PHY682 Special Topics in Solid-State Physics: Quantum Information Science

Lecture time: 2:40-4:00PM Monday & Wednesday

Today 10/19:

1. Final project presentation: topic selection in HW5
2. Brief review of quantum error correction and topological quantum computation
3. Week 9: Quantum computing by evolution and by measurement

Review: Correctable Conditions

- P is projection to code space:

$P = \sum_{\text{codes}} |\text{code}\rangle\langle\text{code}|$ $\quad E_i$: set of errors

$$P E_i^\dagger E_j P = \alpha_{ij} P$$

α Hermitian matrix

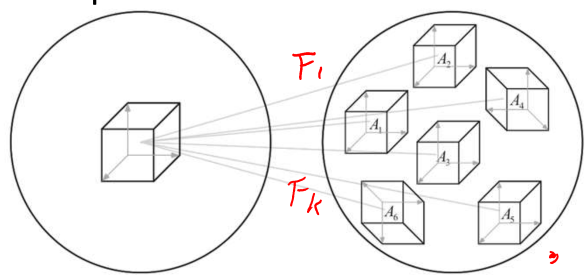
$$\text{Diagonalize} : \alpha_{ij} = \sum_k d_k u_{ik} u_{jk}^*$$

In terms of a new basis $F_k \equiv \sum_j u_{jk}^* E_j$, the condition is equivalent to

$$P F_k^\dagger F_l P = d_k \delta_{kl} P$$

$k \neq l$: $\rightarrow 0$

Errors take to orthogonal subspace → correctable



use n physical qubits indep

n−k constraints

⇒ k qubits encoded

- Theorem 10.8 (Nielsen & Chuang): (Error-correction conditions for stabilizer codes) Let S be the stabilizer for a stabilizer code C(S). Suppose $\{E_j\}$ is a set of operators in G_n such that $E^\dagger_j E_k$ not in N(S)−S for all j and k. Then $\{E_j\}$ is a correctable set of errors for the code C(S).

in this context

either in S or in G_n but not in N(S)

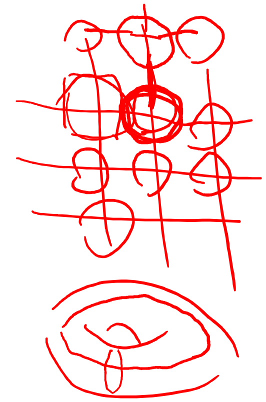
$g \in S \quad g|\text{code}\rangle = |\text{code}\rangle$

$g, g' \in S \quad gg' = g'g$

$I \notin S \quad g =$ product Pauli.

Note: N(S) is normalizer group of S: contains elements E of G_n that preserve S, i.e. ∀ g ∈ S ➔ E g E† ∈ S [In this case, N(S) is equal to the centralizer Z(S), the group that commutes with all elements in S]

Review: Group theory and stabilizer group



- Group theory: Pauli Group G_n: the group generated by product of Pauli operators

$$G_1 = \{\pm 1, \pm i\} \times \{I, X, Y, Z\}$$

$$G_2 = \{\pm 1, \pm i\} \times \{I, X, Y, Z\} \otimes \{I, X, Y, Z\}$$

ψ is in code space

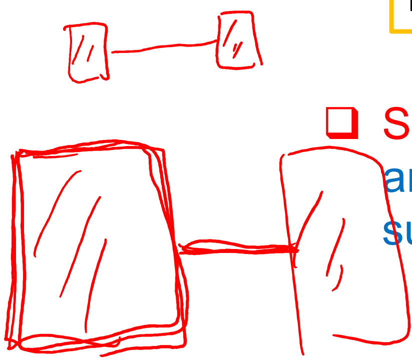
$g|\psi\rangle = |\psi\rangle$

one constraint

$\prod_s A_s = 1$

Recall a group G: (1) has identity element 1, such that 1 x g= g x 1; (2) for any g, there is an inverse g^{-1}, such that g^{-1} x g = g x g^{-1} = 1 . Abelian: g x g' = g' x g (group multiplication is commutative); (3) g x g' is in G if each is; (4) associativity: (a x b) x c = a x (b x c).

surface



- Stabilizer group S, an Abelian subgroup of G, such that

$Z\,Z\,Z$

code space

$$g|\psi\rangle = |\psi\rangle, \ \forall g \in S$$

is extended object

- Example: toric code

This defines the stabilizer group

$E_j^\dagger E_k \notin N(S)-S$

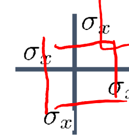
- Star operators:

$\prod_p B_p = 1$

another constraint

N^2

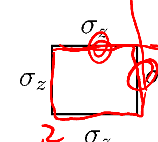
$$A_s = \prod_{j \in s} \sigma_x^{[j]}$$



$n_s = N^2$

- Plaquette operators:

$$B_p = \prod_{j \in \partial(p)} \sigma_z^{[j]}$$



N^2

\# edges $= 2N^2$

2 encoded qubits

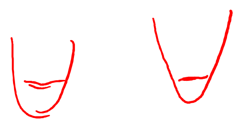
Note: for Abelian group S its normalizer N(S) is equal to the centralizer Z(S), the group that commutes with all elements in S

Review: Anyon models we have learned

- Toric anyon model: 1, e, m, f (abelian)
  - Fusion: *e and m fuse to f (e x m = f); e x f = m, m x f = e*
  - Vacuum I is identity: *I x e = e, I x m = m, I x f = f*
  - Same anyons fuse to vacuum: e x e = I = m x m = f x f

- Fibonacci anyon model: Only one nontrivial anyon: τ

  → Fusion: $\tau \times \tau = 1 + \tau$



p-wave sc.

condense of (fermion) cooper pairs

- Ising anyon model: 1, ψ, σ

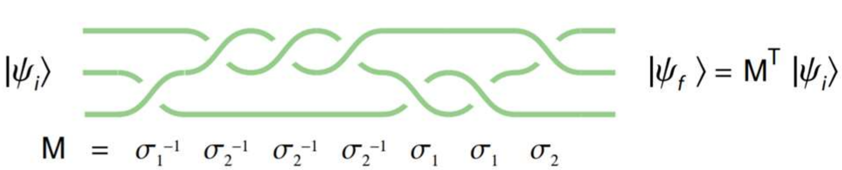
  → Fusion:

