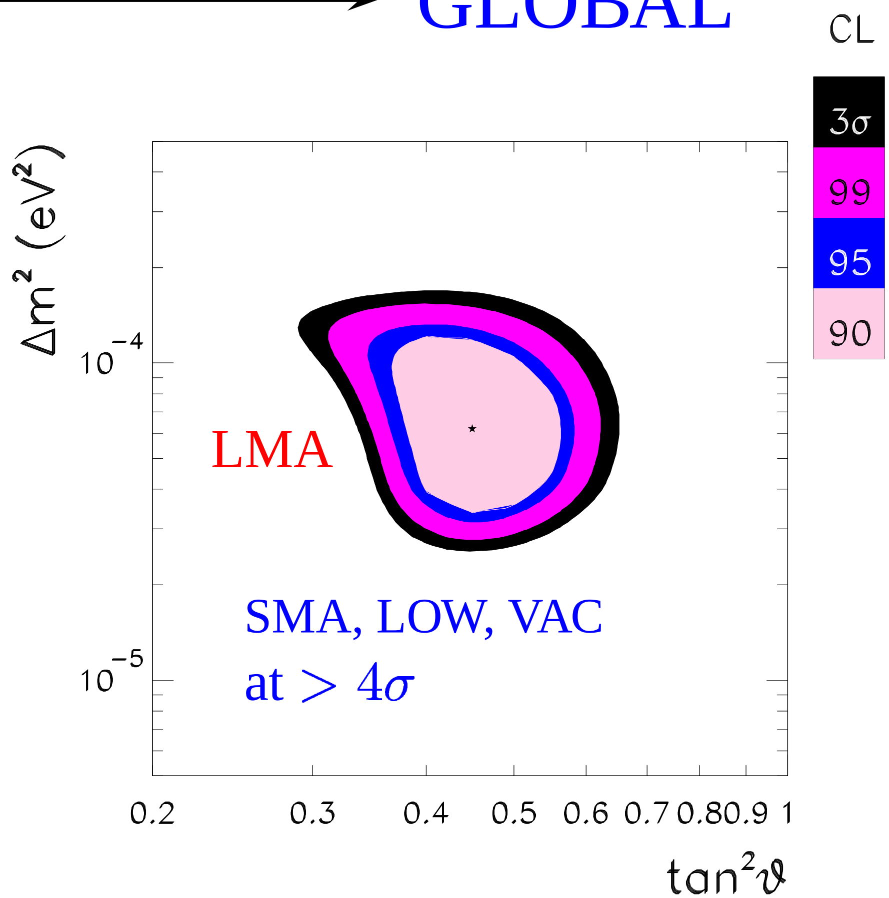
Seasonal Variation

Nothing beyond $1/R^2$

Solar Neutrinos: Oscillation Solutions

RATES ONLY —(SK and SNO E and t dependence)→ GLOBAL





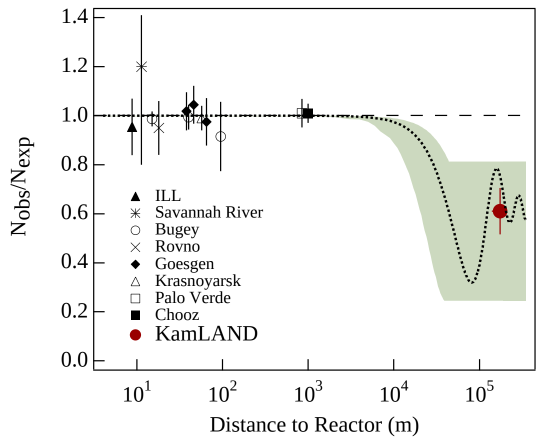
$$\Delta m^2 = \left(6.3^{+2.3}_{-1.9}\right) \times 10^{-5}\ \mathrm{eV}^2\ (1\sigma)$$

