PHY682 Special Topics in Solid-State Physics: Quantum Information Science

Lecture time: 2:40-4:00PM Monday & Wednesday

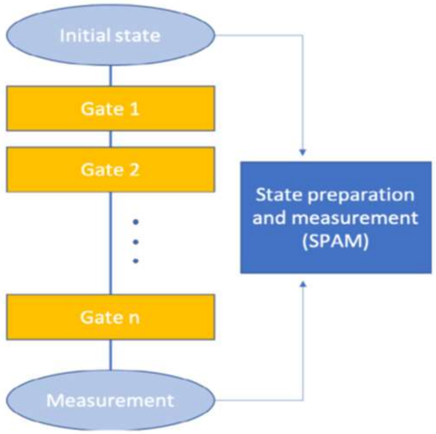
Today 11/9:

1. Final presentation discussion (within each group)
2. Finish Week 10's topics (today on tomographic tools)
3. Begin Week 11's topics (quantum phase estimation and applications)

Tomographic tools

- ❑ Crucial to ensure proper functioning of QC and correctness of results

  - ❖ State preparation
    - ⟹ State tomography
  - ❖ Gate operations
    - ⟹ Process tomography
  - ❖ Measurement (i.e. detectors)
    - ⟹ Detector tomography

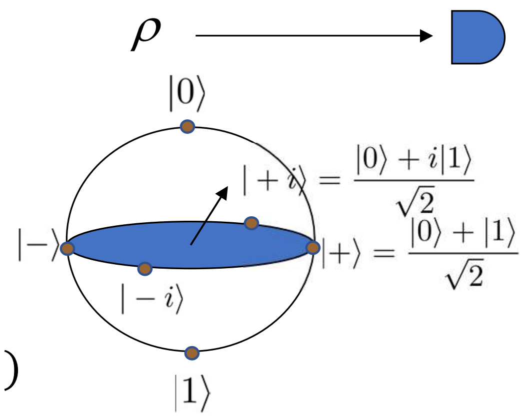


- ❑ Note: detector tomography is often ignored, but important to extract correct computational outcomes

Quantum state tomography

- Estimate unknown state (given multiple copies)
- One qubit

$$\rho = \frac{1}{2}(I + r_x\sigma_x + r_y\sigma_y + r_z\sigma_z)$$
$$r_x = tr(\rho\sigma_x) = tr(\rho|+\rangle\langle+|) - tr(\rho|-\rangle\langle-|)$$
$$r_y = tr(\rho\sigma_y) = tr(\rho|+i\rangle\langle+i|) - tr(\rho|-i\rangle\langle-i|)$$
$$r_z = tr(\rho\sigma_z) = tr(\rho|0\rangle\langle0|) - tr(\rho|1\rangle\langle1|)$$



Bloch sphere

➔ If one can measure the qubit in all three bases, can extract Bloch vector $\vec{r}$

- Multi-qubits:

$$\rho_{2-qubit} = \frac{1}{4}\sum_{\mu\nu} r_{\mu\nu}\sigma_\mu \otimes \sigma_\nu\,, \qquad r_{\mu\nu} = tr\left(\rho_{2-qubit}\,\sigma_\mu \otimes \sigma_\nu\right)$$

➔ Measure in product of bases (i.e. coincidence)

Quantum process tomography

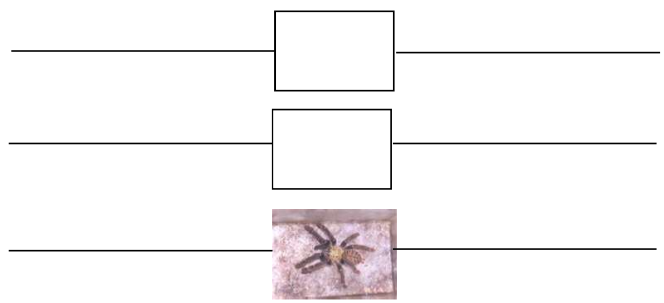
- Estimate unknow process (Black box)

$$\mathrm{E}: \rho \to \sum_i E_i \rho E_i^\dagger$$

(quantum operations)

- Possible application: “debugging” quantum gates

quantum circuit:

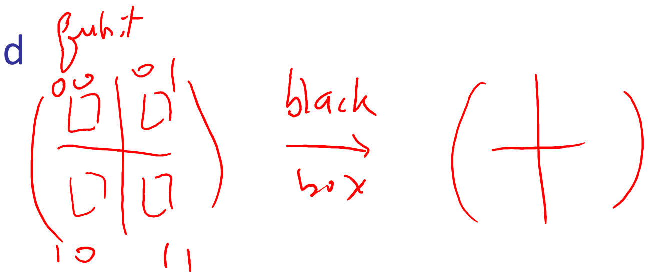


Q: From measuring a limited number of different input states (but unlimited supply of each), is it possible to predict the result for a general input state?