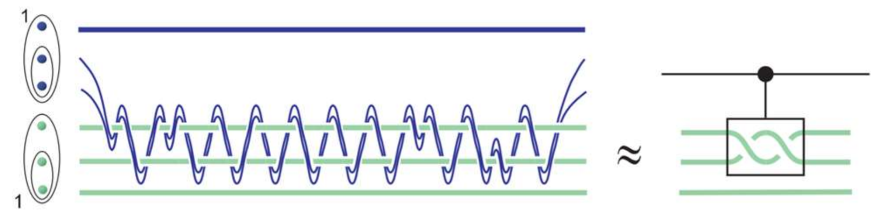
$$1 \times 1 = 1,\ 1 \times \psi = \psi,\ 1 \times \sigma = \sigma$$

$$\psi \times \psi = 1,\ \boxed{\psi \times \sigma = \sigma},\ \boxed{\sigma \times \sigma = 1 + \overset{f}{\psi}}$$

Review: Topological Quantum Computation

- Fibonacci is powerful (but hard to find the physical system):

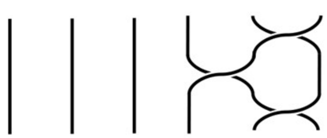




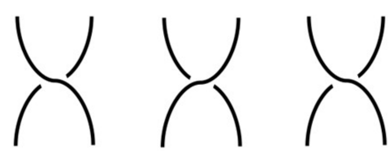
- Ising anyon likely to achieve but not universal (need magic state distillation)

$X_1 = (R_{23})^2$

$U_{H,2} = R_{56} R_{45} R_{56}$

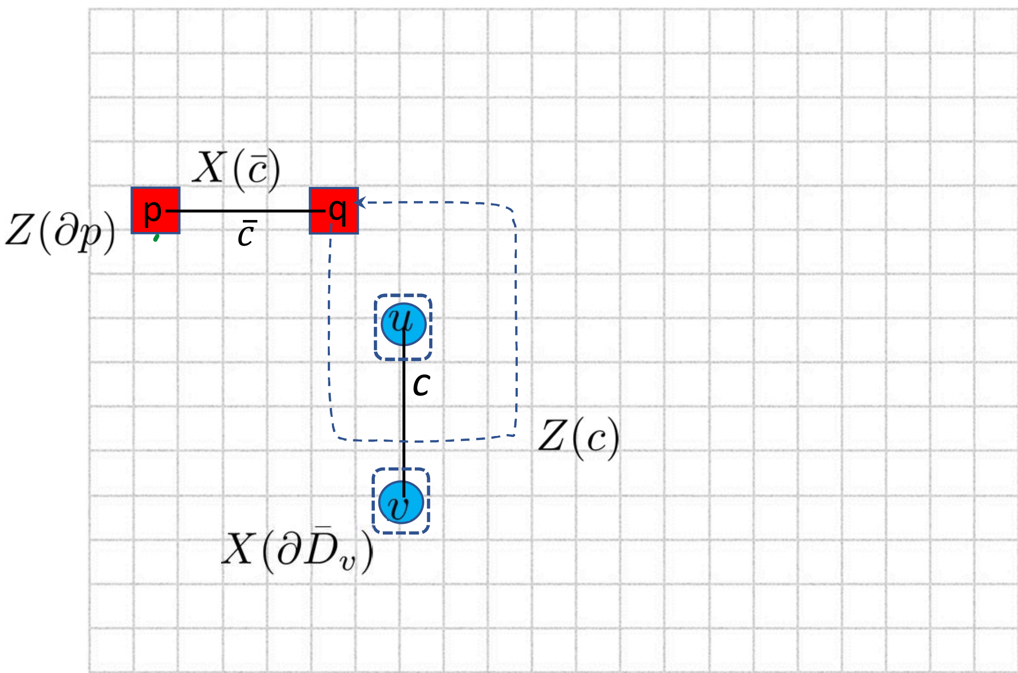


$U_{CZ} = R_{12}^{-1} R_{56}^{-1} R_{34}$

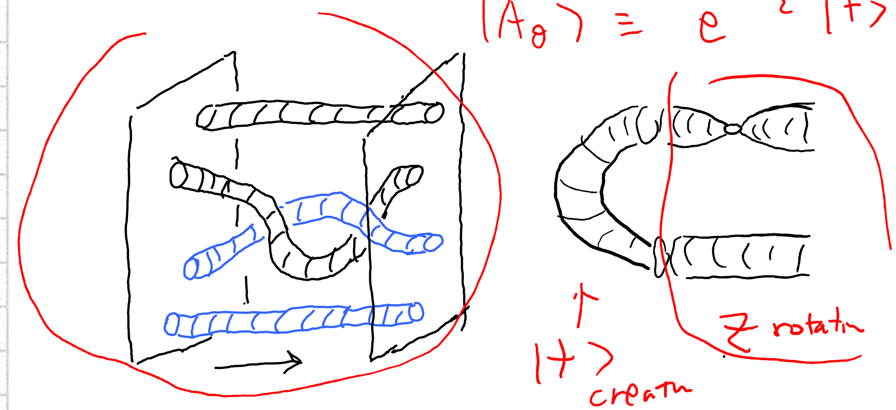


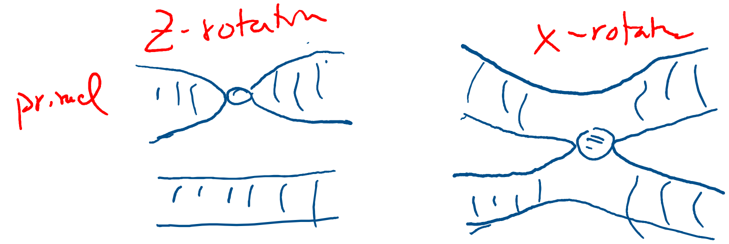
$$
\begin{aligned}
|00\rangle &= |\sigma\sigma;1\rangle|\sigma\sigma;1\rangle|\sigma\sigma;1\rangle, \\
|10\rangle &= |\sigma\sigma;\psi\rangle|\sigma\sigma;\psi\rangle|\sigma\sigma;1\rangle, \\
|01\rangle &= |\sigma\sigma;1\rangle|\sigma\sigma;\psi\rangle|\sigma\sigma;\psi\rangle, \\
|11\rangle &= |\sigma\sigma;\psi\rangle|\sigma\sigma;1\rangle|\sigma\sigma;\psi\rangle,
\end{aligned}
$$