$$\tan^2\theta = 0.45^{+0.05}_{-0.04}$$

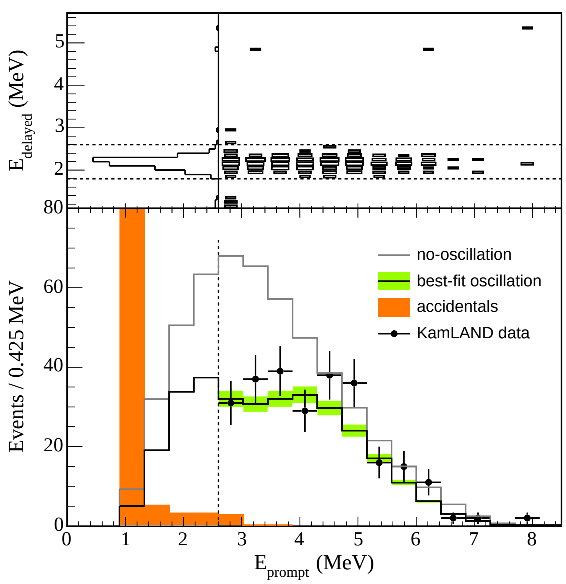
Terrestrial Test of LMA: KamLAND

- Search on $\overline{\nu_e}$ at L$\sim$ 180 km reactors, $E_{\overline{\nu}}$ $\sim$ few MeV: $\bar{\nu}_e + p \to n + e^+$

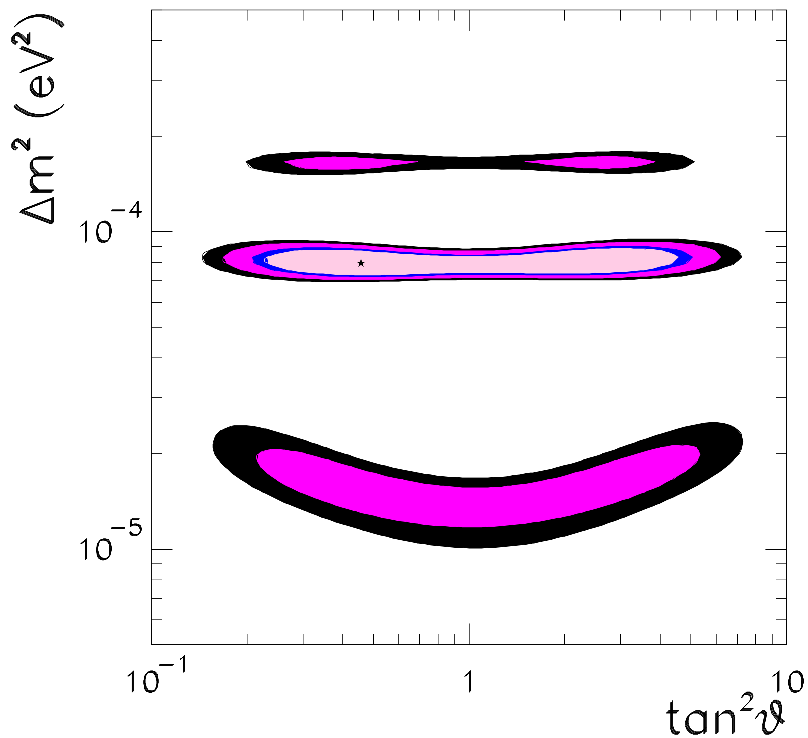
2002: Deficit $R_{\rm KLAND} = 0.611 \pm 0.094$