Three different ways of implementing quantum process tomography (PT)

(I) Standard Quantum PT (SQPT)

- Idea: look at how each element gets transformed



$$E: |j\rangle\langle k| \rightarrow \sum_i E_i |j\rangle\langle k| E_i^\dagger$$

$\left|j\right\rangle\left\langle k\right|$ for single qubit are

$$|0\rangle\langle 0|$$

$$|0\rangle\langle 1| = |+\rangle\langle +| + i|+i\rangle\langle +i| - \frac{1+i}{2}|0\rangle\langle 0| - \frac{1+i}{2}|1\rangle\langle 1|$$

$$|1\rangle\langle 0| = |+\rangle\langle +| - i|+i\rangle\langle +i| - \frac{1-i}{2}|0\rangle\langle 0| - \frac{1-i}{2}|1\rangle\langle 1|$$

$$|1\rangle\langle 1|$$

➔ We only need four different inputs $|0\rangle, |1\rangle, |+\rangle, |+i\rangle$

$\rho \in \{|0\rangle, |1\rangle, |+\rangle, |+i\rangle\}$

ρ'

to figure out the unknown action

(II) Entanglement assisted PT (EAPT)

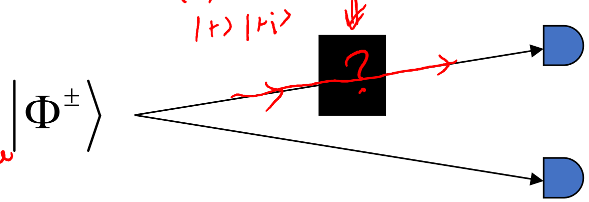
- Q: Can we do better? Send in same state?

Yes, sending a Bell state! $\left|\Phi^{+}\right\rangle \equiv \frac{1}{\sqrt{2}}\left(\left|00\right\rangle + \left|11\right\rangle\right)$

$$\rho_{\Phi^{+}} = \frac{1}{4}\left(I \otimes I + \sigma_x \otimes \sigma_x - \sigma_y \otimes \sigma_y + \sigma_z \otimes \sigma_z\right)$$

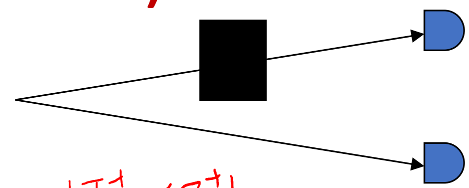
$$\xrightarrow{E} \frac{1}{4}\left(E(I) \otimes I + E(\sigma_x) \otimes \sigma_x - E(\sigma_y) \otimes \sigma_y + E(\sigma_z) \otimes \sigma_z\right)$$

- By measuring output state, we can infer E on the complete set of matrices, thus on arbitrary input state



Question: how can we use "remote-state preparation" to understand this?

(III) Ancilla assisted PT (AAPT)



- Is entanglement necessary? $\rho_{\text{unentangled}}$?

- No, we can use an unentangled state!

$\rho(v) = v|\Phi^+\rangle\langle\Phi^+| + \frac{1-v}{4} I$

$v \le \frac{1}{3}$ not ent.

  - For example, use an unentangled Werner state

$$\rho_{Werner}(1/3) \equiv \frac{1}{6} I \otimes I + \frac{1}{3}\rho_{\Phi^+} = \begin{pmatrix} \frac{1}{3} & 0 & 0 & \frac{1}{6} \\ 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} \end{pmatrix}$$

