Summary I

- In the SM:
  - Accidental global symmetry: $B \times L_e \times L_\mu \times L_\tau \leftrightarrow m_\nu \equiv 0$
  - neutrinos are left-handed ($\equiv$ helicity -1): $m_\nu = 0 \Rightarrow$ chirality $\equiv$ helicity
  - No distinction between Majorana or Dirac Neutrinos
- If $m_\nu \neq 0 \rightarrow$ Need to extend SM

  $\rightarrow$ different ways of adding m_ν to the SM
  - *breaking* total lepton number ($L = L_e + L_\mu + L_\tau$) $\rightarrow$ Majorana ν: $\nu = \nu^C$
  - *conserving* total lepton number $\rightarrow$ Dirac ν: $\nu \neq \nu^C$

  $\rightarrow$ Lepton Mixing$\equiv$ breaking of $L_e \times L_\mu \times L_\tau$
- From direct searches of ν-mass: $m_\nu \leq \mathcal{O}(eV)$

  Question: How to search for $m_\nu \ll \mathcal{O}(eV)$?

  Answer: Tomorrow....

Plan of Lectures

I. Standard Neutrino Properties and Mass Terms (Beyond Standard)

II. Neutrino Oscillations

III. The Data and Its Interpretation

IV. Some Missing Pieces and The Meaning of All This

Plan of Lecture II

Neutrino Oscillations

Vacuum Oscillations

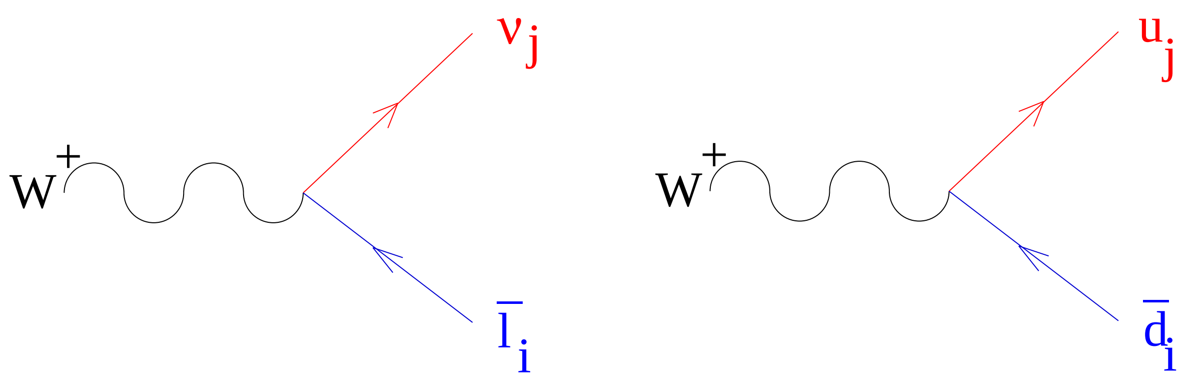
Matter Effects: MSW

Effects of ν Mass

- Neutrino masses can have kinematic effects

- Also if neutrinos have a mass the charged current interactions of leptons are not diagonal (same as quarks)

$$\frac{g}{\sqrt{2}} W^+_\mu \sum_{ij} \left(U^{ij}_{LEP}\, \overline{\ell^i}\, \gamma^\mu\, L\, \nu^j + U^{ij}_{CKM}\, \overline{U^i}\, \gamma^\mu\, L\, D^j \right) + h.c.$$



- SM gauge invariance *does not imply* $U(1)_{L_e} \times U(1)_{L_\mu} \times U(1)_{L_\tau}$ symmetry

$$\frac{g}{\sqrt{2}} W_\mu^+ \sum_{ij} \left(U_{LEP}^{ij} \, \overline{\ell^i} \, \gamma^\mu \, L \, \nu^j + U_{CKM}^{ij} \, \overline{U^i} \, \gamma^\mu \, L \, D^j \right) + h.c.$$

- We want to know:
  - * How many, N, neutral states ν_i and their masses m_i
  - * Their mixing: $\#$ angles $= \frac{N(N-1)}{2} = \begin{cases} 1 \text{ for } N=2 \\ 3 \text{ for } N=3 \\ 6 \text{ for } N=4 \end{cases}$
  - * Their CP properties:

Dirac: $\nu^C \neq \nu$ $\quad \#$ phases $= \frac{(N-1)(N-2)}{2} = \begin{cases} 0 \text{ for } N=2 \\ 1 \text{ for } N=3 \\ 3 \text{ for } N=4 \end{cases}$

Majorana: $\nu^C = \nu$ $\quad \#$ extra phases $= (N-1) = \begin{cases} 1 \text{ for } N=2 \\ 2 \text{ for } N=3 \\ 3 \text{ for } N=4 \end{cases}$

$$U_{\alpha j}^{\rm Maj} = U_{\alpha j}^{\rm Dir} \times {\rm e}^{-i\eta_j}$$

Effects of ν Mass: Flavour Transitions

- Flavour ($\equiv$ Interaction) basis (production and detection): ν_e, ν_μ and ν_τ
- Mass basis (free propagation in space-time): ν_1, ν_2 and ν_3 $\dots$
- In general interaction eigenstates $\neq$ propagation eigenstates

$$U(\nu_1, \nu_2, \nu_3) = (\nu_e, \nu_\mu, \nu_\tau)$$

$\Rightarrow$ Flavour is not conserved during propagation $\Rightarrow$ ν can be detected with different (or same) flavour than produced

- The probability $P_{\alpha\beta}$ of producing neutrino with flavour α and detecting with flavour β has to depend on:
  - Misalignment between interaction and propagation states ($\equiv U$)
  - Difference between propagation eigenvalues
  - Propagation distance

Vacuum Oscillations

- If neutrinos have mass, a weak eigenstate $|\nu_\alpha\rangle$ produced in $l_\alpha + N \rightarrow \nu_\alpha + N'$ is a linear combination of the mass eigenstates ($|\nu_i\rangle$)

$$|\nu_\alpha\rangle = \sum_{i=1}^{n} U_{\alpha i}\,|\nu_i\rangle$$

U is the unitary mixing matrix.

- After a distance L (or time t) it evolves

$$|\nu(t)\rangle = \sum_{i=1}^{n} U_{\alpha i}|\nu_i(t)\rangle$$

- it can be detected with flavour β with probability

$$P_{\alpha\beta} = |\langle\nu_\beta(t)|\nu_\alpha(0)\rangle|^2 = |\sum_{i=1}^{n} U_{\alpha i}U^*_{\beta i}\langle\nu_i(t)|\nu_i(0)\rangle|^2$$

Vacuum Oscillations

- The probability

$$P_{\alpha\beta} = |\langle \nu_\beta(t)|\nu_\alpha(0)\rangle|^2 = |\sum_{i=1}^{n} U_{\alpha i} U^*_{\beta i} \langle \nu_i(t)|\nu_i(0)\rangle|^2$$

- We call E_i the neutrino energy and m_i the neutrino mass

- Under the approximations:

$(\mathbf{1})$ $|\nu\rangle$ is a *plane wave* $\Rightarrow$ $|\nu_i(t)\rangle = \mathrm{e}^{\,-i\,E_i t}|\nu_i(0)\rangle$

$$P_{\alpha\beta} = \delta_{\alpha\beta} - 4\sum_{j\neq i}^{n} \mathrm{Re}[U^\star_{\alpha i} U_{\beta i} U_{\alpha j} U^\star_{\beta j}] \sin^2\left(\frac{\Delta_{ij}}{2}\right) + 2\sum_{j\neq i} \mathrm{Im}[U^\star_{\alpha i} U_{\beta i} U_{\alpha j} U^\star_{\beta j}] \sin\left(\Delta_{ij}\right)$$

with $\Delta_{ij} = (E_i - E_j)t$

$(\mathbf{2})$ *relativistic* ν

$$E_i = \sqrt{p_i^2 + m_i^2} \simeq p_i + \frac{m_i^2}{2E_i}$$

$(\mathbf{3})$ Assuming $p_i \simeq p_j = p \simeq E$

$$\frac{\Delta_{ij}}{2} = 1.27 \frac{m_i^2 - m_j^2}{\mathrm{eV}^2} \frac{L/E}{\mathrm{Km/GeV}}$$

Vacuum Oscillations

- The oscillation probability:

$$P_{\alpha\beta} = \delta_{\alpha\beta} - 4\sum_{j\neq i}^{n} \mathrm{Re}[U^{\star}_{\alpha i}U_{\beta i}U_{\alpha j}U^{\star}_{\beta j}]\sin^2\left(\frac{\Delta_{ij}}{2}\right) + 2\sum_{j\neq i}\mathrm{Im}[U^{\star}_{\alpha i}U_{\beta i}U_{\alpha j}U^{\star}_{\beta j}]\sin(\Delta_{ij})$$

$$\frac{\Delta_{ij}}{2} = \frac{(E_i - E_j)L}{2} = 1.27\frac{(m_i^2 - m_j^2)}{\mathrm{eV}^2}\frac{L/E}{\mathrm{Km/GeV}}$$

– The first term $\delta_{\alpha\beta} - 4\sum_{j\neq i}^{n} \mathrm{Re}[U^{\star}_{\alpha i}U_{\beta i}U_{\alpha j}U^{\star}_{\beta j}]\sin^2\left(\frac{\Delta_{ij}}{2}\right)$ equal for $\overline{\nu}$ $(U \to U^*)$

$\to$ conserves CP

– The last piece $2\sum_{j\neq i}\mathrm{Im}[U^{\star}_{\alpha i}U_{\beta i}U_{\alpha j}U^{\star}_{\beta j}]\sin(\Delta_{ij})$ opposite sign for $\overline{\nu}$

$\to$ violates CP

- $P_{\alpha\beta}$ depends on Theoretical Parameters
  - $\Delta m^2_{ij} = m_i^2 - m_j^2$ The mass differences
  - $U_{\alpha j}$ The mixing angles (and Dirac phases)

  and on Two *Experimental* Parameters:
  - E The neutrino energy
  - L Distance ν source to detector

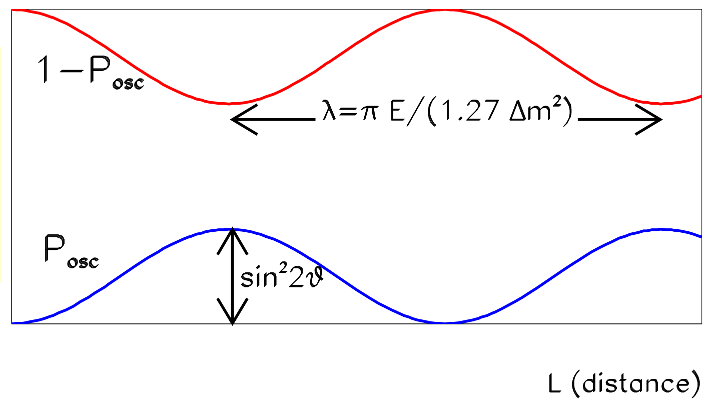
- No information on mass scale nor Majorana phases

2-ν Oscillations

- For 2-ν: $U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ $\qquad \Delta m^2 = m_2^2 - m_1^2$

$P_{osc} = \sin^2(2\theta)\,\sin^2\left(\frac{\Delta m^2 L}{4E}\right)$ Appear

$P_{\alpha\alpha} = 1 - P_{osc}$ Disappear



- $\Delta m^2 \to -\Delta m^2$ *and* $\theta \to -\theta + \frac{\pi}{2}$ is only a redefinition $\nu_1 \leftrightarrow \nu_2$

  $\Rightarrow$ We can chose the convention $\Delta m^2 > 0$ and $0 \leq \theta \leq \frac{\pi}{2}$

- Moreover P_{osc} is symmetric under $\Delta m^2 \to -\Delta m^2$ *or* $\theta \to -\theta + \frac{\pi}{2}$

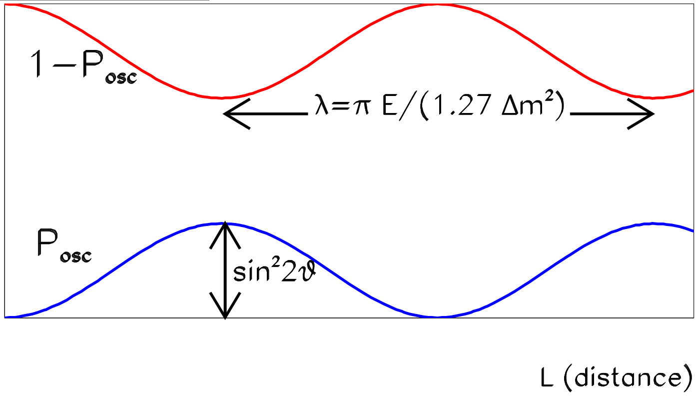
  $\Rightarrow$ We can chose the convention $\Delta m^2 > 0$ and $0 \leq \theta \leq \frac{\pi}{4}$

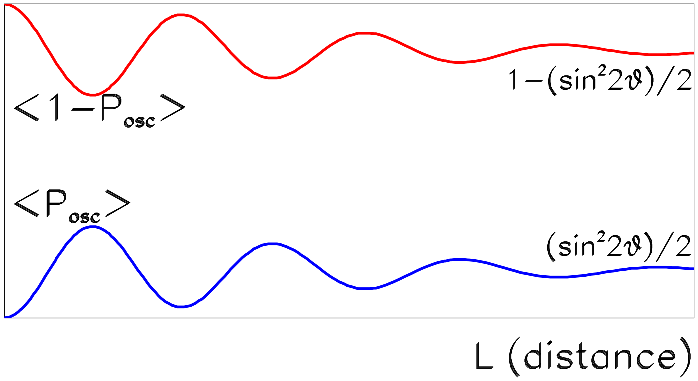
  This only happens for 2ν vacuum oscillations

2-ν Oscillations

$$P_{osc} = \sin^2(2\theta)\,\sin^2\left(\frac{\Delta m^2 L}{E}\right) \quad \text{Appear}$$

$$P_{\alpha\alpha} = 1 - P_{osc} \quad \text{Disappear}$$





- In real experiments neutrinos are not monochromatic

$$\Rightarrow \langle P_{\alpha\beta}\rangle = \int dE_\nu \frac{d\Phi}{dE_\nu}\sigma_{CC}(E_\nu)P_{\alpha\beta}(E_\nu)$$

