PHY682 Special Topics in Solid-State Physics: Quantum Information Science

Lecture time: 2:40-4:00PM Monday & Wednesday

Today 11/18:

1. Final lecture: finishing Week 13's topics (quantum simulations and metrology)

Heisenberg's uncertainty principle

$\bar{A} \equiv \langle\psi|\hat{A}|\psi\rangle$

$\langle\psi|(\hat{A}-\bar{A})^2|\psi\rangle$

$\equiv$

- For observables A & B that do not commute

$$\Delta A \Delta B \geq |\langle\psi|[A,B]|\psi\rangle|/2$$

$(\Delta A)^2 \equiv \langle\psi|\hat{A}^2|\psi\rangle - \langle\psi|\hat{A}|\psi\rangle^2$

$(\Delta B)^2 \equiv \langle\psi|\hat{B}^2|\psi\rangle - \langle\psi|\hat{B}|\psi\rangle^2$

e.g. position x and momentum p:

$$[x,p] = i\hbar \rightarrow \Delta x \Delta p \geq \hbar/2$$

- Heisenberg initially regarded this as a relationship between the precision of a measurement and the disturbance it creates

- Correct interpretation: (intrinsic uncertainty) if we prepare a large number of quantum systems in identical states, ψ, and then perform measurements of A on some of those, and of B in others, the statistical uncertainties satisfy above inequality

- Rozema et al. performed an experiment that refuted the original interpretation [Phys. Rev. Lett. 109, 100404 (2012)]

Heisenberg's uncertainty principle

- For observables A & B that do not commute

$$\Delta A \Delta B \geq |\langle\psi|[A,B]|\psi\rangle|/2$$

- In contrast, classical mechanics assumes measurement of position and momentum (or speed) can be made separately and arbitrarily precise (i.e. no fundamental limit) $\longrightarrow$ classical

- In quantum mechanics, the act of measurement changes the system [quantum back action], and thus repeated measurement cannot obtain initial properties (e.g. observing the location of a particle 'localizes' the particle).

- It is possible to formulate some uncertainty relation between measurement precision and disturbance → need to carefully define measurement noise & disturbance and/or introduce new inequality [e.g. Ozawa, Ann of Phys. 2003, Hofmann, PRA 2013]

Deriving the Uncertainty Principle

- Consider two Hermitian operators A and B: $A^\dagger = A,\ B^\dagger = B$

uncertainty relation does not hold if A & B are not hermitian

$[a, a^\dagger] = 1$

$\Rightarrow$ conclude $\cancel{\Delta a \Delta a^\dagger \geq \frac{1}{2}}$

$$AB = \frac{1}{2}(AB + BA) + \frac{1}{2}(AB - BA) = \frac{1}{2}\{A, B\} + \frac{1}{2}[A, B]$$

$$\langle\psi|AB|\psi\rangle = \frac{1}{2}\langle\psi|\{A, B\}|\psi\rangle + \frac{1}{2}\langle\psi|[A, B]|\psi\rangle =: x + y\, i$$

$= \langle\psi|\{A, B\}|\psi\rangle^*$

real

pure imaginary

$[A, B]^\dagger = -[A, B]$

$$\frac{1}{4}|\langle\psi|\{A, B\}|\psi\rangle|^2 + \frac{1}{4}|\langle\psi|[A, B]|\psi\rangle|^2 = |\langle\psi|AB|\psi\rangle|^2 \leq \langle\psi|A^2|\psi\rangle\langle\psi|B^2|\psi\rangle$$

$\left(\frac{a+a^\dagger}{2}, \frac{a-a^\dagger}{2i}\right)$

$x^2 + y^2$

Schwarz ineq.

$|\vec{a}\cdot\vec{b}|^2 \leq |\vec{a}|^2|\vec{b}|^2$

$\frac{1}{4}|\langle\psi|[A,B]|\psi\rangle|^2 \leq \langle\psi|A^2|\psi\rangle\langle\psi|B^2|\psi\rangle$

- Replace A by A - <A> and B by B-<B>:

$[A - \langle A\rangle, B - \langle B\rangle] = [A, B]$

$$\frac{1}{4}|\langle\psi|[A, B]|\psi\rangle|^2 \leq \langle\psi|(A - \langle A\rangle)^2|\psi\rangle\langle\psi|(B - \langle B\rangle)^2|\psi\rangle$$

$$\Delta A \Delta B \geq |\langle\psi|[A, B]|\psi\rangle|/2$$

Consequence: standard quantum limit

- Position-momentum uncertainty $\Delta x \Delta p \geq \hbar/2$

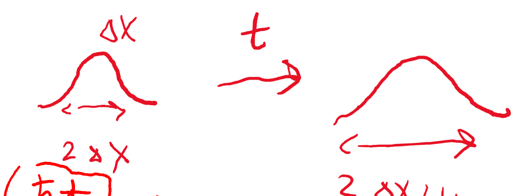
➔ As time evolves, the particle wavepacket expands

$$\Delta v = \Delta p/m \geq \hbar/(2m\Delta x)$$

$\hbar/2\Delta x$

$$\boxed{\Delta x(t) \sim \Delta x + \frac{\hbar t}{2m\Delta x}}$$

$\Delta v\, t$



$\rightarrow \Delta x \approx \frac{\hbar t}{2m\Delta x} \Rightarrow \Delta x|_{min} = \sqrt{\frac{\hbar t}{2m}}$