2004: Significant Energy Distortion



Oscillation Analysis

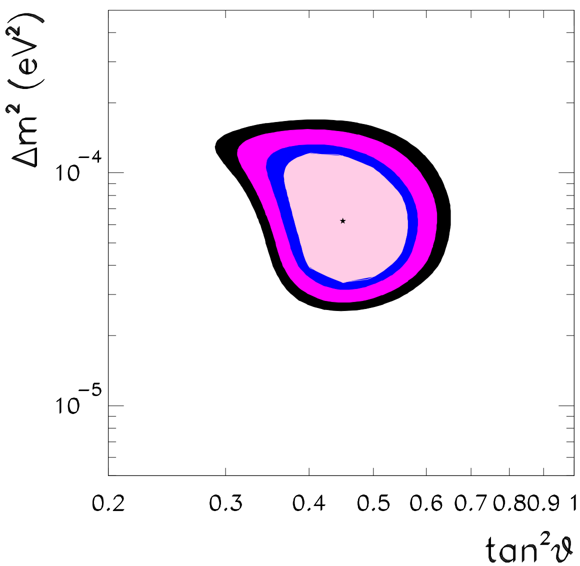


$$\Delta m^2 = \left(7.9^{+0.4}_{-0.3}\right) \times 10^{-5}\ {\rm eV}^2\ (1\sigma)$$