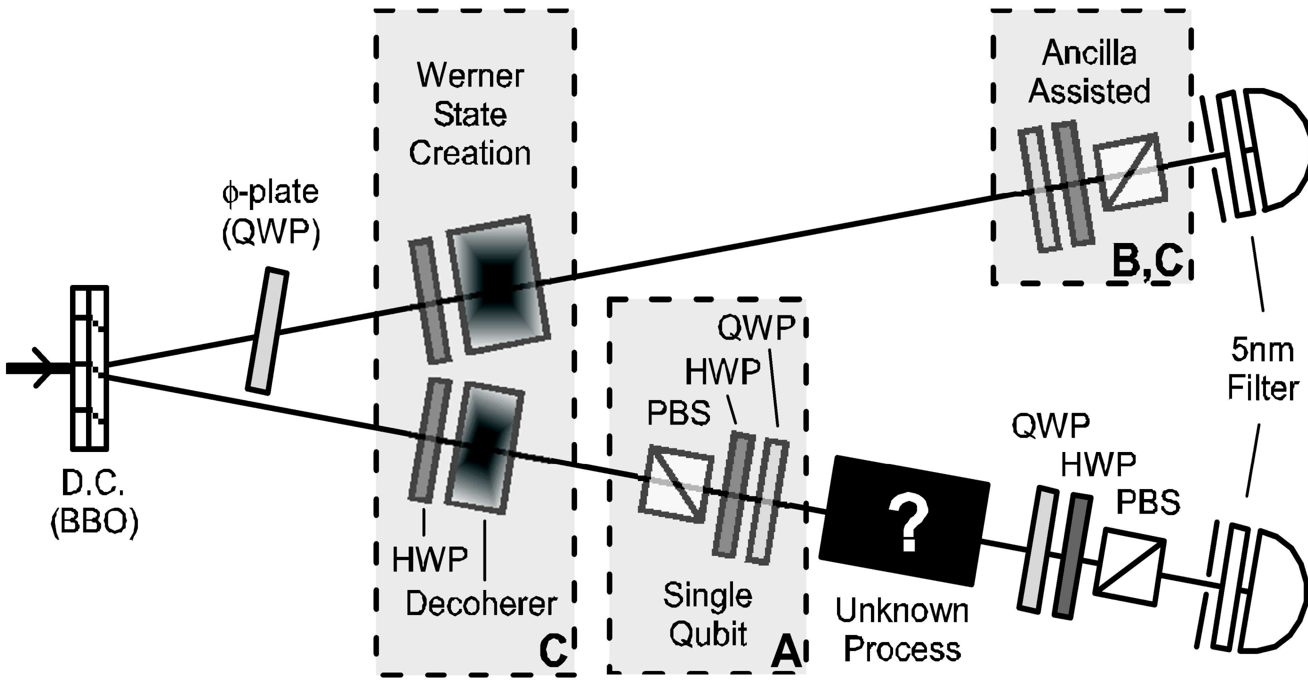
$$= \frac{1}{4}\left(I \otimes I + \frac{1}{3}\sigma_x \otimes \sigma_x - \frac{1}{3}\sigma_y \otimes \sigma_y + \frac{1}{3}\sigma_z \otimes \sigma_z \right)$$

$$\xrightarrow{E} \left(\frac{1}{4}\right)\left(E(I) \otimes I + \left(\frac{1}{3}\right) E(\sigma_x) \otimes \sigma_x - \left(\frac{1}{3}\right) E(\sigma_y) \otimes \sigma_y + \left(\frac{1}{3}\right) E(\sigma_z) \otimes \sigma_z \right)$$

  - Due to 1/3, the noise is expected to be higher than that of using a Bell state

Experimental setup for process tomography

Altepeter, Branning, Jeffrey, **Wei**, Kwiat, Thew, O'Brien, Nielsen, White, Phys. Rev. Lett. 90, 193601 (2003)