➔ If we minimize this with respect to Δx, we find the standard quantum limit:

$$(\Delta x)_{\rm SQL} = \frac{\hbar t}{2m(\Delta x)_{\rm SQL}} = \sqrt{\frac{\hbar t}{2m}}$$

total # of particles

- Phase-number uncertainty $[\hat{\phi}, \hat{N}] = i \;\rightarrow \Delta\phi \geq 1/(2\Delta N) \geq 1/N_{\rm tot}$

➔ Heisenberg limit: the uncertainty in measuring a phase is limited by the total number of photons used

Dealing with uncertainty

[See e.g. Giovannetti, Lloyd & Maccone, Science 06, 1330 (2004)]

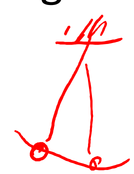
- ❑ Monitors only one out of a set of incompatible observables

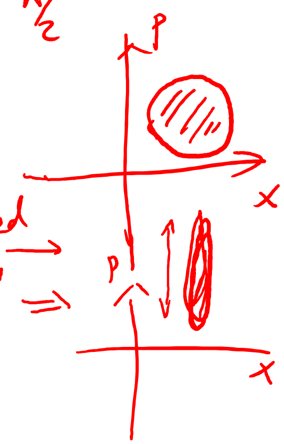
- ❑ Employ a quantum state in which the uncertainty in the to-be-monitored observable is small (at the cost of a very large uncertainty in complementary observables)

coherent

$\Delta x \Delta p \geq \hbar/2$

LIGO





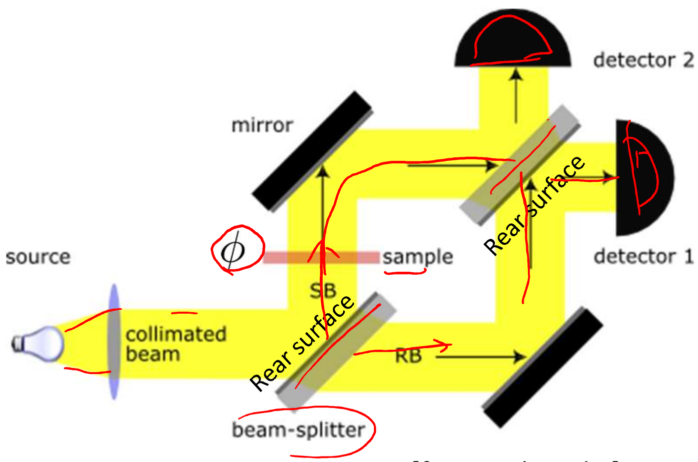
squeezed states

- ❑ Typical source of errors:
  - ➢ Environment-induced noise from vacuum fluctuations (the so-called shot noise)
  - ➢ Dynamically induced noise in e.g. the position measurement of a free mass (the so-called standard quantum limit)

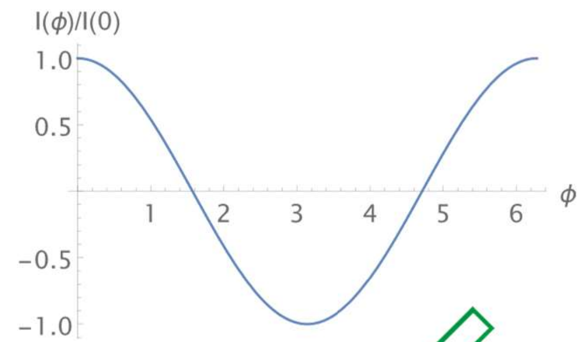
➔ Not fundamental limit though

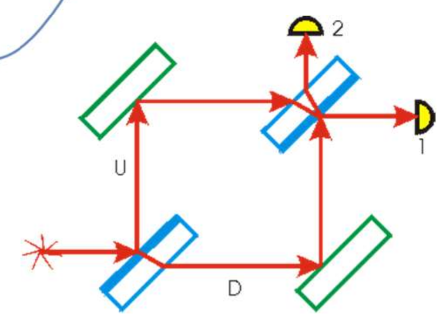
$\Delta A \, \Delta B$

Mach-Zehnder interferometer



[from Wikipedia]



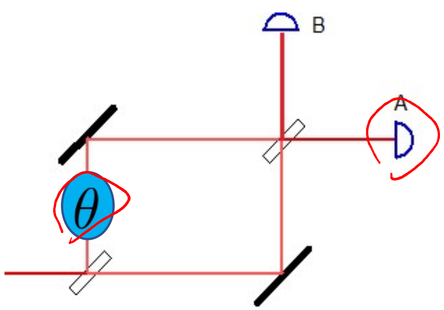


- Can understand using interference of classical waves:

Reflection from rear surface of beam-splitter and from mirror gives a π phase shift. Going through the beam splitter once accumulates a path phase n 2πd/λ (n is the refraction index)

- ✓ Waves from two paths arriving at detector 1 add constructively, and those arriving at detector 2 add destructively in absence of a sample
- ✓ Assume sample gives a phase shift ϕ, this modifies output intensities at 1 & 2

Recall: Homework 1 problem



Special rule: "pinball going through acquires amplitude $1/\sqrt{2}$; being deflected acquires $i/\sqrt{2}$

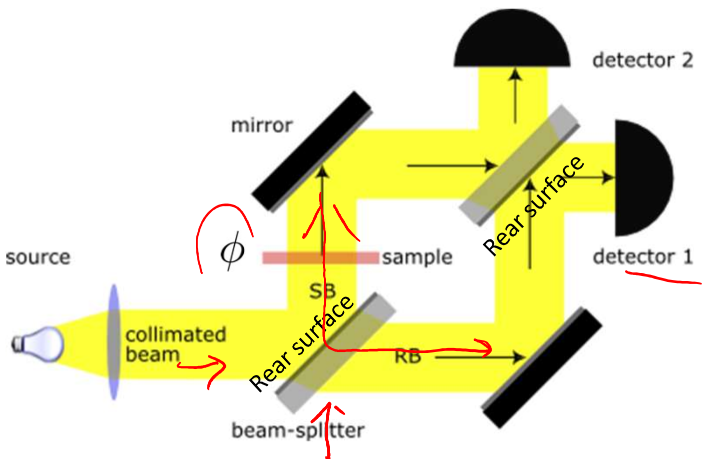
We add a phase shifter and assume that the path gains additional factor exp(iθ) relative to the other path. Repeat the calculation in class but to take account of this phase factor and calculate the probability is $P_A(\theta)$ of the quantum pinball appearing at port A.

$$P_A(\theta) = \left| \frac{1}{\sqrt{2}} + \right|^2 \qquad P_B(\theta) = \frac{1-\cos\theta}{2}$$

$$\left| \frac{i}{\sqrt{2}} e^{i\theta} \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \cdot 1 \cdot \frac{i}{\sqrt{2}} \right|^2 = \left| \frac{i}{2} \left(1 + e^{i\theta}\right) \right|^2 = \frac{1+\cos\theta}{2}$$

$$I_A(\theta) = I_0 \left(\frac{1+\cos\theta}{2} \right) = \cos^2\frac{\theta}{2} \qquad I_B(\theta) = I_0 \left(\frac{1-\cos\theta}{2} \right) = \sin^2\frac{\theta}{2}$$

Mach-Zehnder interferometer



[from Wikipedia]

In terms of photon counting, to establish intensities at 1 and 2:

$$I_1 = I_0 \cos^2(\phi/2),\ \ I_2 = I_0 \sin^2(\phi/2)$$

one counts the clicks from photo-detectors, if photons are independent then, the uncertainty in the normalized intensity is $= \cos^2\frac{\phi}{2} - \sin^2\frac{\phi}{2} = \cos\phi$

$$\Delta(I_1 - I_2)/I_0 = \sin\phi\Delta\phi \propto 1/\sqrt{N}$$

$\Delta(\cdot\cdot) = (-\sin\phi)\Delta\phi \propto \frac{1}{\sqrt{N}}$ $\quad \Delta\phi \propto \frac{1}{\sqrt{N}}$

➔ The uncertainty in the phase induced by the sample is limited by the number of photons used (i.e. the shot-noise limit) $\quad (\Delta\phi)_{\text{shot-noise}} \propto 1/\sqrt{N}$

- ❑ Can the shot-noise limit be overcome?

  Yes, by using e.g. squeezed vacuum in the original vacuum (no-input) port

  $$\Delta\phi \propto 1/N^{3/4}$$ [Caves, PRD 1981; Barnett, Fabre, Maitre, Eur. Phys. J. D (2003)]

- ❑ Can be further improved using entangled states to the Heisenberg limit $\quad \Delta\phi \propto 1/N$

NOON state for interferometry

[see e.g. Dowling, arXiv:0904.0163]

$$|\mathrm{N00N}\rangle \equiv (|N_A, 0_B\rangle + |0_A, N_B\rangle)/\sqrt{2} \xrightarrow{\text{Phase shift } \phi} (e^{iN\phi}|N_A, 0_B\rangle + |0_A, N_B\rangle)/\sqrt{2}$$