$$\tan^2\theta = 0.46^{+0.20}_{-0.15} \quad \left(2.2^{+1.0}_{-0.6}\right)$$

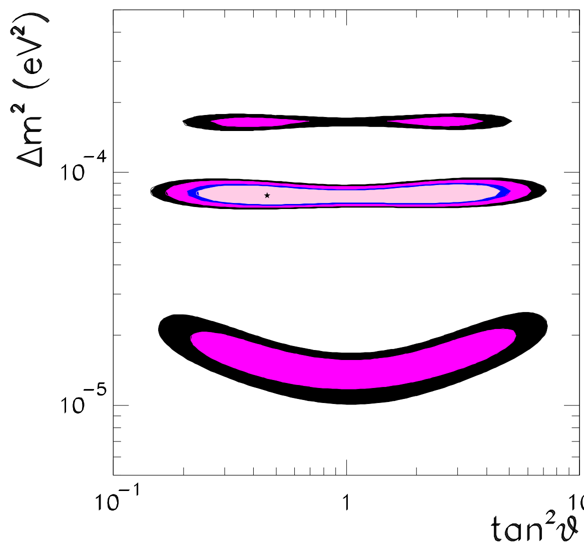
Solar

$\nu_e \to \nu_{\text{active}}$

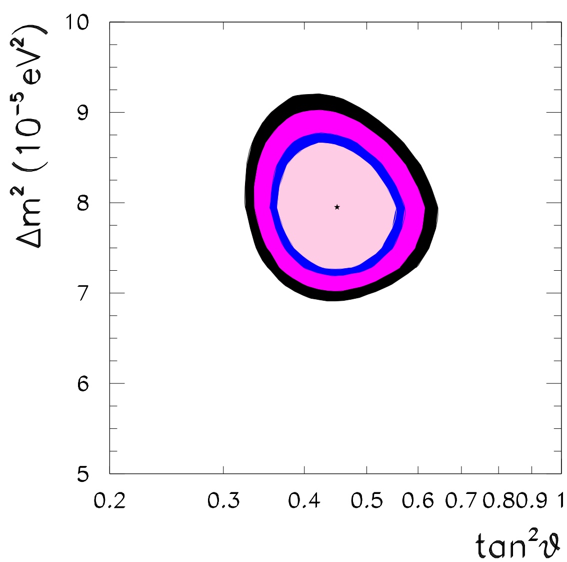


+ KamLAND

$\bar{\nu}_e \not\to \bar{\nu}_e$



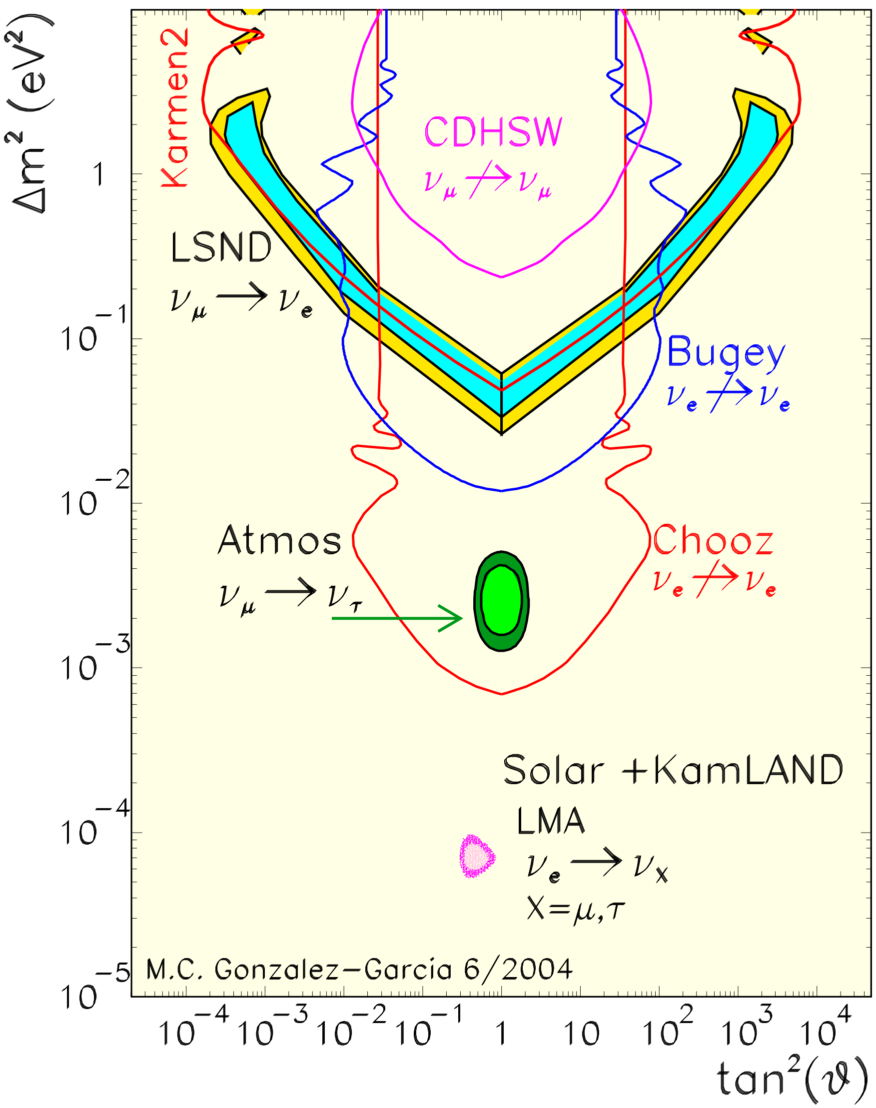
ν_e oscillation parameters compatible with $\overline{\nu}_e$: *Sensible* to assume CPT: $P_{ee} = P_{\bar{e}\bar{e}}$



$$\Delta m^2_\odot = \left(8^{+0.4}_{-0.5}\right) \times 10^{-5}\ \text{eV}^2 \quad (1\sigma)$$

$$\tan^2\theta_\odot = 0.45^{+0.05}_{-0.05}$$

Two Neutrino Oscillations: Summary



- How to fit all this together?

– 3 oscillation signals in 3 different scales

$$\Delta m^2_{\rm SOLAR} \ll \Delta m^2_{\rm ATM} \ll \Delta m^2_{\rm LSND}$$

– Mixing of ν_e, ν_μ, ν_τ $\to$ 2 mass diff

$\to$ Explain only two evidences:

For example Solar + Atmos

- Theorists have tried hard to fit LSND in:

– Adding a fourth *sterile* neutrino

– Breaking CPT…

…but nothing works well

The Naked True:

If Miniboone finds a signal we have no good theory of what to do

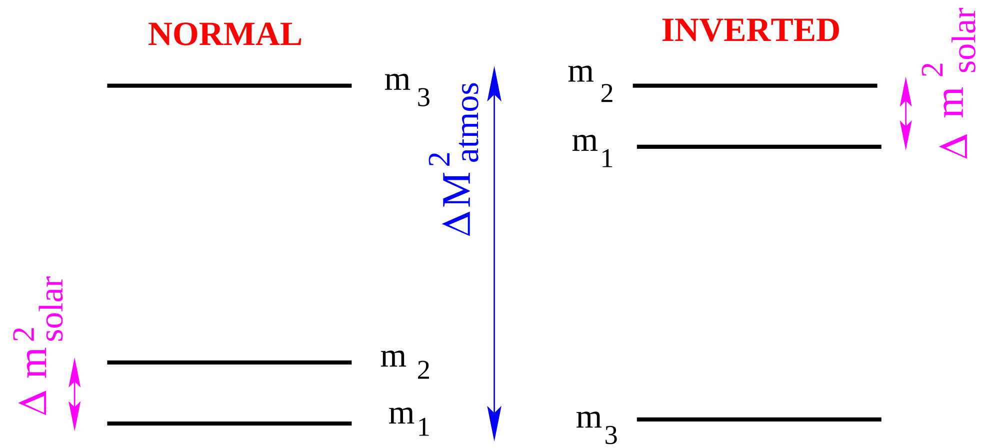
Ergo I am going to ignore LSND

Solar+Atmospheric+Reactor+LBL 3ν Oscillations

U: 3 angles, 1 CP-phase
+ (2 Majorana phases)

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{21} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Two mass schemes



2ν oscillation analysis $\Rightarrow \Delta m^2_{21} = \Delta m^2_{\odot} \ll \Delta M^2_{atm} \simeq \pm\Delta m^2_{32} \simeq \pm\Delta m^2_{31}$

Generic 3ν mixing effects:

– Interference of two wavelength oscillations

– Effects due to θ_{13}

– Difference between Inverted and Normal

– CP violation due to phase δ

In Present Data:

2 wavelengths *Unobservable*
θ_{13} *Only a limit*
N versus I *Below sensitivity*
CP violation *Unobservable*

• But all these 3ν effects within reach of planned experiments

Global Analysis: Three Neutrino Oscillations

3 σ ranges:

$$7 \le \frac{\Delta m^2_{21}}{10^{-5}\mathrm{eV}^2} \le 9.1 \qquad 1.9 \le \frac{\Delta m^2_{32}}{10^{-3}\mathrm{eV}^2} \le 3.25$$

$$0.34 \le \tan^2\theta_{12} \le 0.62 \quad 0.49 \le \tan^2\theta_{23} \le 2.2 \quad \sin^2\theta_{13} \le 0.045$$

$$-\pi \le \delta \le \pi$$

$$|U_{\rm LEP}| = \begin{pmatrix} 0.79-0.86 & 0.50-0.61 & <0.20 \\ 0.19-0.53 & 0.39-0.72 & 0.58-0.82 \\ 0.22-0.55 & 0.43-0.74 & 0.55-0.81 \end{pmatrix}$$

with structure

$$|U_{\rm LEP}| \simeq \begin{pmatrix} \frac{1}{\sqrt{2}}(1+\mathcal{O}(\lambda)) & \frac{1}{\sqrt{2}}(1-\mathcal{O}(\lambda)) & \epsilon \\ -\frac{1}{2}(1-\mathcal{O}(\lambda)+\epsilon) & \frac{1}{2}(1+\mathcal{O}(\lambda)-\epsilon) & \frac{1}{\sqrt{2}} \\ \frac{1}{2}(1-\mathcal{O}(\lambda)-\epsilon) & -\frac{1}{2}(1+\mathcal{O}(\lambda)-\epsilon) & \frac{1}{\sqrt{2}} \end{pmatrix} \quad \begin{matrix} \lambda \sim 0.2 \\ \epsilon \lesssim 0.2 \end{matrix}$$

very different from quark's

$$|U_{\rm CKM}| \simeq \begin{pmatrix} 1 & \mathcal{O}(\lambda) & \mathcal{O}(\lambda^3) \\ \mathcal{O}(\lambda) & 1 & \mathcal{O}(\lambda^2) \\ \mathcal{O}(\lambda^3) & \mathcal{O}(\lambda^2) & 1 \end{pmatrix} \qquad \lambda \sim 0.2$$