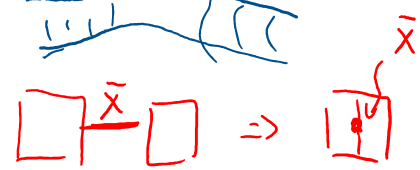
Review: Surface code QC is closer to reality (than you think)



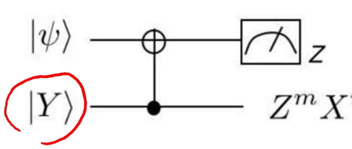
$|A_\theta\rangle = e^{-iZ\frac{\theta}{2}}|+\rangle$



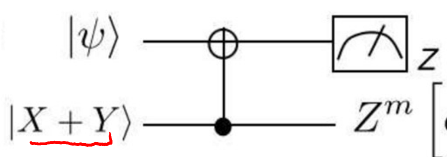




$S = \begin{pmatrix} 1 & \\ 0 & i \end{pmatrix}$ gate



$Z^m X^m e^{-i(\pi/4)X}|\psi\rangle$ ↙ X-basis

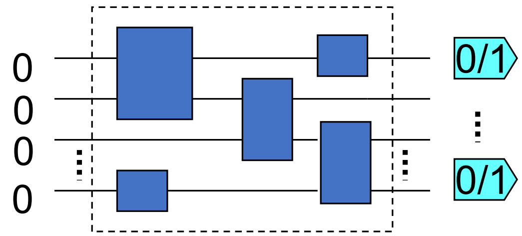


$Z^m\left[e^{-i(\pi/4)X}\right]^m e^{-i(\pi/8)X}|\psi\rangle$

Week 9: Quantum computing by evolution and by measurement: Other frameworks of quantum computation: adiabatic and measurement-based; D-Wave's quantum annealers

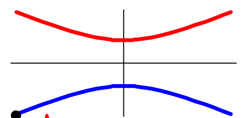
(Frameworks of) Quantum Computation

I. Circuit:



- ✓ Major scheme by most labs: IBM, Intel Rigetti, IonQ, Alibaba

II. Adiabatic:

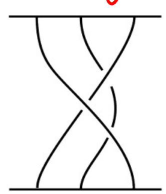


via evolving Schrödinger eq

$$H(t) = \left(1 - \frac{t}{T}\right) H_{\text{initial}} + \frac{t}{T} H_{\text{final}}$$

- ✓ Approach by D-Wave

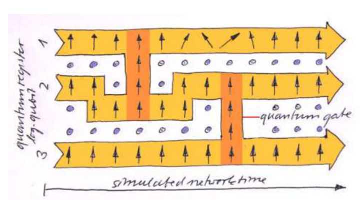
III. Topological:



quantum gates = braiding anyons

- ✓ Approach by Microsoft, ← Ising anyon
  Google uses a hybrid of III and I
  (circuit version of IV) ← surface code

IV. Measurement -based:



local measurement is the only operation needed

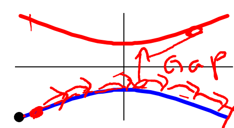
- ✓ Used in photonic systems, such as PsiQuantum

Adiabatic Quantum Computation

- Quantum Computation by Adiabatic Evolution:
  engineer a time dependent Hamiltonian

[Kadowaki & Nishimori, PRE 58, 5355 (1998); Averin, Solid State Comm; Farhi, Goldstone, Gutmann & Sipser,quant-ph/0001106; Grant & Humble, review article in Oxford Research Encyclopedia of Physics 2019]

$$H(t) = \left(1 - \frac{t}{T}\right) H_{\text{initial}} + \frac{t}{T} H_{\text{final}}$$



$$i\hbar \frac{d}{dt}|\psi(t)\rangle = \hat{H}(t)|\psi(t)\rangle$$

solution of SE

$$|\psi(t)\rangle = e^{-iH(t_N)dt} e^{-iH(t_2)dt} e^{-iH(t_1)dt} |\psi_{\text{initial}}\rangle, \quad dt = T/N, \ N \to \infty$$

$$=: \{\text{Time order}\}\, e^{-i\int_0^T dt' H(t')} |\psi_{\text{initial}}\rangle$$

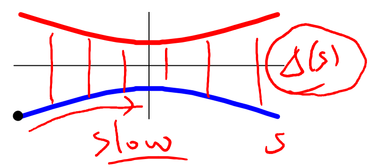
- Known to achieve universal QC [Aharonov et al. '07, Lidar & Mitchell '07]

➔ Can turn any quantum circuit and construct a time-dependent Hamiltonian H(t) to achieve the same computation

Adiabatic Quantum Computation

- Quantum Computation by Adiabatic Evolution:
  engineer a time dependent Hamiltonian

$$H(t) = \left(1 - \frac{t}{T}\right) H_{\text{initial}} + \frac{t}{T} H_{\text{final}}$$



- Requirement: H(t) needs to have an energy gap and the evolution needs to be slow ➔ How large should T be?