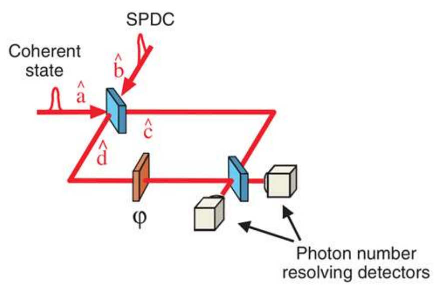
N00N

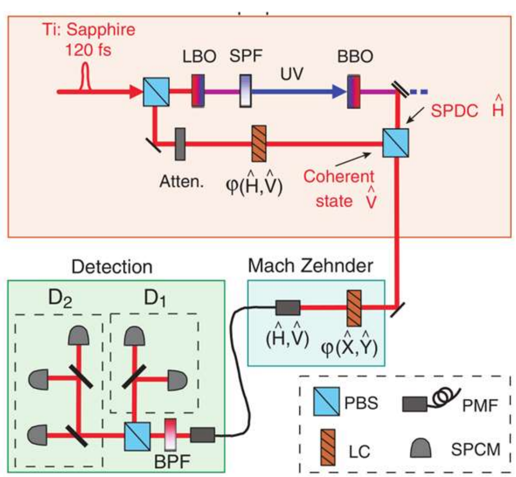
If one can interfere the two paths, e.g. after beam splitting only measure N photon detection ➔ N times faster oscillation

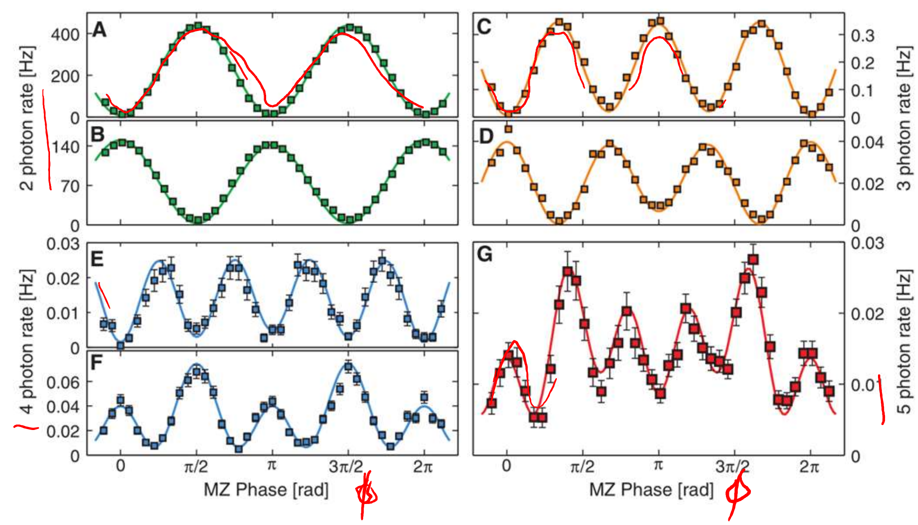
$$|1+e^{iN\phi}|^2 \quad \text{vs} \quad |1+e^{i\phi}|^2$$

➢ Can reach the Heisenberg limit $\Delta\phi \propto 1/N$

[Generation by Afek, Ambar& Silberberg, Science 2010]



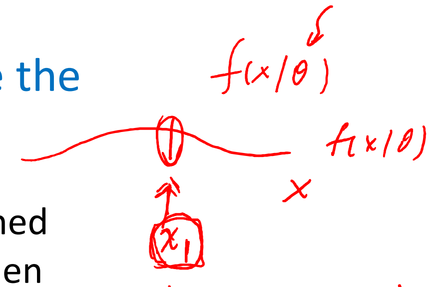




Next, we will introduce a mathematical formalism based on Fisher information and Cramer-Rao bound

Fisher information and Cramer-Rao bound

- Goal: to estimate an unknown parameter θ and to estimate the uncertainty

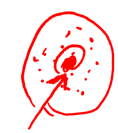


  - **Setting**: Given a θ, the measurement that gives some value x is assumed to follow a probability distribution $f(x|\theta)$. From a given reading x_1, then we estimate the value of θ to be $t(x_1)$

↳ assume estimate is $t(x_1)$

$$\langle t\rangle_\theta = \int dx\, f(x|\theta)\, t(x) = \theta$$

with sufficient statistics, we get the exact value of θ



✓ Fisher information: how the log likelihood function varies with the unknown parameter

$$I(\theta) \doteq \int dx\, f(x|\theta) \left(\frac{\partial}{\partial\theta}\log f(x|\theta)\right)^2 = \left\langle \left(\frac{\partial}{\partial\theta}\log f(x|\theta)\right)^2 \right\rangle_\theta$$

✓ Cramer-Rao bound for n independent samples or experiments:

$\Delta t \geq \frac{1}{\sqrt{n}} \frac{1}{\sqrt{I(\theta)}}$

$$\Delta^2 t \geq \frac{1}{nI(\theta)}$$

[You might have learned this in AMS 571 Mathematical statistics]

Proving Cramer-Rao bound

$$I(\theta) = \left\langle \left(\frac{\partial}{\partial\theta} \log f(x|\theta) \right)^2 \right\rangle_\theta$$

$$\Delta^2 \frac{\partial}{\partial\theta} L = \left\langle \left(\frac{\partial}{\partial\theta} L \right)^2 \right\rangle$$

- First, consider $n_{sample} = 1$ and introduce covariance between two functions $g(x)$ and $h(x)$:

$$\mathrm{Cov}[g, h] \equiv \langle (g - \langle g \rangle)(h - \langle h \rangle) \rangle$$