Experimental Results

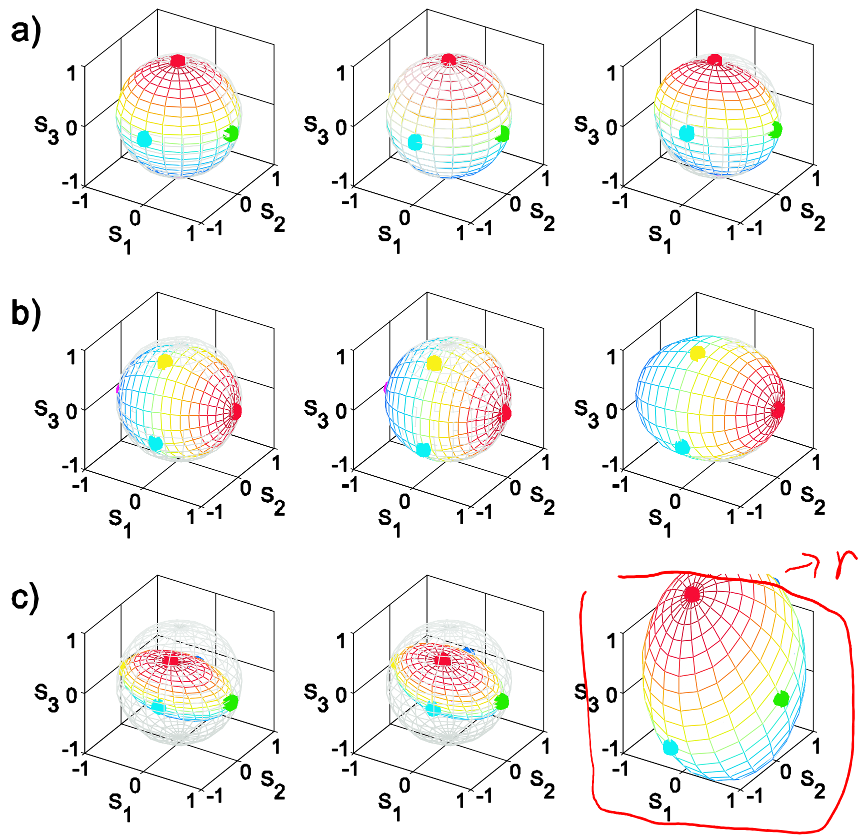
[Altepeter et al., PRL'03]

SQPT EAPT AAPT

Identity

Quarter waveplate

Partial decoherence



$\to r > 1 \Rightarrow$ not physical

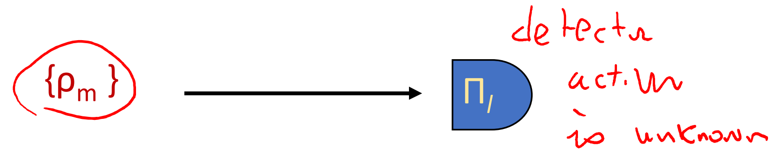
$\mathcal{E}(\rho) = \sum_i E_i^* \rho E_i^+$

using the same degree of freedom to create Werner ρ(!)

Detector tomography

[Fiurasek, PRA 64, 024102 (2001)]

- Assumption: an informationally complete set of test states $\{\rho_m\}$ be prepared with error smaller than error in measurement

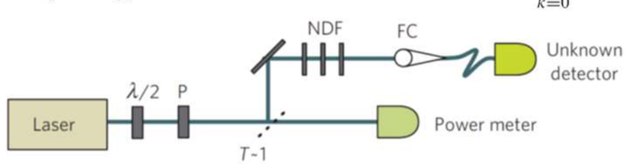


- Goal: infer when detector outcome l clicks, what is measured?

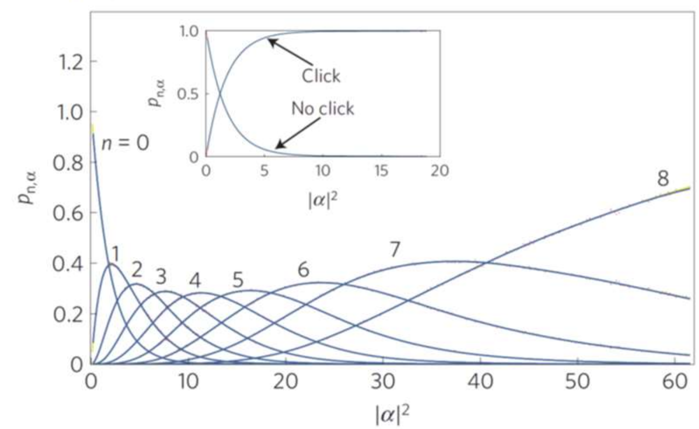
- Recent application in characterizing photon detectors