Ground
final eigenstate

- Time factor: $\widetilde{H}(s = t/T) \equiv H(t)$

  - For T→∞, $|\psi(0)\rangle = |\tilde{E}_0(0)\rangle, \; |\langle \tilde{E}_0(1)|\psi(T)\rangle|^2 \to 1$ (adiabatic theorem)

  - If we aim to have $\| |\tilde{E}_0(1)\rangle - |\psi(T)\rangle \| < \epsilon$ evolved state [Teufel '03]

    we must have

$$T \geq \frac{2}{\epsilon}\left[c_1 \frac{\|\dot{\tilde{H}}(0)\|}{\Delta(0)^2} + c_2 \frac{\|\dot{\tilde{H}}(1)\|}{\Delta(1)^2} + \int_0^1 ds \left((3c_1^2 + c_1 + c_3) \frac{\|\dot{\tilde{H}}\|}{\Delta(s)^3} + c_2 \frac{\|\ddot{\tilde{H}}\|}{\Delta(s)^2} \right) \right]$$

where Δ(s) is spectral gap

AQC: time factor

- Time factor: $\widetilde{H}(s = t/T) \equiv H(t)$
  - For T→∞, $|\psi(0)\rangle = |\tilde{E}_0(0)\rangle, \ |\langle \tilde{E}_0(1)|\psi(T)\rangle|^2 \to 1$
  - If we aim to have $\| |\tilde{E}_0(1)\rangle - |\psi(T)\rangle \| < \epsilon$ [Teufel '03]

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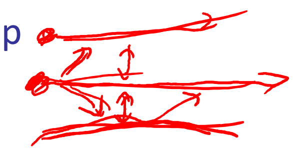
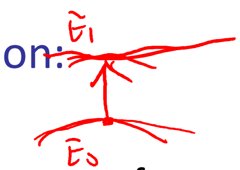
where Δ(s) is spectral gap



- Less precise but easier to remember condition:



1st ↙GS

$$\frac{\max |\langle E_1(s)| dH(t)/dt |E_0(s)\rangle|}{\min \Delta(s)^2} \leq \epsilon$$

➔ The rate of energy coupling to excited state is slow

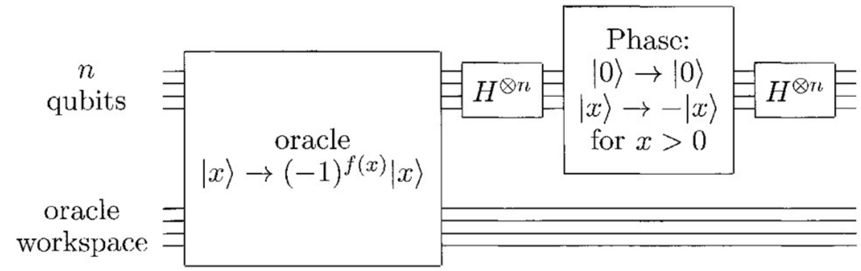
- We will apply the AQC to Grover's search algorithm

Review of Grover searching

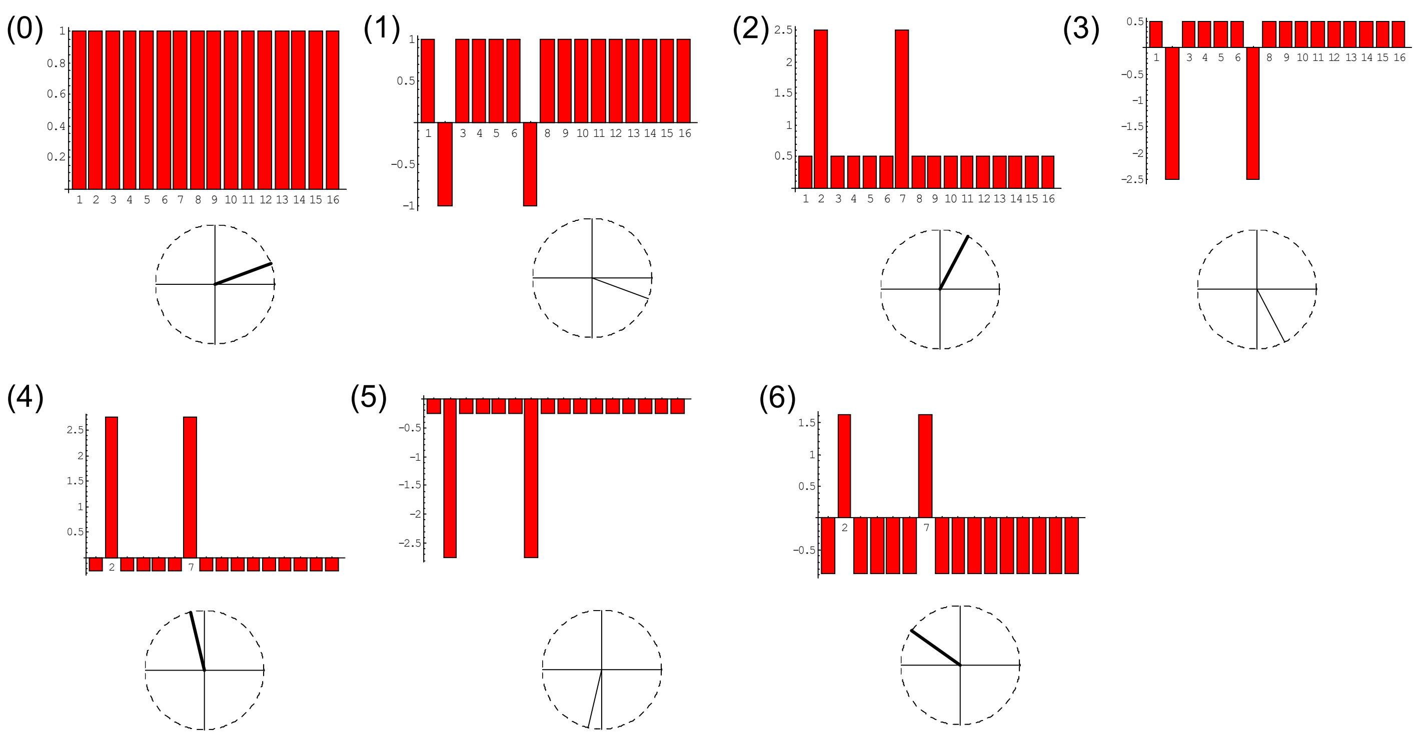
Perform iteration:

(i) Sign on marked targets

(ii) Reflection w.r.t mean



Suppose: items 2 and 7 marked



Review: Analysis of one Grover step

(i) Sign on marked targets
[equivalent to reflection w.r.t. the unmarked "plane"]

$$\hat{O}_f = \sum_x (-1)^{f(x)} |x\rangle\langle x| = I - 2 \sum_{x \in \text{marked}} |x\rangle\langle x|$$

(ii) Reflection w.r.t mean

$$U_s = 2|s\rangle\langle s| - I = H^{\otimes n}(2|0\ldots 0\rangle\langle 0 \ldots 0| - I)H^{\otimes n}$$

$$|s\rangle = |++\cdots+\rangle = \frac{1}{\sqrt{N = 2^n}} \sum_{x=0}^{2^n-1} |x\rangle$$

$$|\alpha\rangle \equiv \sum_k \alpha_k |k\rangle \longrightarrow 2|s\rangle\langle s|\alpha\rangle - |\alpha\rangle \qquad \alpha_k \longrightarrow 2\frac{1}{N}\sum_j \alpha_j - \alpha_k = 2\langle\alpha\rangle - \alpha_k$$

- One Grover iteration is a unitary operation that is equivalent to a rotation:

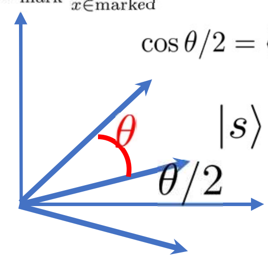
$$\hat{G} \equiv U_s \hat{O}_f$$

$$|\psi_{\text{marked}}\rangle \equiv \frac{1}{\sqrt{N_{\text{mark}}}} \sum_{x \in \text{marked}} |x\rangle$$

$$\cos\theta/2 = \langle s|\psi_{unmarked}\rangle = \frac{\sqrt{(N - N_{\text{mark}})}}{\sqrt{N}}$$

with the angle satisfying

$$\sin\theta = 2\frac{\sqrt{N_{\text{mark}}(N - N_{\text{mark}})}}{N}$$



$$|\psi_{\text{unmarked}}\rangle \equiv \frac{1}{\sqrt{N - N_{\text{mark}}}} \sum_{x \in \text{unmarked}} |x\rangle$$

Time complexity of Grover Algorithm

- One Grover iteration is a unitary operation that is equivalent to a rotation:

$$\hat{G} \equiv U_s \, \hat{O}_f$$

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$$\cos\theta/2 = \langle s|\psi_{unmarked}\rangle = \frac{\sqrt{(N - N_{\mathrm{mark}})}}{\sqrt{N}}$$

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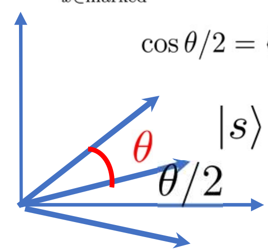


$$|\psi_{\mathrm{unmarked}}\rangle \equiv \frac{1}{\sqrt{N - N_{\mathrm{mark}}}} \sum_{x \in \mathrm{unmarked}} |x\rangle$$

- Assume number of marked items smaller than N/2, and approximate

$$\theta \approx 2\frac{\sqrt{N_{\mathrm{mark}}(N - N_{\mathrm{mark}})}}{N}$$

➔ Number of iterations to reach an angle π/2:

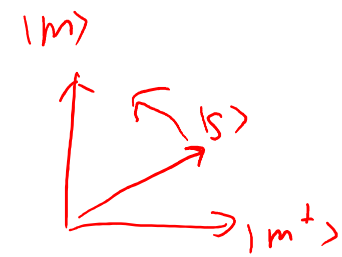
$$N_{\mathrm{iter}}\,\theta + \frac{\theta}{2} \approx \frac{\pi}{2} \qquad\qquad N_{\mathrm{iter}} \approx \frac{\pi}{2\theta} - \frac{1}{2} \approx \left[\frac{1}{4}\sqrt{\frac{N}{N_{\mathrm{mark}}}}\right]$$

➔ For N=4, only one marked item: θ=π/3, one iteration reaches the target with probability 1

Search by adiabatic evolution

- ❑ Choose initial Hamiltonian is such that |s> is the ground state $|s\rangle = |++\cdots+\rangle = \frac{1}{\sqrt{N=2^n}} \sum_{x=0}^{2^n-1} |x\rangle$

$$H_0 = I - |s\rangle\langle s|$$



- ❑ Assume one marked item ***m*** ➔ choose final Hamiltonian

$$H_m = I - |m\rangle\langle m| \qquad \langle m|s\rangle = 1/\sqrt{N} =: a$$

|m>

|s>

- ❑ Time dependent Hamiltonian

$$H(t) = \left(1 - \frac{t}{T}\right) H_0 + \frac{t}{T} H_m = I - (1 - \frac{t}{T})|s\rangle\langle s| - \frac{t}{T}|m\rangle\langle m|$$

Time-dependent Hamiltonian

$$H(t) = \left(1-\frac{t}{T}\right)H_0 + \frac{t}{T}H_m = I - \underbrace{(1-\frac{t}{T})|s\rangle\langle s| - \frac{t}{T}|m\rangle\langle m|}_{} \qquad \langle m|s\rangle = 1/\sqrt{N} =: a$$

- In the two-dimensional subspace spanned by |m> & |s> or equivalently |m> & |m⊥>

$|m\rangle \to |m^\perp\rangle$? $|s\rangle$ — component along $|m\rangle \to |m^\perp\rangle$

$$|m^\perp\rangle = c(|s\rangle - a|m\rangle),\ c = 1/\sqrt{1-a^2},\ a = \langle m|s\rangle = 1/\sqrt{N}$$

needed to normalize $|m^\perp\rangle$

$\langle m|m^\perp\rangle = c(\underbrace{\langle m|s\rangle}_{a} - a\underbrace{\langle m|m\rangle}_{1}) = 0$

$$H(t) = \begin{pmatrix} \langle m|H(t)|m\rangle & \langle m|H(t)|m^\perp\rangle \\ \langle m^\perp|H(t)|m\rangle & \langle m^\perp|H(t)|m^\perp\rangle \end{pmatrix} = I - \begin{pmatrix} (1-t/T)a^2 + t/T & (1-t/T)ca(1-a^2) \\ (1-t/T)ca(1-a^2) & (1-t/T)c^2(1-a^2)^2 \end{pmatrix}$$