Specializing this to (i) $t(x)$ whose average is $\langle t \rangle = \theta$ and (ii) $\frac{\partial}{\partial\theta} L(x|\theta)$ [with $L(x|\theta) \equiv \log f(x|\theta)$] whose average is 0:

$$\left\langle \frac{\partial}{\partial\theta} L(x|\theta) \right\rangle = \int dx\, \cancel{f(x|\theta)} \frac{\partial}{\partial\theta} L(x|\theta) = \int dx \frac{\partial}{\partial\theta} f(x|\theta) = \frac{\partial}{\partial\theta}(1) = 0$$

$$\frac{\partial}{\partial\theta} L(x|\theta) = \frac{\partial f}{\partial\theta} \Big/ \cancel{f}$$

$$\mathrm{Cov}\left[t(x), \frac{\partial}{\partial\theta} L(x|\theta)\right] \equiv \left\langle (t(x) - \theta) \frac{\partial}{\partial\theta} L(x|\theta) \right\rangle$$

$$= \left\langle t(x) \frac{\partial}{\partial\theta} L(x|\theta) \right\rangle \quad = \int dx\; t(x) \frac{\partial}{\partial\theta} f(x|\theta) = 1$$

$$= \frac{\partial}{\partial\theta} \int dx\, t(x) f(x|\theta) = \frac{\partial}{\partial\theta} \theta = 1$$

- Next, use Cauchy-Schwarz inequality: $\Delta^2 g\, \Delta^2 h = \langle (g - \langle g \rangle)^2 \rangle \langle (h - \langle h \rangle)^2 \rangle \geq |\langle (g - \langle g \rangle)(h - \langle h \rangle) \rangle|^2$

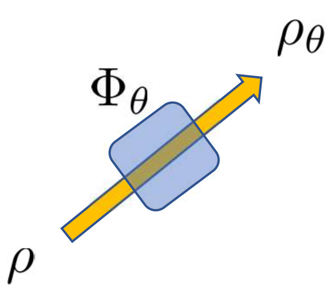
$\Delta^2 t = \langle (\Delta t)^2 \rangle$ $\quad \Delta^2 t\, \Delta^2 \left(\frac{\partial}{\partial\theta} L \right) \geq 1$

Note: $\boxed{\Delta^2 \frac{\partial}{\partial\theta} L = I(\theta)}$ $\quad$ Generalize to n independent samples: $\Delta^2 t \geq \frac{1}{n_{\text{sample}} I(\theta)}$

$\Delta^2 t\, I \geq 1$

Quantum Fisher information—Intuitive picture

- Using a quantum state ρ that is under some quantum operation Φ_θ : $\rho_\theta = \Phi_\theta(\rho)$
- Want to estimate parameter θ. How do we generalize $\frac{\partial}{\partial\theta}L(x|\theta)$? Where $L(x|\theta) \equiv \log f(x|\theta)$?



$f(x|\theta) = tr(\rho_\theta \Pi_x)$

Intuitively $f(x|\theta) \sim \rho_\theta$ (more precisely measured w.r.t. an operator Π_x), then

$\frac{\partial}{\partial\theta}\log f(x|\theta) \sim \frac{\partial}{\partial\theta}\rho_\theta / \rho_\theta \sim D_\theta$ If $\frac{\partial}{\partial\theta}\rho_\theta = D_\theta\, \rho_\theta$

hand waving

Thus $$I(\theta) = \left\langle \left(\frac{\partial}{\partial\theta}\log f(x|\theta)\right)^2 \right\rangle_\theta \Longrightarrow \mathrm{Tr}(\rho_\theta D_\theta^2)$$

- It turns out D_θ is defined more precisely via

rigorous

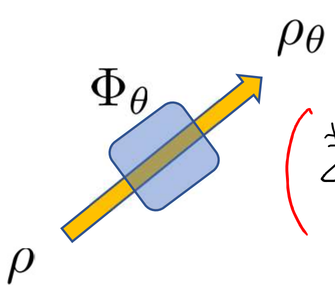
$\{D_\theta, \rho_\theta\}/2$

$$\frac{\partial}{\partial\theta}\rho_\theta = \{\rho_\theta, D_\theta\}/2 =: A \qquad \{A, B\} \equiv AB + BA$$

- Quantum Fisher information is thus $I_Q(\rho,\theta) = \boxed{\mathrm{Tr}(\rho_\theta D_\theta^2)}$
- Quantum Cramer-Rao bound $\Delta^2 t \geq \dfrac{1}{n_{\mathrm{sample}} I_Q(\rho,\theta)}$

Quantum Fisher information (cont'd)

- Using a quantum state ρ that is under some quantum operation Φ_θ : $\rho_\theta = \Phi_\theta(\rho)$



D_θ: An important operator related to how ρ_θ changes w.r.t. θ

regard this as definition of D_θ

$$\langle i|\frac{\partial}{\partial\theta}\rho_\theta|j\rangle = \{\rho_\theta, D_\theta\}/2 =: \langle i|A|j\rangle \qquad \{A,B\} \equiv AB + BA$$