- Maximal sensitivity for $\Delta m^2 \sim E/L$
  - $\Delta m^2 \ll E/L \Rightarrow$ No time to oscillate
  $$\Rightarrow \langle \sin^2(1.27\Delta m^2 L/E)\rangle \simeq 0 \to \langle P_{osc}\rangle \simeq 0$$
  - $\Delta m^2 \gg E/L \Rightarrow$ Averaged oscillations
  $$\Rightarrow \langle \sin^2(1.27\Delta m^2 L/E)\rangle \simeq \frac{1}{2} \to \langle P_{osc}\rangle \simeq \frac{1}{2}\sin^2(2\theta)$$

To allow observation of neutrino oscillations:

– Nature has to be good: $\theta \not\ll 0$

– Need the right set up ($\equiv$right $\langle \frac{L}{E} \rangle$) for Δm^2

Source	E (GeV)	L (Km)	Δm^2 (eV^2)
Solar	10^{-3}	10^7	10^{-10}
Atmospheric	0.1–10^2	10–10^3	10^{-1}–10^{-4}
Reactor	10^{-3}	SBL: 0.1–1	10^{-2}–10^{-3}
		LBL: 10–10^2	10^{-4}–10^{-5}
Accelerator	10	SBL: 0.1	$\gtrsim 0.01$
		LBL: 10^2– 10^3	10^{-2}–10^{-3}

Equations of Motion for Weak Eigenstates

- ν oscillations can also be understood from the eq. of motion of weak eigenstates
- A state mixture of 2 neutrino species $|\nu_e\rangle$ and $|\nu_X\rangle$ or equivalently of $|\nu_1\rangle$ and $|\nu_2\rangle$

$$\Phi(x) = \Phi_e(x)|\nu_e\rangle + \Phi_X(x)|\nu_X\rangle = \Phi_1(x)|\nu_1\rangle + \Phi_2(x)|\nu_2\rangle$$

- Evolution of $\Phi(x) =$ is given by the Dirac Equations. Calling $\Phi_i(x) = \nu_i(x)\phi_i$

In the relativistic limit $\sqrt{E^2 - m_i^2} \simeq E - \frac{m_i^2}{2E}$

$$-i\frac{\partial}{\partial x}\begin{pmatrix}\nu_1(x)\\ \nu_2(x)\end{pmatrix} = \begin{pmatrix} E - \frac{m_1^2}{2E} & 0 \\ 0 & E - \frac{m_2^2}{2E}\end{pmatrix}\begin{pmatrix}\nu_1(x)\\ \nu_2(x)\end{pmatrix}$$

For initial conditions $\nu_\alpha(0) = 1$ and $\nu_\beta(0) = 0$

and final state $\nu_\alpha(L) = 0$ and $\nu_\beta(L) = 1$

Neutrinos in Matter:Effective Potentials

- In SM the characteristic ν-p interaction cross section

$$\sigma \sim \frac{G_F^2 E^2}{\pi} \sim 10^{-43}\mathrm{cm}^2 \quad \text{at } \mathrm{E}_\nu \sim \mathrm{MeV}$$

- So if a beam of $\Phi_\nu \sim 10^{10} \nu' s$ was aimed at the Earth only 1 would be deflected

  so it seems that for neutrinos *matter does not matter*

- But that cross section is for *inelastic* scattering

  Does not contain forward elastic *coherent* scattering

- In *coherent* interactions ⇒ ν and medium remain unchanged

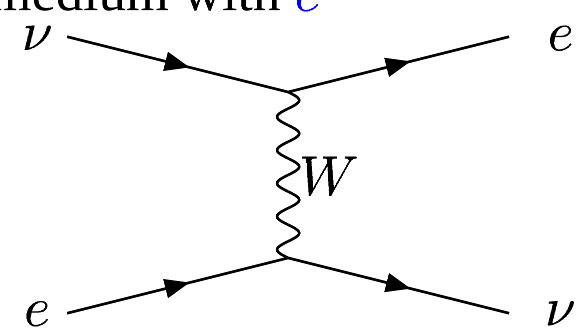
  Interference of scattered and unscattered ν waves

Neutrinos in Matter:Effective Potentials

• Coherence $\Rightarrow$ decoupling of ν evolution equation from equations of the medium.

• The effect of the medium is described by an effective potential depending on density and composition of matter

• For example for ν_e in medium with e^-



$$V_{CC} = \sqrt{2} G_F N_e$$

$N_e \equiv$ electron number density

• The effective potential has opposite sign for neutrinos y antineutrinos

• Other potentials for ν_e $(\overline{\nu}_e)$ due to different particles in medium

medium	V_C	V_N
e^+ and e^-	$\pm\sqrt{2}G_F(N_e - N_{\bar{e}})$	$\mp\frac{G_F}{\sqrt{2}}(N_e - N_{\bar{e}})(1-4\sin^2\theta_W)$
p and $\bar{p}$	0	$\pm\frac{G_F}{\sqrt{2}}(N_p - N_{\bar{p}})(1-4\sin^2\theta_W)$
n and $\bar{n}$	0	$\mp\frac{G_F}{\sqrt{2}}(N_n - N_{\bar{n}})$
Neutral $(N_e = N_p)$	$\pm\sqrt{2}G_F N_e$	$\mp\frac{G_F}{\sqrt{2}} N_n$

Neutrinos in Matter: Evolution Equation

Evolution Eq. for $|\nu\rangle = \nu_1|\nu_1\rangle + \nu_2|\nu_2\rangle \equiv \nu_e|\nu_e\rangle + \nu_X|\nu_X\rangle$ $(X = \mu, \tau, \text{sterile})$

(a) In vacuum in the mass basis: $-i\frac{\partial}{\partial x}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix} = E - \begin{pmatrix}\frac{m_1^2}{2E} & 0\\ 0 & \frac{m_2^2}{2E}\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix}$

(b) In vacuum in the weak basis

$$-i\frac{\partial}{\partial x}\begin{pmatrix}\nu_e\\ \nu_X\end{pmatrix} = E - \frac{m_1^2+m_2^2}{2E} - \begin{pmatrix}-\frac{\Delta m^2}{4E}\cos 2\theta & \frac{\Delta m^2}{4E}\sin 2\theta\\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta\end{pmatrix}\begin{pmatrix}\nu_e\\ \nu_X\end{pmatrix}$$

(c) In matter (e, p, n) in weak basis

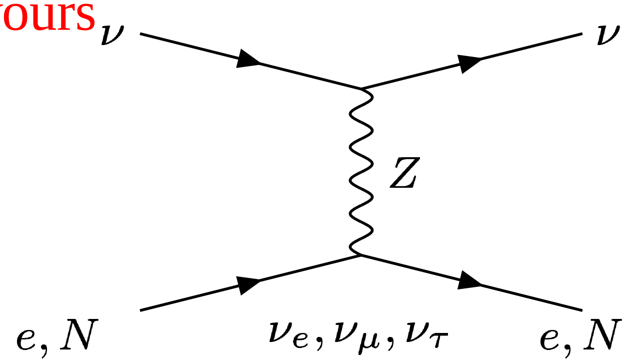
$$-i\frac{\partial}{\partial x}\begin{pmatrix}\nu_e\\ \nu_X\end{pmatrix} = E - V_X - \frac{m_1^2+m_2^2}{2E} - \begin{pmatrix}V_e - V_X - \frac{\Delta m^2}{4E}\cos 2\theta & \frac{\Delta m^2}{4E}\sin 2\theta\\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta\end{pmatrix}\begin{pmatrix}\nu_e\\ \nu_X\end{pmatrix}$$