Same

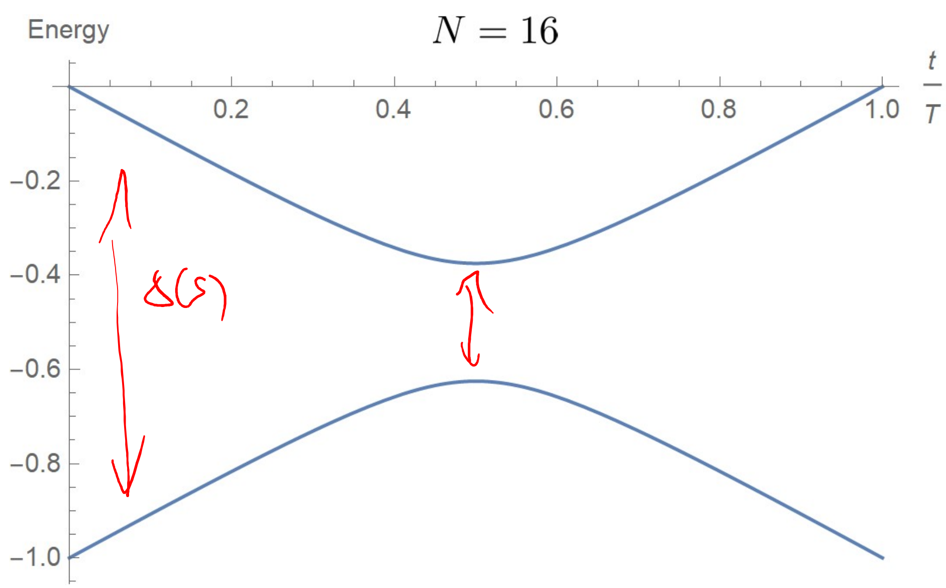
$-(1-\frac{t}{T})\underbrace{\langle m|s\rangle\langle s|m\rangle}_{a^2} - \frac{t}{T}\underbrace{\langle m|m\rangle\langle m|m\rangle}_{1}$

- 2x2 matrix → easily diagonalized

→ eigenvalues ⇒ $\Delta(t/T)$

Gap of the Grover's Hamiltonian

$$H(t) = \left(1 - \frac{t}{T}\right) H_0 + \frac{t}{T} H_m = I - (1 - \frac{t}{T})|s\rangle\langle s| - \frac{t}{T}|m\rangle\langle m| \qquad \langle m|s\rangle = 1/\sqrt{N} =: a$$



$= s$

$1 - \frac{t}{T}$ $\qquad$ $\frac{t}{T}$

our time runs linearly

Gap: $\Delta(s = t/T) = \sqrt{1 - 4(1 - 1/N)(1 - s)s}$ $\qquad$ $\min\limits_s \Delta(s) = 1/\sqrt{N}$

Time Factor

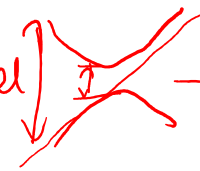
$$\min_s \Delta(s) = 1/\sqrt{N}, \quad s = t/T \qquad H(t) = \left(1-\frac{t}{T}\right)H_0 + \frac{t}{T}H_m = I - \left(1-\frac{t}{T}\right)|s\rangle\langle s| - \frac{t}{T}|m\rangle\langle m|$$

- We find minimum gap scales as 1/√N, however, the 'time' evolves linearly

❖ Recall condition of "adiabaticity"

$$\frac{\max |\langle E_1(s)|dH(t)/dt|E_0(s)\rangle|}{\min \Delta(s)^2} \le \epsilon$$

$1/T$



$$dH(t)/dt = \frac{1}{T}(|s\rangle\langle s| - |m\rangle\langle m|) \qquad \Longrightarrow \max |\langle E_1(s)|dH(t)/dt|E_0(s)\rangle| \sim 1/T$$

$$T \ge 1/(\min \Delta(s))^2/\epsilon \sim N/\epsilon$$

➔ No speedup ☹

$1-\frac{t}{T}$ $\qquad \frac{t}{T}$

❖ Should run faster outside minimum gap region, e.g. take

$s \neq t \qquad s(t)$

$$ds/dt = \epsilon\Delta(s)^2 = \epsilon\Big(1 - 4(1-1/N)(1-s)s\Big)$$

New time schedule

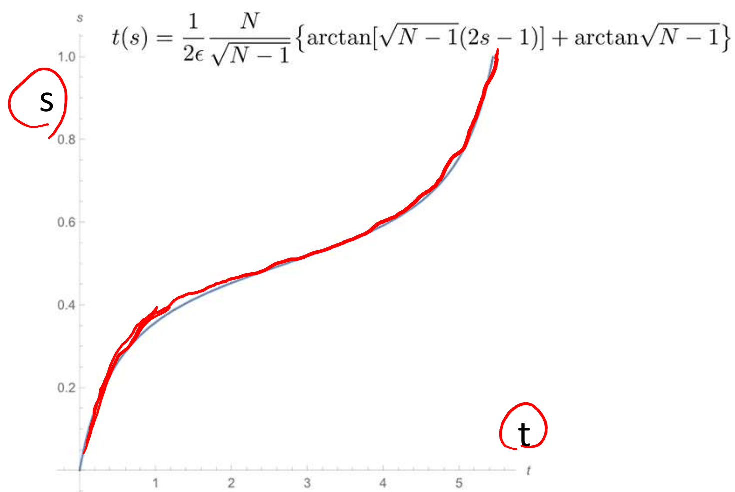
$$H(t) = \Big(1 - s(t)\Big)H_0 + s(t)H_m = I - \Big(1 - s(t)\Big)|s\rangle\langle s| - s(t)|m\rangle\langle m|$$