$\rho_\theta|j\rangle = \lambda_j|j\rangle$

$$\left(\sum_{i,j}|i\rangle\langle i|\ \frac{1}{2}(\rho_\theta D_\theta + D_\theta\rho_\theta)|j\rangle\langle j|\right) = \sum_{ij}\frac{1}{2}|i\rangle\langle j|(\lambda_i+\lambda_j)\langle i|D_\theta|j\rangle$$

if $\lambda_i+\lambda_j=0 \Rightarrow$ leave out this

$$A_{ij} = \langle i|\partial\rho_\theta/\partial\theta|j\rangle$$

$\Downarrow$ find $\langle i|D_\theta|j\rangle$

$$D_\theta = 2\sum_{\substack{i,j\\ \lambda_i+\lambda_j\neq 0}}\frac{A_{ij}}{\lambda_i+\lambda_j}|i\rangle\langle j|, \text{ where } \rho_\theta = \sum_i \lambda_i|i\rangle\langle i|$$

$\Rightarrow$ can evaluate $I(\theta) = \mathrm{Tr}(\rho_\theta D_\theta^2)$,

- The corresponding quantum Cramer-Rao bound implies that [proof omitted, but uses $f(x|\theta)$=Tr $(\rho_\theta\,\Pi_x)$]

$$\Delta^2 t \geq \frac{1}{n_{\mathrm{sample}} I_Q(\rho,\theta)} \qquad I_Q(\rho,\theta) = \mathrm{Tr}(\rho_\theta D_\theta^2)$$

Quantum Fisher information: unitary channel

$$\rho_\theta = U(\theta)\rho U^\dagger(\theta), \quad U(\theta) \equiv e^{-i\theta \hat{G}}$$

hermitian

e.g. pure state $|\psi(\theta)\rangle = U(\theta)|\psi\rangle$

$$I_Q(\rho,\theta) = \mathrm{Tr}(\rho_\theta D_\theta^2) \qquad D_\theta = 2\sum_{i,j} \frac{\langle i|\partial\rho_\theta/\partial\theta|j\rangle}{\lambda_i + \lambda_j}|i\rangle\langle j|$$

- Quantum Fisher information in this case is independent of θ:

$$I_Q(\rho,\theta,\hat{G}) = 2\sum_{i,j}\frac{(\lambda_i-\lambda_j)^2}{\lambda_i+\lambda_j}|\langle i_0|G|j_0\rangle|^2, \quad \rho = \sum_i \lambda_i |i_0\rangle\langle i_0|$$

- ✓ For pure state [exercise]:

$$I_Q(|\psi\rangle,\theta,\hat{G}) = 4(\langle\psi|\hat{G}^2|\psi\rangle - \langle\psi|\hat{G}|\psi\rangle^2) = 4(\Delta G)^2$$

- The corresponding quantum Cramer-Rao bound implies that

$$\Delta^2 t \geq \frac{1}{n_{\mathrm{sample}} I_Q(\rho,\theta)}$$

Some derivation

$$\rho_\theta = U(\theta)\rho U^\dagger(\theta), \quad U(\theta) \equiv e^{-i\theta\hat{G}}$$

$$I_Q(\rho,\theta) = \mathrm{Tr}(\rho_\theta D_\theta^2) \quad D_\theta = 2\sum_{i,j} \frac{\langle i|\partial\rho_\theta/\partial\theta|j\rangle}{\lambda_i + \lambda_j}|i\rangle\langle j|$$

- Quantum Fisher information in this case is independent of θ:

$$\partial\rho_\theta/\partial\theta = -iU_\theta[\hat{G},\rho]U^\dagger(\theta) \quad \langle i|\partial\rho_\theta/\partial\theta|j\rangle = -i\langle i|U_\theta[\hat{G},\rho]U^\dagger(\theta)|j\rangle$$

$$D_\theta = -i2U_\theta\left(\sum_{i,j}\langle i_0|[\hat{G},\rho]|j_0\rangle\frac{1}{\lambda_i+\lambda_j}|i_0\rangle\langle j_0|\right)U^\dagger(\theta) = -i2U_\theta B U^\dagger(\theta)$$

$$B \equiv \sum_{i,j}\langle i_0|[\hat{G},\rho]|j_0\rangle\frac{1}{\lambda_i+\lambda_j}|i_0\rangle\langle j_0| = \sum_{i,j}\langle i_0|\hat{G}|j_0\rangle\frac{\lambda_j-\lambda_i}{\lambda_i+\lambda_j}|i_0\rangle\langle j_0|$$

$$I_Q(\rho,\theta) = \mathrm{Tr}(\rho_\theta D_\theta^2) = -4\mathrm{Tr}(\rho B^2) \qquad \rho = \sum_i \lambda_i|i_0\rangle\langle i_0|$$

$$B^2 = \sum_{i,j,k}\langle i_0|\hat{G}|j_0\rangle\frac{\lambda_j-\lambda_i}{\lambda_i+\lambda_j}\langle j_0|\hat{G}|k_0\rangle\frac{\lambda_k-\lambda_j}{\lambda_j+\lambda_k}|i_0\rangle\langle k_0|$$