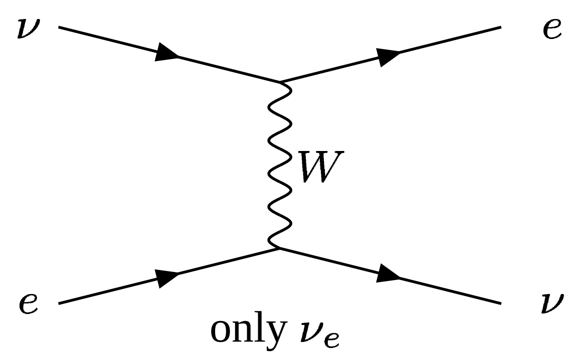
(c)$\neq$ (b) because different flavours have different interactions

For example $X = \mu, \tau$:

$$V_{CC} = V_e - V_X = \sqrt{2}G_F N_e$$

(opposite sign for $\overline{\nu}$)



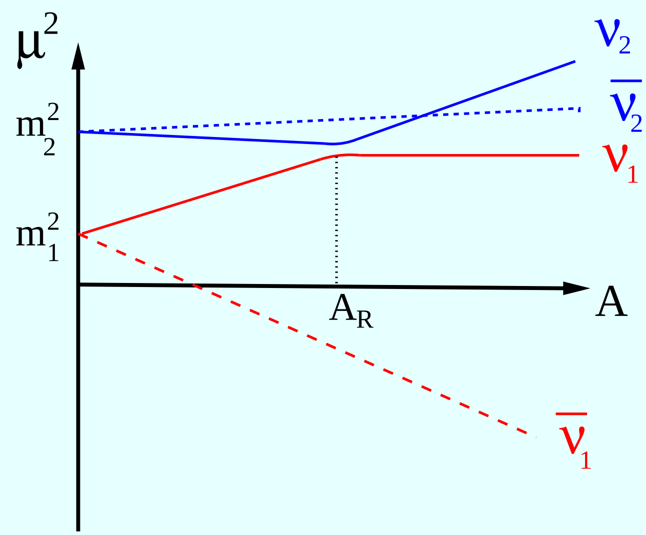


⇒ Effective masses and mixing are different than in vacuum

⇒ If matter density varies along ν trajectory the effective masses and mixing vary too

The effective masses: ($A = 2E(V_e - V_X)$)

$$\mu_{1,2}(x) = \frac{m_1^2 + m_2^2}{2} + E(V_e + V_X)$$
$$\pm \frac{1}{2}\sqrt{(\Delta m^2 \cos 2\theta - A)^2 + (\Delta m^2 \sin 2\theta)^2}$$

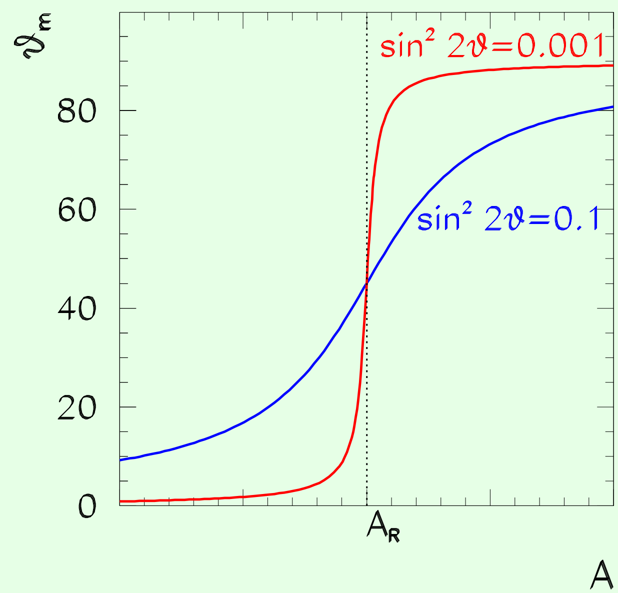


At *resonant* potential: $A_R = \Delta m^2 \cos 2\theta$

Minimum $\Delta\mu^2 = \mu_2^2 - \mu_1^2$

The mixing angle in matter

$$\tan 2\theta_m = \frac{\Delta m^2 \sin 2\theta}{\Delta m^2 \cos 2\theta - A}$$



∗ At $A = 0$ (vacuum) ⇒ $\theta_m = \theta$

∗ At $A = A_R \Rightarrow \theta_m = \frac{\pi}{2}$

∗ At $A >> A_R \Rightarrow \theta_m = \frac{\pi}{2} - \theta$

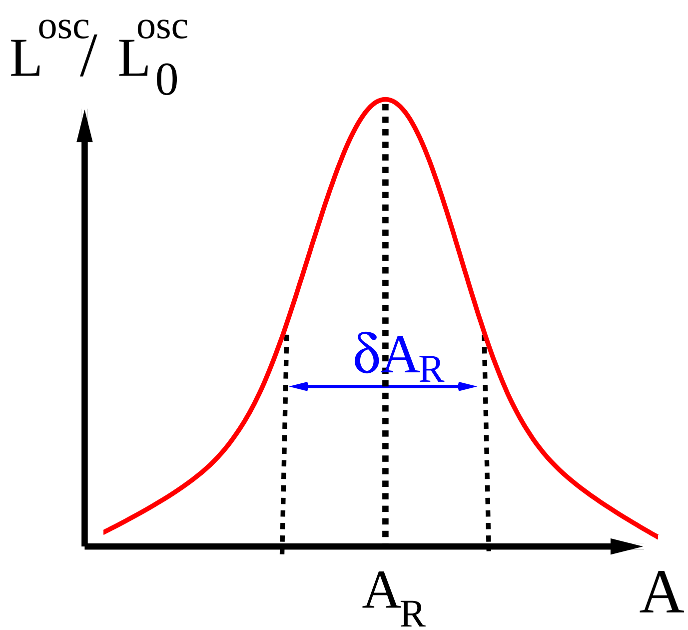
The oscillation length in vacuum

$$L_0^{osc} = \frac{4\pi E}{\Delta m^2}$$

The oscillation length in matter

$$L^{osc} = \frac{L_0^{osc}}{\sqrt{(\Delta m^2 \cos 2\theta - A)^2 + (\Delta m^2 \sin 2\theta)^2}} \equiv \frac{4\pi E}{\Delta \mu^2}$$

L^{osc} presents a resonant behaviour



At the resonant point

$$L_R^{osc} = \frac{L_0^{osc}}{\sin 2\theta}$$

The width of the resonance in potential:

$$\delta V_R = \frac{\Delta m^2 \sin 2\theta}{E}$$

The width of the resonance in distance:

$$\delta r_R = \frac{\delta V_R}{|\frac{dV}{dr}|_R}$$

- In terms of the mass eigenstates in matter: $$\begin{pmatrix}\nu_e\\ \nu_X\end{pmatrix} = U[\theta_m(x)]\begin{pmatrix}\nu_1^m(x)\\ \nu_2^m(x)\end{pmatrix}$$

