Figure 1.5 *Real-life application of the IK loop model, here at the dilute polymer point $n_{\rm loop} \to 0$ in regime I ($\gamma = \frac{3\pi}{4}$). Photo from the shopping mall EKZ Wien Mitte (Vienna, Austria).*

All three models can be solved in a unified way using coordinate Bethe ansatz [42]. The algebraic Bethe ansatz is also known [42, 43]. We henceforth focus on the second, Izergin–Korepin model.

1.5.2 Algebraic Bethe ansatz for the IK model

The monodromy matrix (1.78) for the IK model is now a 3×3 matrix,

$$T(u) = \begin{bmatrix} A_1(u) & B_1(u) & B_2(u) \\ C_1(u) & A_2(u) & B_3(u) \\ C_2(u) & C_3(u) & A_3(u) \end{bmatrix}, \tag{1.197}$$

where each entry is an operator on the quantum spaces $(\mathbb{C}^3)^{\otimes N}$ for a chain of length N. Starting from the pseudo-vacuum $|\Uparrow\rangle$, we can produce algebraic Bethe ansatz states as in (1.100), but acting now with the *three* types of creation operators $B_j(u_i)$. We note that $B_1(u_i)$ and $B_3(u_i)$ each create *one* particle ($|\uparrow\rangle \to |0\rangle$ or $|0\rangle \to |\downarrow\rangle$), while $B_2(u_i)$ creates *two* particles ($|\uparrow\rangle \to |\downarrow\rangle$). Similarly, the $C_j(u_i)$ are annihilation operators of one or two particles.

The aim is now to construct n-particle states which are eigenstates of the transfer matrix