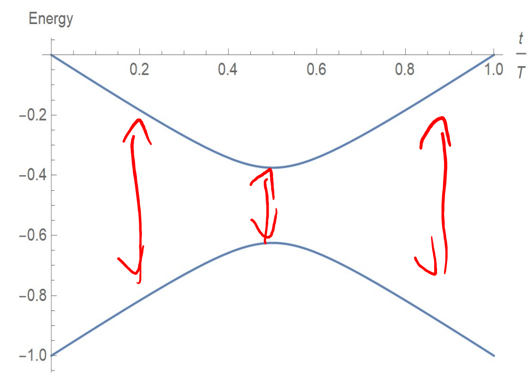
$$\boxed{ds/dt = \epsilon\Delta(s)^2 = \epsilon\Big(1 - 4(1 - 1/N)(1 - s)s\Big)}$$

→ find $s(t)$ $t(s)$

$$T = \int_0^T dt = \int_0^1 \frac{ds}{\epsilon\Big(1 - 4(1 - 1/N)(1 - s)s\Big)} = \boxed{\frac{1}{\epsilon}\frac{N}{\sqrt{N-1}}\arctan\sqrt{N-1}} \approx \frac{\pi}{2\epsilon}\sqrt{N} \text{ for large } N$$

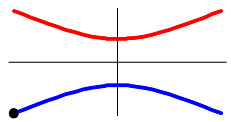
➔ Speedup ☺





Adiabatic vs. "Zeno" approach

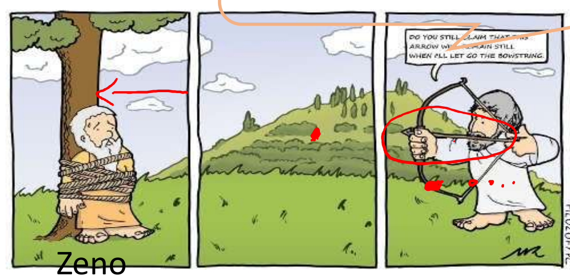
❑ Adiabatic:



$$H(t) = \left(1 - \frac{t}{T}\right) H_{\text{initial}} + \frac{t}{T} H_{\text{final}}$$

❑ It is also possible to use measurement, i.e. Zeno effect

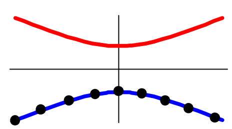
Do you still claim that this arrow will remain still ….



Zeno

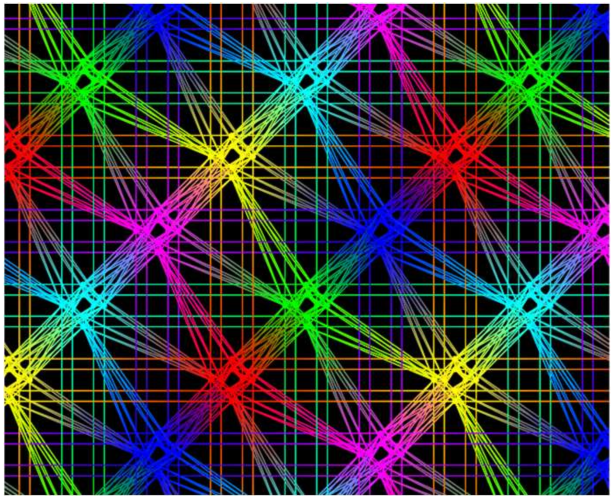
❑ "Quantum simulations of classical annealing processes" by Somma, Boixo, Barnum and Knill [PRL101,130504 (2008)]

- Measurement needs to **project to eigenstates of H(t)** [see e.g. Chen &Wei, PRA 101, 032339 (2020)]
- Ground state at t=T can be arrived by such Zeno measurement on H(t) for a sequence of t=0,Δt, 2Δt, …, T



D-Wave’s quantum annealers

- ❑ Has 5,000-qubit fifth generation quantum annealer
- ❑ These qubits are much noisier than other circuit-based ones (such as in IBM, Google, Rigetti, etc.)

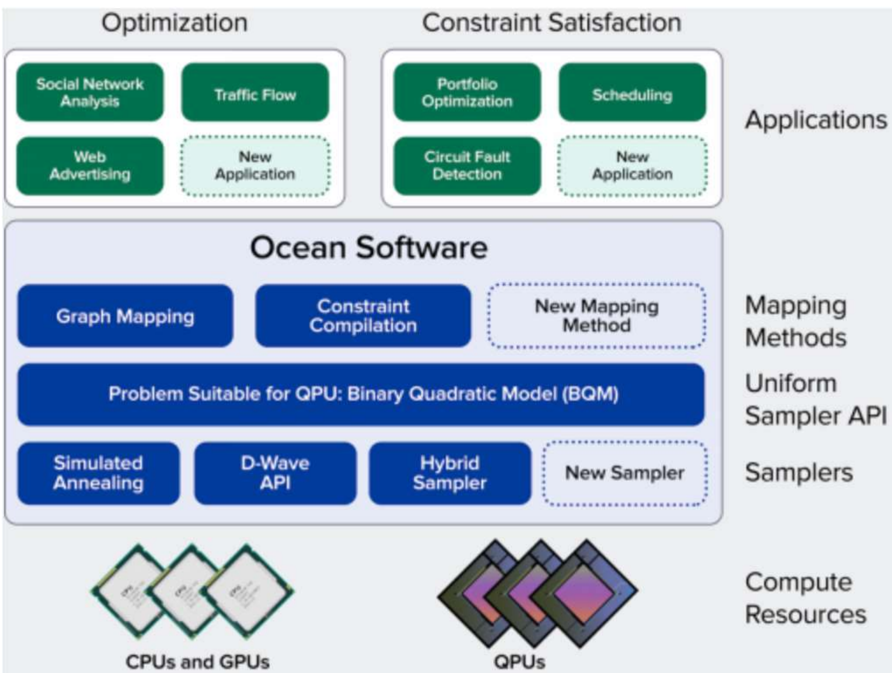


- ❖ Qubits are coupled as in a `Chimera’ graph
  → high local connectivity
- ❖ Detailed description at
  https://docs.dwavesys.com/docs/latest/c_gs_4.html

https://arstechnica.com/science/2019/09/d-wave-announces-the-next-generation-of-its-quantum-annealer/