- For constant potential θ_m and μ_i are constant along ν evolution

  $\Rightarrow$ the evolution is determined by masses and mixing in matter as

$$P_{osc} = \sin^2(2\theta_m)\,\sin^2\left(\frac{\Delta\mu^2 L}{2E}\right)$$

- For varying potential: $$\begin{pmatrix}\dot\nu_e\\ \dot\nu_X\end{pmatrix} = \dot U[\theta_m(x)]\begin{pmatrix}\nu_1^m(x)\\ \nu_2^m(x)\end{pmatrix} + U[\theta_m(x)]\begin{pmatrix}\dot\nu_1^m(x)\\ \dot\nu_2^m(x)\end{pmatrix}$$

$\Rightarrow$ the evolution equation in flavour basis (removing diagonal part)

$$i\begin{pmatrix}\dot\nu_e\\ \dot\nu_X\end{pmatrix} = \frac{1}{2E}\begin{pmatrix} A-\frac{\Delta m^2}{2}\cos 2\theta & \frac{\Delta m^2}{2}\sin 2\theta \\ \frac{\Delta m^2}{2}\sin 2\theta & \frac{\Delta m^2}{2}\cos 2\theta\end{pmatrix}\begin{pmatrix}\nu_e\\ \nu_X\end{pmatrix}$$

$\Rightarrow$ the evolution equation in instantaneous mass basis

$$i\begin{pmatrix}\dot\nu_1^m\\ \dot\nu_2^m\end{pmatrix} = \frac{1}{2E}U^\dagger(\theta_m)\begin{pmatrix} A-\frac{\Delta m^2}{2}\cos 2\theta & \frac{\Delta m^2}{2}\sin 2\theta \\ \frac{\Delta m^2}{2}\sin 2\theta & \frac{\Delta m^2}{2}\cos 2\theta\end{pmatrix}U(\theta_m)\begin{pmatrix}\nu_1^m\\ \nu_2^m\end{pmatrix} - i\,U^\dagger\dot U(\theta_m)\begin{pmatrix}\nu_1^m\\ \nu_2^m\end{pmatrix}$$

$$\Rightarrow\; i\begin{pmatrix}\dot\nu_1^m\\ \dot\nu_2^m\end{pmatrix} = \frac{1}{4E}\begin{pmatrix} -\Delta\mu^2(x) & -4\,i\,E\,\dot\theta_m(x) \\ 4\,i\,E\,\dot\theta_m(x) & \Delta\mu^2(x)\end{pmatrix}\begin{pmatrix}\nu_1^m\\ \nu_2^m\end{pmatrix}$$

- The evolution equation in instantaneous mass basis

$$i \begin{pmatrix} \dot{\nu}_1^m \\ \dot{\nu}_2^m \end{pmatrix} = \frac{1}{4E} \begin{pmatrix} -\Delta\mu^2(x) & -4\, i\, E\, \dot{\theta}_m(x) \\ 4\, i\, E\, \dot{\theta}_m(x) & \Delta\mu^2(x) \end{pmatrix} \begin{pmatrix} \nu_1^m \\ \nu_2^m \end{pmatrix}$$

$\Rightarrow$ It is not diagonal $\Rightarrow$ Instantaneous mass eigenstates $\neq$ eigenstates of evolution

$\Rightarrow$ Transitions $\nu_1^m \rightarrow \nu_2^m$ can occur $\equiv$ *Non adiabaticity*

- For $\Delta\mu^2(x) \gg 4\, E\, \dot{\theta}_m(x)$ $\left[\frac{1}{V}\frac{dV}{dx}\Big|_R \ll \frac{\Delta m^2}{2E}\frac{\sin^2 2\theta}{\cos 2\theta} \right]$ $\equiv$ Slowly varying matter potent

$\Rightarrow$ ν_i^m behave approximately as *evolution eigenstates*

$\Rightarrow$ ν_i^m do not mix in the evolution This is the *adiabatic* transition approximation

The adiabaticity condition

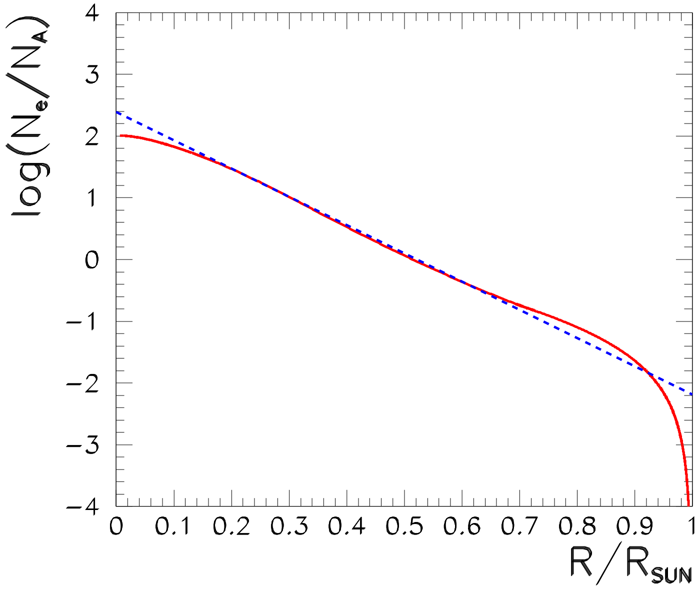
$$\frac{1}{V}\frac{dV}{dx}\Big|_R \ll \frac{\Delta m^2}{2E}\frac{\sin^2 2\theta}{\cos 2\theta} \equiv 2\pi\, \delta r_R \gg L_R^{osc}$$

$\Rightarrow$ Many oscillations take place in the resonant region

Neutrinos in The Sun : MSW Effect

- Solar neutrinos are ν_e produced in the core ($R \lesssim 0.3 R_\odot$) of the Sun

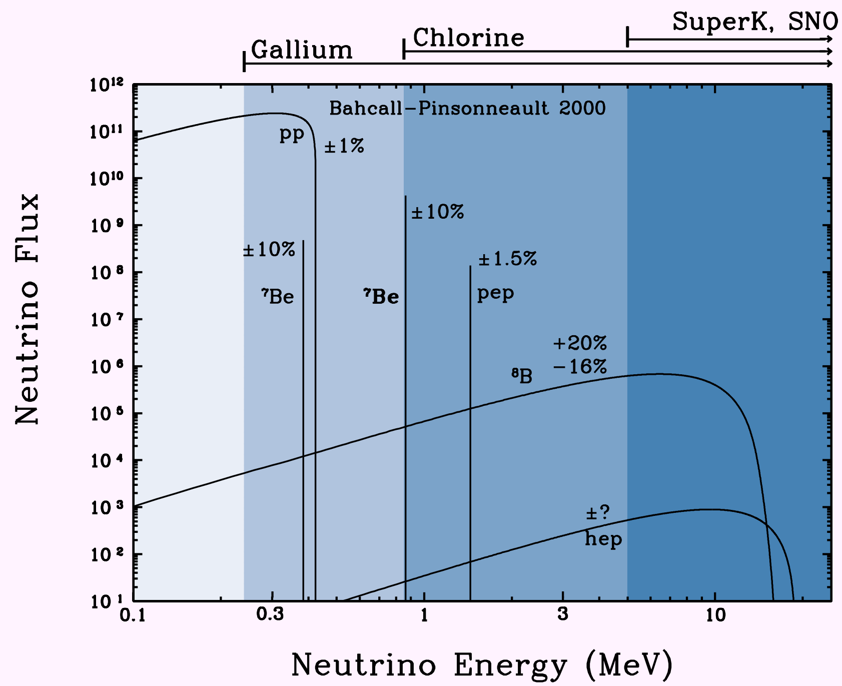
The solar matter density



$$V_{CC} = \sqrt{2} G_F N_e \sim 10^{-14} \frac{N_e}{N_A} \text{ eV}$$

At core: $V_{CC,0} \sim 10^{-14}$–10^{-12} eV

The energy spectrum of solar $\nu_e' s$



$E_\nu \sim 0.1$–10 MeV

- For $\nu_e \leftrightarrow \nu_{\mu(\tau)}$, in vacuum $\nu_e = \cos\theta\, \nu_1 + \sin\theta\, \nu_2$