[Lundeen et al, Nat Phys 5, 27 (2008)]

$$\pi_n = \sum_{k=0}^{\infty} \theta_k^{(n)} |k\rangle\langle k|$$

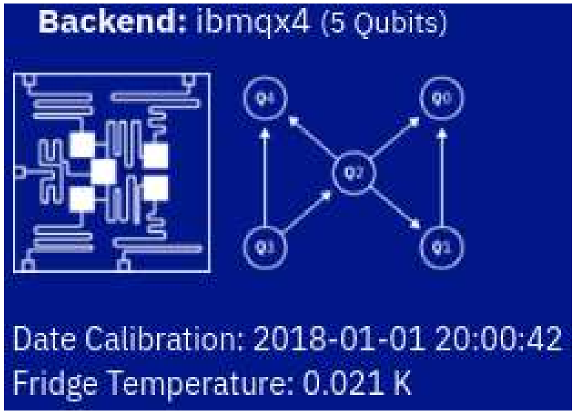


$$p_{n,\rho} = \mathrm{tr}[\rho\pi_n]$$

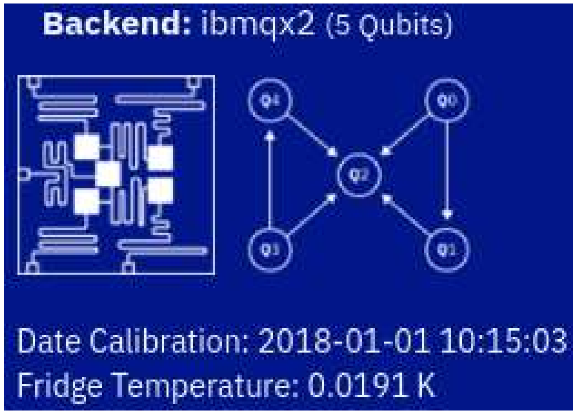


Carry out detector tomography on IBM Q Machines

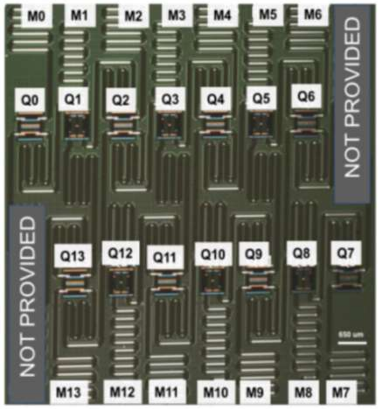
Tenerife



Yorktown



IBM Q 16 Melbourne (14 qubits)

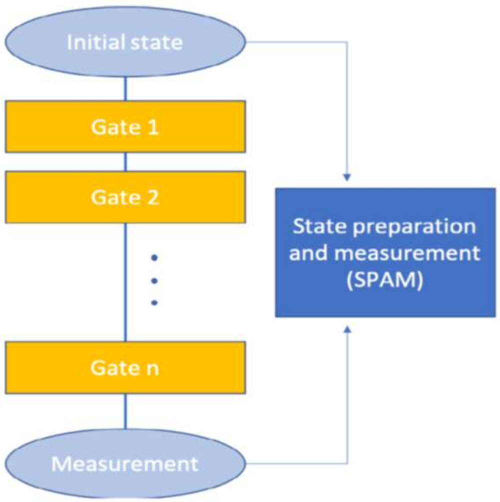


- Transmon qubits

  (transmission line shunted plasma oscillation qubit)

- Coupled by resonators
- Qubit operations by microwave pulses; readout by measuring transmitted radiation

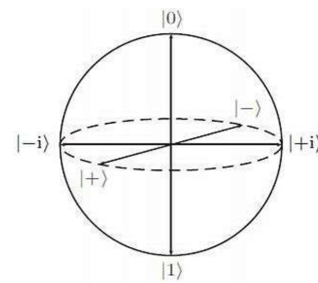
Detector tomography

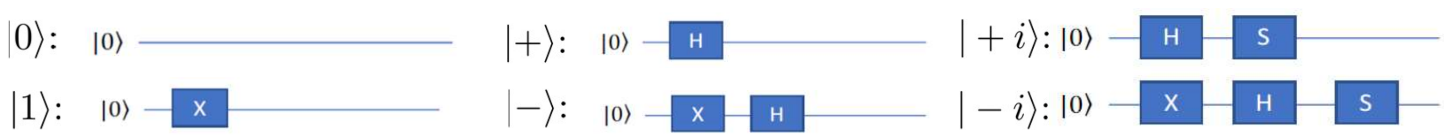


[Fiurasek, PRA 64, 024102 (2001)]

- Assumption: an informationally complete set of test states $\{\rho_m\}$ be prepared with error smaller than error in measurement