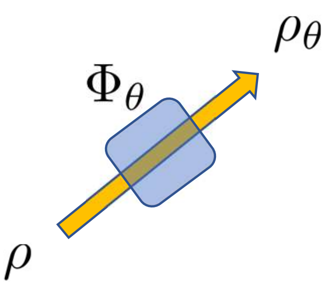
$$I_Q = -4\mathrm{Tr}(\rho B^2) = -4\sum_{i,j}\langle i_0|\hat{G}|j_0\rangle\frac{\lambda_j-\lambda_i}{\lambda_i+\lambda_j}\langle j_0|\hat{G}|i_0\rangle\frac{\lambda_i-\lambda_j}{\lambda_j+\lambda_i}\lambda_i$$

$$= 2\sum_{i,j}\frac{(\lambda_i-\lambda_j)^2}{\lambda_i+\lambda_j}|\langle i_0|G|j_0\rangle|^2 = 2\sum_{i,j}\frac{(\lambda_i-\lambda_j)^2}{\lambda_i+\lambda_j}|\langle i|G|j\rangle|^2$$

Examples---Single Mode

$a^\dagger(a^\dagger a+1)a = a^\dagger a^\dagger a a + a^\dagger a$

$\langle\alpha|(a^\dagger a)(a^\dagger a)|\alpha\rangle = \alpha^{*2}\alpha^2 + |\alpha|^2$
$= |\alpha|^4 + |\alpha|^2$



$$\rho_\theta = U(\theta)\rho U^\dagger(\theta), \quad U(\theta) \equiv e^{-i\theta\hat{G}} \qquad \hat{G} = \hat{a}^\dagger\hat{a}/2$$

$$I_Q(|\psi\rangle, \theta, \hat{G}) = 4(\langle\psi|\hat{G}^2|\psi\rangle - \langle\psi|\hat{G}|\psi\rangle^2) = 4(\Delta G)^2 \qquad \Delta t \geq \frac{1}{\sqrt{n_{\text{sample}}}\sqrt{I_Q(\rho,\theta)}}$$

- Coherence state: $\rho = |\alpha\rangle\langle\alpha| \qquad \hat{a}|\alpha\rangle = \alpha|\alpha\rangle$

$\Delta G^2 = \langle\alpha|(\frac{a^\dagger a}{2})^2|\alpha\rangle - \langle\alpha|\frac{a^\dagger a}{2}|\alpha\rangle^2$
$= \frac{1}{4}(|\alpha|^4 + |\alpha|^2 - (\alpha^*\alpha)^2) = \frac{1}{4}|\alpha|^2$

$I_Q = |\alpha|^2$ ~ average number of photons ➔ shot-noise limit

$\Delta t \geq \frac{1}{n_{samp}\sqrt{|\alpha|^2}}$

- Fock state: $\rho = |n\rangle\langle n| \qquad \hat{a}|n\rangle = \sqrt{n}|n\rangle$

$I_Q = 0$ → not useful to use $|n\rangle$ in estimating θ

- Superposition: $\rho = |\psi\rangle\langle\psi| \qquad |\psi\rangle = (|0\rangle + |n\rangle)/\sqrt{2}$

$I_Q = n^2$ ➔ Heisenberg limit

$\Delta t \geq \frac{1}{n_{sample}\, n}$

When input is pure and there are multiple parameters

$$|\psi_\theta\rangle = U(\theta)|\psi\rangle \qquad |\partial_\theta \psi_\theta\rangle \equiv \frac{\partial}{\partial\theta}|\psi_\theta\rangle$$

- Quantum Fisher information for a pure state (expression much simpler):

$$I_Q(|\psi\rangle, \theta) = 4(\langle\partial_\theta\psi_\theta|\partial_\theta\psi_\theta\rangle - |\langle\psi_\theta|\partial_\theta\psi_\theta\rangle|^2)$$

- Can generalize to multiple parameters (Quantum Fisher matrix):

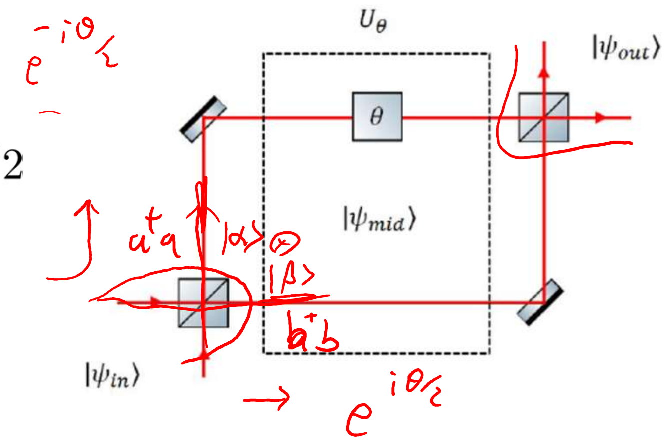
$$(I_Q)_{i,j} = 4\mathrm{Re}(\langle\partial_{\theta_i}\psi_\theta|\partial_{\theta_j}\psi_\theta\rangle - \langle\psi_\theta|\partial_{\theta_i}\psi_\theta\rangle\langle\partial_{\theta_j}\psi_\theta|\psi_\theta\rangle)$$

→ Is it related to Berry curvature?