- For 10^{-9} eV$^2 \lesssim \Delta m^2 \lesssim 10^{-4}$ eV^2 $\Rightarrow$ $2E_\nu V_{CC,0} > \Delta m^2 \cos 2\theta$

$\Rightarrow$ ν can cross resonance condition in its way out of the Sun

For $\theta \ll \frac{\pi}{4}$: In vacuum $\nu_e = \cos\theta\,\nu_1 + \sin\theta\,\nu_2$ is mostly ν_1

In Sun core $\nu_e = \cos\theta_{m,0}\,\nu_1 + \sin\theta_{m,0}\,\nu_2$ is mostly ν_2

If $\frac{(\Delta m^2/\mathrm{eV}^2)\sin^2 2\theta}{(E/\mathrm{MeV})\cos 2\theta} \gg 3\times 10^{-9}$

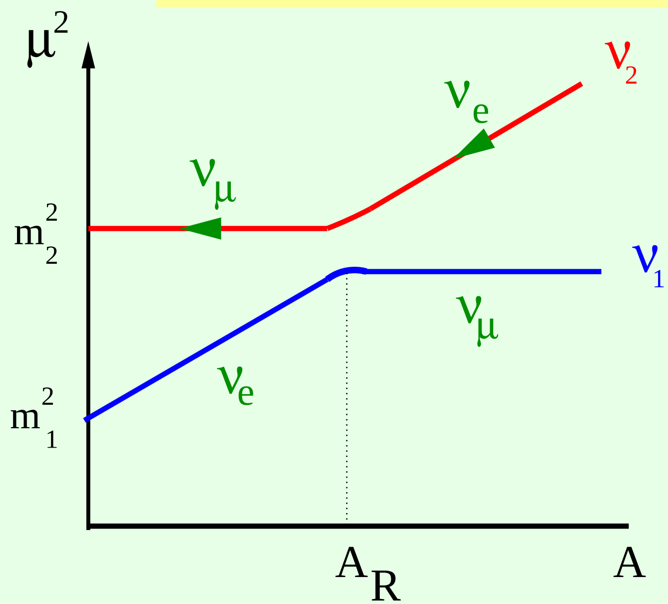
$\Rightarrow$ Adiabatic transition

$*$ ν is mostly ν_2 before and after resonance

$*$ $\theta_m \downarrow$ *dramatically* at resonance

$\Rightarrow$ ν_e component $\downarrow \Rightarrow P_{ee}\downarrow$

This is the MSW effect



$$P_{ee} = \frac{1}{2}\left[1 + \cos 2\theta_{m,0}\cos 2\theta\right]$$

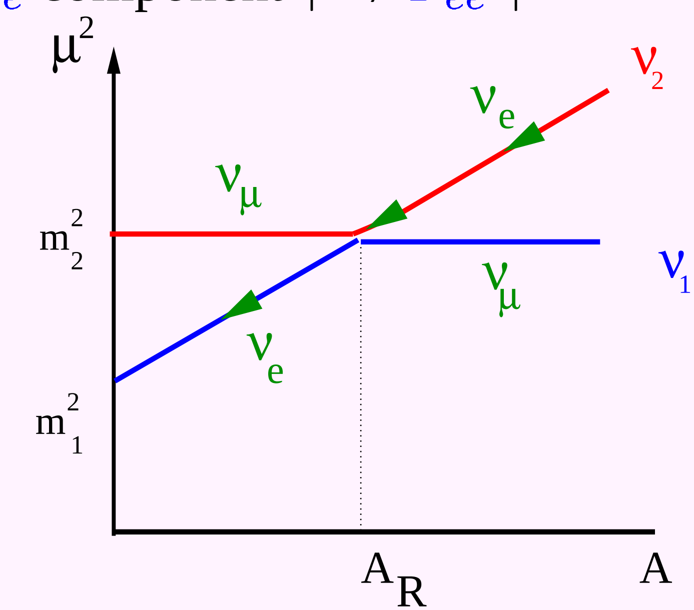
If $\frac{(\Delta m^2/\mathrm{eV}^2)\sin^2 2\theta}{(E/\mathrm{MeV})\cos 2\theta} \lesssim 3\times 10^{-9}$

$\Rightarrow$ Non-Adiabatic transition

$*$ ν is mostly ν_2 till the resonance

$*$ At resonance the state can jump into ν_1 (with probability P_{LZ})

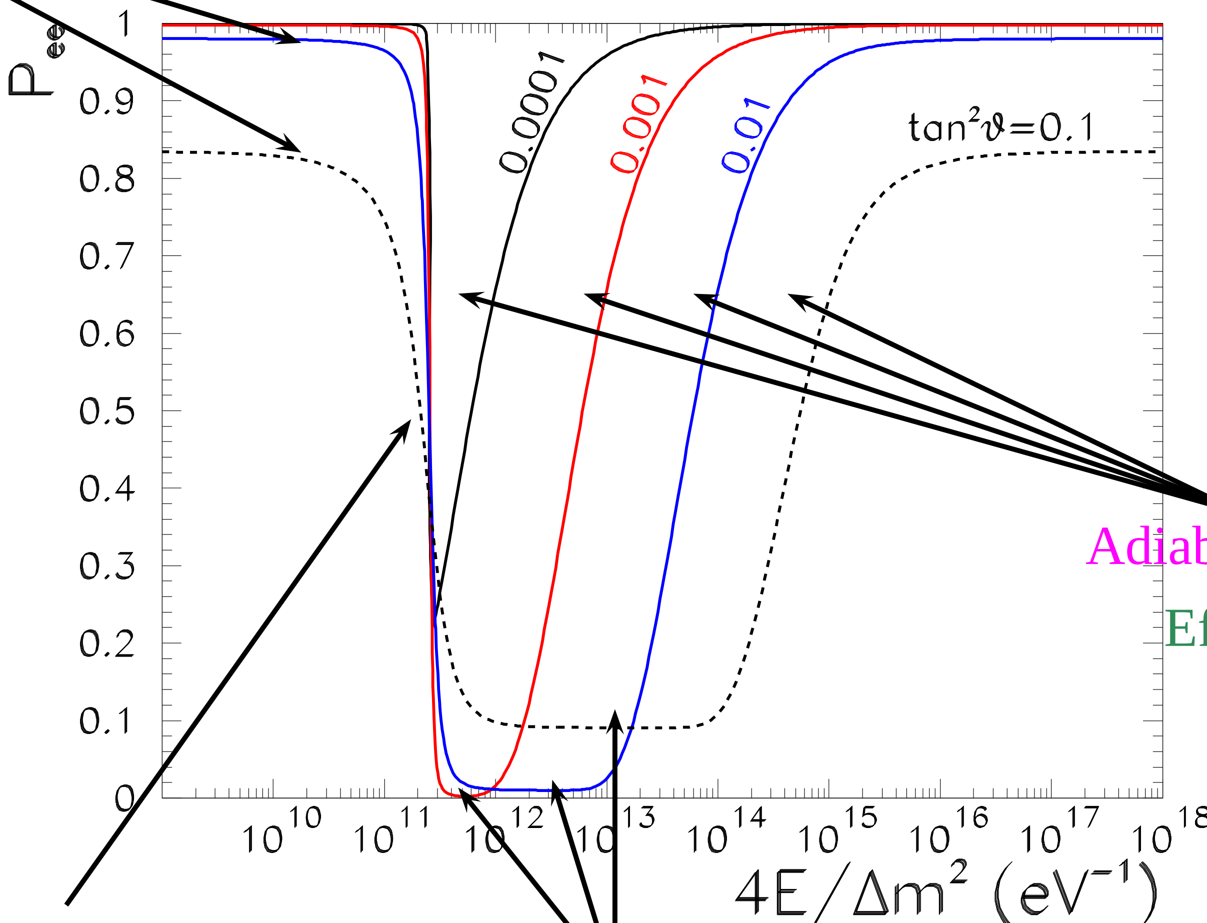
$\Rightarrow$ ν_e component $\uparrow \Rightarrow P_{ee}\uparrow$