- In IBM Q machines, measurement has large errors, but
  ground state $|0\rangle$ preparation and single-qubit gates are of high fidelity so
  can prepare $\{\rho_m = 0,1,+,-,+i,-i\}$





- Goal: to characterize a 2-outcome positive operator valued measure (POVM): $\{\Pi_0, \Pi_1\}$

Maximum Likelihood for Detector Tomography*

[Fiurasek, PRA 64, 024102 (2001)]

- Prepare states ρ_m to characterize POVM elements $\pi_l \geq 0$
  - Probability $p_{lm}=\mathrm{Tr}(\rho_m \pi_l)$ $\approx \frac{f_{lm}}{\sum_{l'} f_{l'm}}$ ← from experimental data

    f_{lm}: number of times input ρ_m such that detector π_l clicks

- Use Maximize the likelihood function to iteratively find π_l

$$\mathcal{L}\{\Pi_l\} = \prod_l \prod_m \left(\mathrm{Tr}[\Pi_l \rho_m]\right)^{f_{lm}}$$ constraint: $$\sum_l \Pi_l = I$$

  - Optimal π_l can be obtained by an iterative process:

$$\Pi_l \leftarrow R_l \Pi_l R_l^\dagger \qquad R_l = \hat{\lambda}^{-1} \sum_m \frac{f_{lm}}{p_{lm}} \rho_m$$

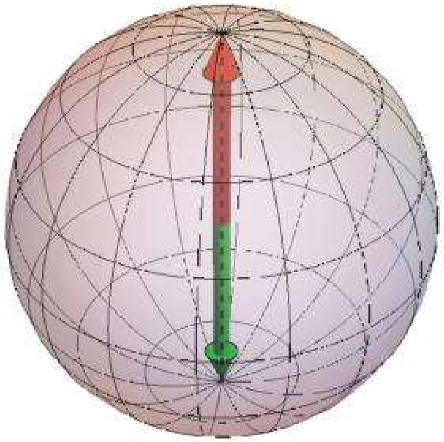
$$\hat{\lambda} = \left(\sum_{m',m,l} \frac{f_{lm}}{p_{lm}} \frac{f_{lm'}}{p_{lm'}} \rho_{m'} \Pi_l \rho_m \right)^{1/2}$$

Perfect projectors onto 0 & 1

- As a reminder, perfect 0 & 1 measurements are

$$\Pi_0 = |0\rangle\langle 0| = \frac{1}{2}I + \frac{1}{2}\sigma_z$$

$$\Pi_1 = |1\rangle\langle 1| = \frac{1}{2}I - \frac{1}{2}\sigma_z$$



Runs on IBMQ

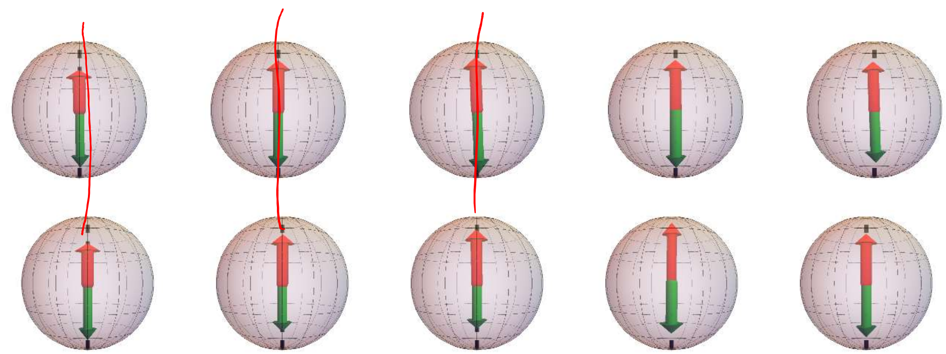
- Prepare states that are ± eigenstates of Pauli operators: {ρ_j =0,1,+,-,+i,-i}

[Chen, Farahzad, Yoo,& Wei, PRA 2019]

- Model of single-qubit detector (POVM):

$$\pi_0 = a_0^{(0)} I + \vec{a}^{(0)} \cdot \vec{\sigma}, \quad \pi_1 = a_0^{(1)} + \vec{a}^{(1)} \cdot \vec{\sigma}$$

- Results on IBM QX4 (5 qubits):
  - Measured individually
  - measured 5 qubits in parallel



- Can be used to mitigate imperfect measurement (next slide)

Arrows indicate the vectors $\vec{a}^{(0)}$ or $\vec{a}^{(1)}$ averaged from 100 different runs;
thickness of arrows represents magnitude of $a_0^{(0)}$ or $a_0^{(1)}$.