D-Wave's software

- Ocean SDK (software development kit): Python-based



https://www.dwavesys.com/quantum-computing

- Solve binary quadratic model (BQM), e.g. Maxcut and Traveling Salesman problems discussed earlier

$$H_P = \sum_{i}^{N} q_i \overset{0/1}{x_i} + \sum_{i<j}^{N} q_{i,j} x_i x_j, \;\; \min H_P = ?$$

  - ✓ Problem input: q_i and q_{ij}

- Method: adiabatic optimization---from a simple Hamiltonian H_0 and connects to problem H_P in Pauli Z: x= (1- $\underset{z}{\sigma}$)/2

$$H_0 = -\sum_{i}^{N} \sigma_i^x$$

$(1-\frac{t}{T})H_0 + \frac{t}{T} H_p$

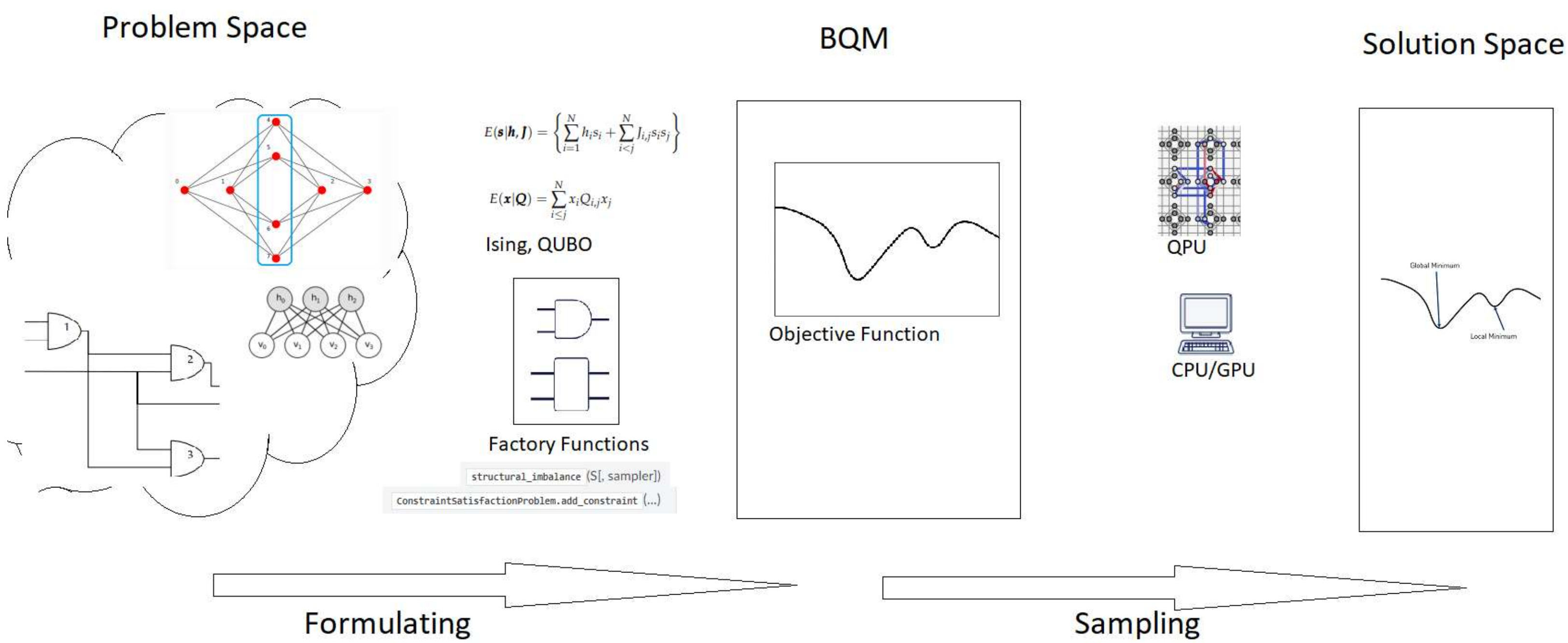
$A(s)\, H_0 + B(s)\, H_p$

$A \to 1$ $s \to 0$
$B \to 0$

$A \to 0$ $s \to 1$
$B \to 1$

Schematic diagram (using D-Wave’s annealer)

https://docs.ocean.dwavesys.com/en/latest/overview/solving_problems.html

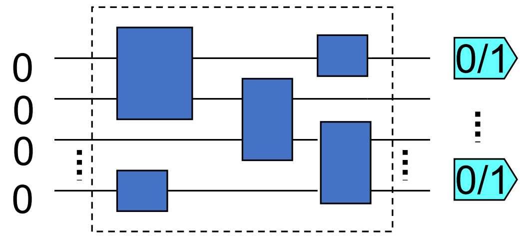


Examples and tutorials: https://docs.ocean.dwavesys.com/en/latest/getting_started.html#demonstrations-and-jupyter-notebooks

(Frameworks of) Quantum Computation

✓ Previously covered:

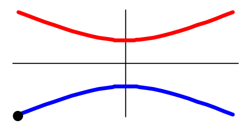
I. Circuit:



✓ Major scheme by most labs: IBM, Intel Rigetti, IonQ, Alibaba

✓ Done with this:

II. Adiabatic:

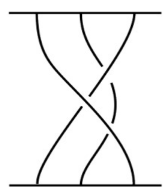


$$H(t) = \left(1 - \frac{t}{T}\right) H_{\text{initial}} + \frac{t}{T} H_{\text{final}}$$

✓ Approach by D-Wave

✓ Previously covered:

III. Topological:

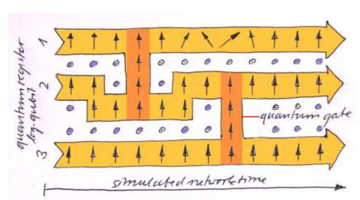


quantum gates = braiding anyons

✓ Approach by Microsoft, Google uses a hybrid of III and I (circuit version of IV)

Next:

IV. Measurement -based:



local measurement is the only operation needed

✓ Used in photonic systems, such as PsiQuantum