$$P_{ee} = \frac{1}{2}\left[1 + (1-2P_{LZ})\cos 2\theta_{m,0}\cos 2\theta\right]$$

Neutrinos in The Sun : MSW Effect

ν does not cross resonance: $P_{ee} = 1 - \frac{1}{2}\sin^2 2\theta > \frac{1}{2}$



Adiabacity breaking

Effect of P_{LZ}

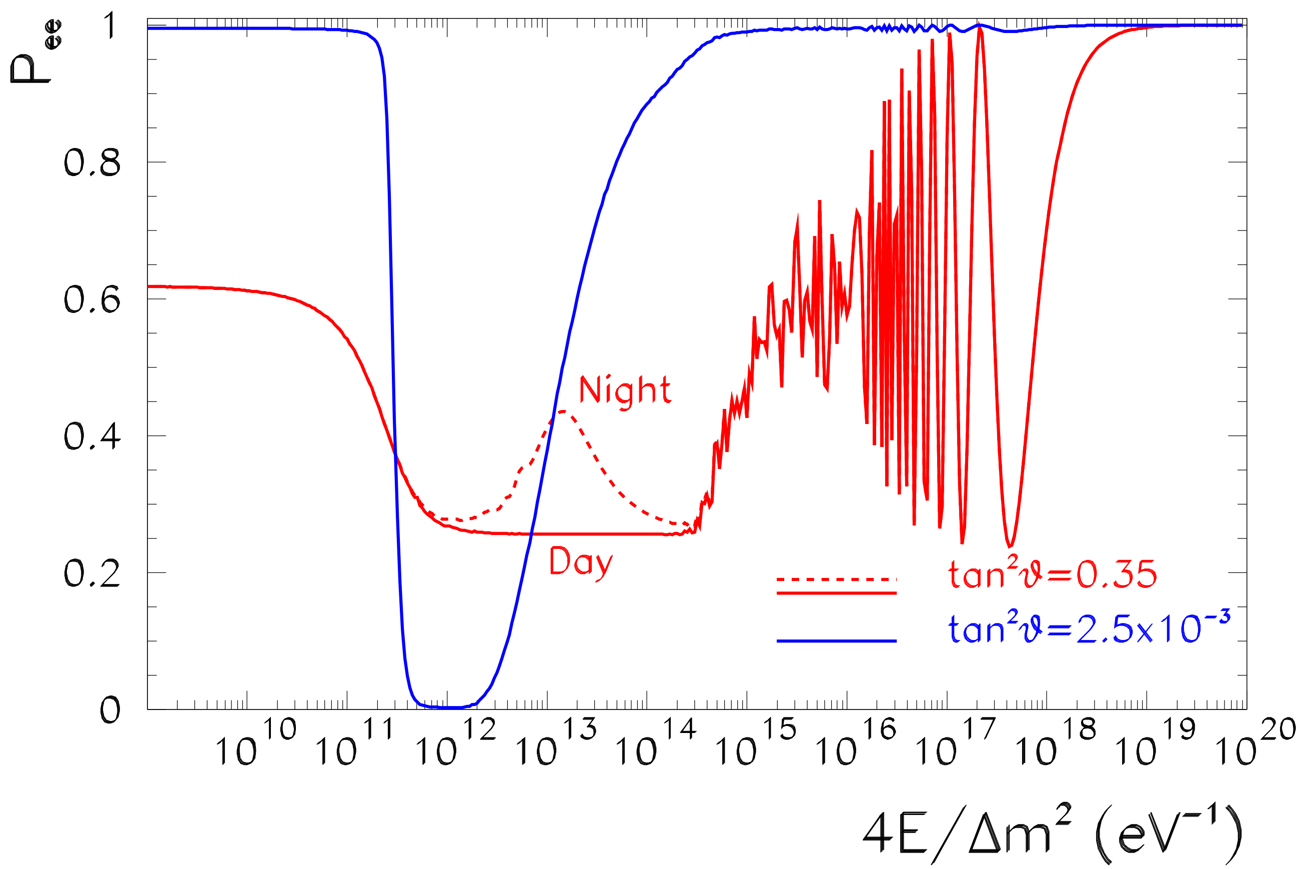
ν crosses resonance

MSW effect

Adiabatic MSW transition $P_{ee} = \sin\theta < \frac{1}{2}$

Neutrinos from The Sun : The Full Story

$$A(\nu_e \to \nu_e) \quad = \quad A_{Sun}(\nu_e \to \nu_1) \times A_{\rm vac}(\nu_1 \to \nu_1) \times A_{Earth}(\nu_1 \to \nu_e)$$
$$+A_{Sun}(\nu_e \to \nu_2) \times A_{\rm vac}(\nu_2 \to \nu_2) \times A_{Earth}(\nu_2 \to \nu_e)$$



ν Oscillations: Parameter Plots

- If experiment does not see oscillations: $\langle P_{osc} \rangle < P_L \rightarrow$ excluded region
- If the experiment detects oscillation $\rightarrow$ allowed region
- For experiments in vacuum, for 2–ν