$|0\rangle\langle 0| \Leftrightarrow$

generalize

$\rho : |0\rangle\langle 0| \xrightarrow{\text{detection}} M_\rho$

$P_0 = \langle 0|\rho|0\rangle$ $\quad P_\rho = Tr(\rho M_\rho)$

Mitigation of imperfect detectors

- Assume detectors act independently (otherwise need multi-detector tomography)

[Chen, Farahzad, Yoo,& Wei, PRA 2019]

Distribution of measurement outcomes

$$\tilde{P}_{(n_0,n_1,\ldots,n_{N-1})} \approx \mathrm{Tr}\Big[\rho \prod_{j=0}^{N-1}\big(a_{0,[j]} + a_{3,[j]}^{(n_j)}\sigma_{3,[j]}\big)\Big]$$

$$= \sum_{(m_0,m_1,\ldots,m_{N-1})} P_{(m_0,m_1,\ldots,m_{N-1})} \prod_{j=0}^{N-1}\left(a_{0,[j]}^{(n_j)} + (-1)^{m_j} a_{3,[j]}^{(n_j)}\right),$$

$$= \sum_{\vec{m}} M_{\vec{n};\vec{m}} P_{\vec{m}}$$

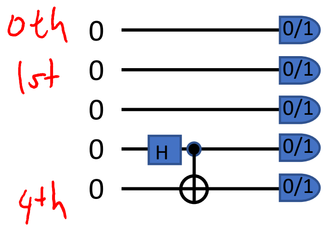
$M_{\vec{n};\vec{m}}$

Distribution of ideal measurement outcomes

- Can invert ideal measurement distribution by inversion (under constraint P be non-negative) [mentioned in an earlier lecture]

Simple application of mitigation

- Task: create a Bell state and measure in 0/1 basis



$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \otimes |000\rangle$$

- Results---the largest two components: $\tilde{P}_{00000} = 0.466, \tilde{P}_{11000} = 0.422$

  Smaller ones: $\tilde{P}_{00001} = 0.013, \tilde{P}_{01000} = 0.042, \tilde{P}_{10000} = 0.032, \tilde{P}_{11001} = 0.011$

- Invert to find P with constraint P ≥ 0: $\tilde{P}_{\vec{n}} = \sum_{\vec{m}} M_{\vec{n};\vec{m}} P_{\vec{m}}$

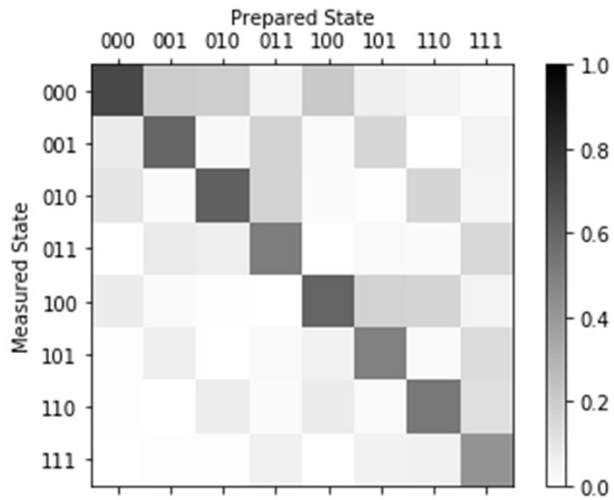
  - ✓ The \`corrected' P has two dominant components, P_{00000} = 0.493 and P_{11000} = 0.507, with other components very small

- Can invert ideal measurement distribution by inversion (under constraint P be non-negative)

IBM has similar implementation

qiskit-iqx-tutorials/qiskit/advanced/**ignis**/ 4_measurement_error_mitigation.ipynb

- Prepare and measure in computational basis



- Since one is concerned with only measurement distribution
- Obtain matrix M, then apply its inverse to mitigate errors

Other "tomographic" tools

- ❑ Disadvantage of current tomorgraphic tools is the number of basis in the measurement scales exponentially with no. of qubits

- ❑ Randomized benchmarking

  ➔ Quantify average error rates for gates

- ❑ Quantum volume

  ➔ Quantify overall quality of quantum circuits

➢ May return to these later (if time permits) and will do Qiskit demo