Summary

- In the SM:
  - Accidental global symmetry: $B \times L_e \times L_\mu \times L_\tau \leftrightarrow m_\nu \equiv 0$
- To include $m_\nu \neq 0$ $\rightarrow$ Need to extend SM

  $\rightarrow$ different ways of adding m_ν to the SM
  - *breaking* total lepton number ($L = L_e + L_\mu + L_\tau$) $\rightarrow$ Majorana ν: $\nu = \nu^C$
  - *conserving* total lepton number $\rightarrow$ Dirac ν: $\nu \neq \nu^C$

  $\rightarrow$ Lepton Mixing$\equiv$ breaking of $L_e \times L_\mu \times L_\tau \Rightarrow \nu$ oscillate in flavour
- Neutrino oscillation searches have shown us
  - $\Delta m^2_{31} \sim 2 \times 10^{-3}$ eV^2 and $\Delta m^2_{21} \sim 8 \times 10^{-5}$ eV^2 $\Rightarrow$ ν's are massive
  - $|U_{\rm LEP}| \simeq \begin{pmatrix} \frac{1}{\sqrt{2}}(1+\mathcal{O}(\lambda)) & \frac{1}{\sqrt{2}}(1-\mathcal{O}(\lambda)) & \epsilon \\ -\frac{1}{2}(1-\mathcal{O}(\lambda)+\epsilon) & \frac{1}{2}(1+\mathcal{O}(\lambda)-\epsilon) & \frac{1}{\sqrt{2}} \\ \frac{1}{2}(1-\mathcal{O}(\lambda)-\epsilon) & -\frac{1}{2}(1+\mathcal{O}(\lambda)-\epsilon) & \frac{1}{\sqrt{2}} \end{pmatrix}$ $\lambda \sim 0.2$, $\epsilon \lesssim 0.2$ $\Rightarrow$ Different from U_{CKM}

Summary

- Still open questions

  Is $\theta_{13} \neq 0$?

  Is there CP violation in the leptons (is $\delta \neq 0, \pi$)?

  Is θ_{23} large or maximal?

  Normal or Inverted mass ordering?

  Are neutrino masses:

  hierarchical: $m_i - m_j \sim m_i + m_j$?

  degenerated: $m_i - m_j \ll m_i + m_j$?

  Dirac or Majorana? what about the Majorana Phases?

  $\ldots$

Summary

• Majorana $\nu' s$ are more *Natural*: appear *generically* if SM is a LE effective theory

If SM is an effective low energy theory, for $E \ll \Lambda_{\rm NP}$

– The same particle content as the SM and same pattern of symmetry breaking

– But there can be non-renormalizable (dim> 4) operators

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \sum_n \frac{1}{\Lambda_{\rm NP}^{n-4}} \mathcal{O}_n$$

First NP effect ⇒ dim=5 operator

There is only one!

$$\mathcal{O}_5 = \frac{Z_{ij}^\nu}{\Lambda_{\rm NP}} \left(\overline{L_{L,i}} \tilde{\phi} \right) \left(\tilde{\phi}^T L_{L,j}^C \right)$$

which after symmetry breaking

induces a ν Majorana mass

$$(M_\nu)_{ij} = Z_{ij}^\nu \frac{v^2}{\Lambda_{\rm NP}}$$

$\mathcal{O}_5$ breaks total lepton and lepton flavour numbers

Implications:

– It is natural that ν mass is the first evidence of NP

– Naturally $m_\nu \ll$ other fermions masses $\sim \lambda^f v$

– $m_\nu > \sqrt{\Delta m^2_{atm}} \sim 0.05$ eV ⇒ $\Lambda_{\rm NP} \lesssim 10^{15}$ GeV