  $\langle P^{\rm vac}_{osc} \rangle = \sin^2(2\theta) \langle \sin^2\left(1.27 \Delta m^2 L/E\right) \rangle$

  is symmetric for $\theta \rightarrow \frac{\pi}{2} - \theta$

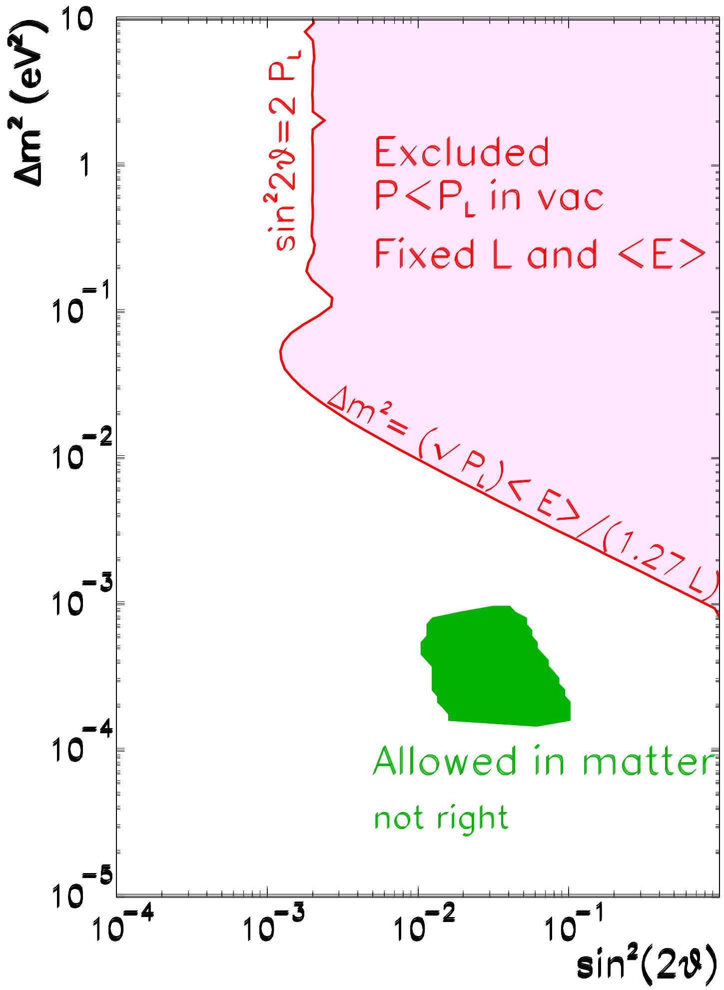
  $\rightarrow$ *old* plots in $(\Delta m^2\,, \sin^2(2\theta))$ plane

  $\rightarrow$ Each point represents 2 physical solutions
- Region for experiments at fixed L and $\langle E \rangle$:
  - $\Delta m^2 \gg 1/\langle L/E \rangle$: vertical at $\sin^2 2\theta = 2\,P_L$
  - $\Delta m^2 \ll 1/\langle L/E \rangle \rightarrow \Delta m^2 \sin 2\theta = 4 \frac{\sqrt{P_L}}{\langle L/E \rangle}$

    $\rightarrow$ diagonal of slope $-\dfrac{1}{2}$ in log-log plot .
- But $P^{\rm mat}_{osc}$ is not symmetric for $\theta \rightarrow \frac{\pi}{2} - \theta$

$\rightarrow \sin^2 2\theta$ is not good

$\rightarrow$ Use $\sin^2\theta$ or $\tan^2\theta$



ν Oscillations: Parameter Plots

- $\langle P_{osc}^{\rm vac}\rangle = \sin^2(2\theta)\langle\sin^2\left(1.27\Delta m^2 L/E\right)\rangle$ is symmetric for $\theta \to \frac{\pi}{2} - \theta$
- $P_{osc}^{\rm mat}$ is not symmetric for $\theta \to \frac{\pi}{2} - \theta$

- If experiment does not see oscillations:

  $\langle P_{osc}\rangle < P_L \to$ excluded region

- If the experiment detects oscillation

  $\to$ allowed region

- If data at fixed $\langle L\rangle$ and $\langle E\rangle$

  (like most laboratory searches)

  $\to$ region is open in large Δm^2

- If data at several $\langle L\rangle$ and/or $\langle E\rangle$

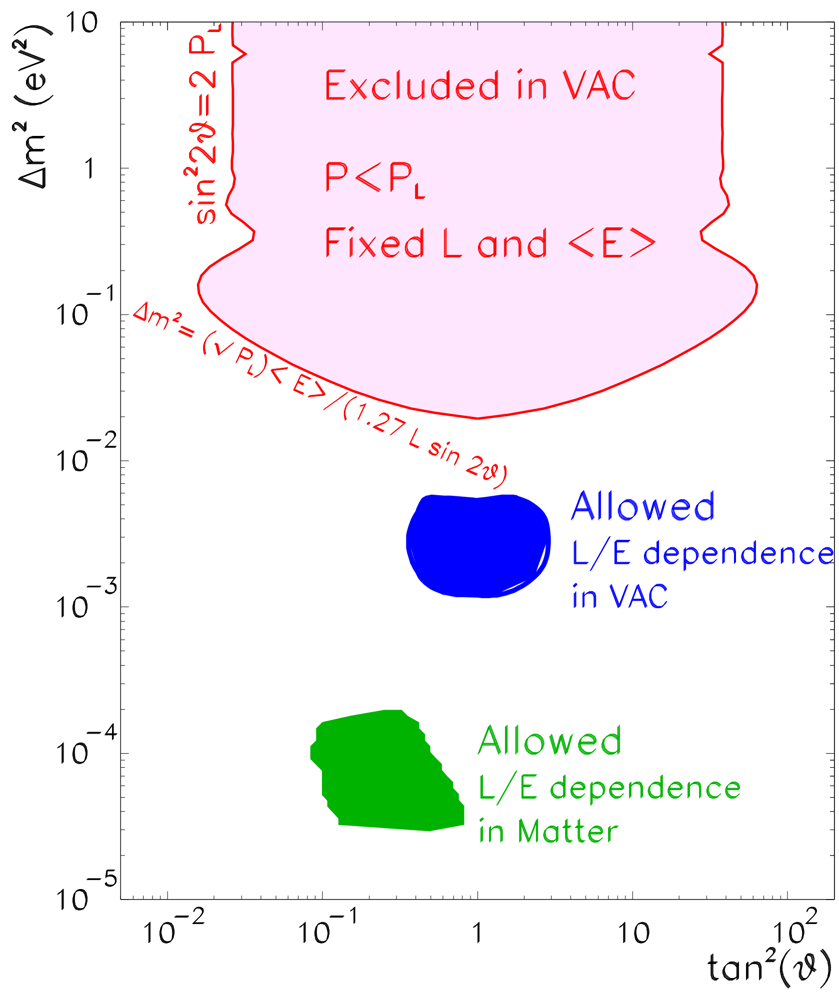
  $\to$ region may be closed

– If no matter effects:

  $\to$ region is symmetric around $\theta = \frac{\pi}{4}$

– If matter effects

  $\to$ region not symmetric around $\theta = \frac{\pi}{4}$



Summary II

- Neutrino masses and mixing $\Rightarrow$ Flavour oscillations
- Experiments observing oscillations $\Rightarrow$ measurement of Δm^2_{ij} and θ_{ij}
- For existing sources and set-ups

Source		Δm^2 (eV^2)
Solar	Vac Osc	10^{-11}–10^{-10}
Solar	MSW	10^{-9}–10^{-4}
Atmospheric	Vac Osc	10^{-1}–10^{-4}
Reactor SBL	Vac Osc	10^{-2}–10^{-3}
Reactor LBL	Vac Osc	10^{-4}–10^{-5}
Accelerator SBL	Vac Osc	$\gtrsim 0.01$
Accelerator LBL	Vac Osc with some matter effects	10^{-2}–10^{-3}

- If $\theta_{ij} \neq 0$ we can find oscillations with 10^{-11} eV$^2 \leq \Delta m^2 \leq 1$ eV^2