Examples---Interferometry

$$\rho_\theta = U(\theta)\rho U^\dagger(\theta), \quad U(\theta) \equiv e^{-i\theta\hat{G}} \qquad \hat{G} = (\hat{a}^\dagger\hat{a} - \hat{b}^\dagger\hat{b})/2$$

$$I_Q(|\psi\rangle, \theta, \hat{G}) = 4(\langle\psi|\hat{G}^2|\psi\rangle - \langle\psi|\hat{G}|\psi\rangle^2) = 4(\Delta G)^2$$



[e.g. see Tan & Jeong, arXiv: 1909.00942]

- There are two modes, and $\hat{G} = (\hat{a}^\dagger\hat{a} - \hat{b}^\dagger\hat{b})/2$ introduces a phase difference

- Coherence state: $|\alpha\rangle \otimes |\beta\rangle \qquad \hat{a}|\alpha\rangle = \alpha|\alpha\rangle \quad \hat{b}|\beta\rangle = \beta|\beta\rangle$

$$\Delta t \geq \frac{1}{\sqrt{n_{\text{sample}}}\sqrt{I_Q(\rho,\theta)}}$$

$$I_Q = |\alpha|^2 + |\beta|^2$$ = total # of photons ⇒ shot-noise limit

- N00N state: $|\text{N00N}\rangle \equiv (|N_A, 0_B\rangle + |0_A, N_B\rangle)/\sqrt{2}$

$I_Q = N^2$ ➔ Heisenberg limit

Congratulations!

You have completed 13 weeks of [25] lectures in QIS

→ 2000 minutes

In Spring 2021: PHY680 Quantum computing course is offered by Prof. Vladimir Korepin [contains more advanced topics]

In the remaining 3 classes, it's all your show!

[student presentation]

QIS syllabus

http://insti.physics.sunysb.edu/~twei/Courses/Fall2020/PHY682/

- ✓ (week 1) **The history of Q:** Overview and review of linear algebra, basics of quantum mechanics, quantum bits and mixed states.
- ✓ (week 2) **From foundation to science-fiction teleportation**: Bell inequality, teleportation of states and gates, entanglement swapping, remote state preparation, superdense coding, and superdense teleportation.
- ✓ (week 3) **Information is physical**---Physical systems for quantum information processing: Superconducting qubits, solid-state spin qubits, photons, trapped ions, and topological qubits
- ✓ (week 4) **Grinding gates in quantum computers**: Quantum gates and circuit model of quantum computation, introduction to IBM's Qiskit, Grover's quantum search algorithm, amplitude amplification.
- ✓ (week 5) **Programming through quantum clouds**: Computational complexity, Quantum programming on IBM's superconducting quantum computers, including VQE on quantum chemistry of molecules, QAOA for optimization, hybrid classical-quantum neural network.
- ✓ (week 6) **Dealing with errors**: Error models, Quantum error correction, topological stabilizer codes and topological phases (including fractons), error mitigations
- ✓ (week 7) **Quantum computing by braiding**: Kitaev's chain, Majorana fermions, anyons and topological quantum computation
- ✓ (week 8) **More topological please**: Topological quantum computation continued, surface code and magic state distillation
- ✓ (week 9) **Quantum computing by evolution and by measurement**: Other frameworks of quantum computation: adiabatic and measurement-based; D-Wave's quantum annealers
- ✓ (week 10) **Quantum entangles**: Entanglement of quantum states, entanglement of formation and distillation, entanglement entropy, Schmidt decomposition, majorization, quantum Shannon theory
- ✓ (week 11) **No clones in quantum**: No cloning of quantum states, non-orthogonal state discrimination, quantum tomographic tools, quantum cryptography: quantum key distribution from transmitting qubits and from shared entanglement
- ✓ (week 12) **Show me your 'phase', Mr. Unitary**: Quantum Fourier Transform, quantum phase estimation, Shor's factoring algorithm, and quantum linear system (such as the HHL algorithm) and programming with IBM Qiskit
- ✓ (week 13) **The quantum 'Matrix':** Quantum simulations and quantum sensing and metrology

Presentation topics & Schedule

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Group 1: “Entanglement-Based Machine Learning on a Quantum Computer”, PhysRevLett.114.110504 (2019)

Group 2: “Universal Blind Quantum Computation” (3 related references)

Group 7: “Quantum Internet”
Ref: The quantum internet by H. J. Kimble, Nature 453, 1023-1030 (2010)

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Group 4: “Unpaired Majorana fermions in quantum wires”
Ref: A Yu Kitaev “Unpaired Majorana fermions in quantum wires”, 2001 Phys.-Usp. 44 131

Group 5: Google’s paper on Quantum Supremacy?

Group 6: “Hybrid Quantum algorithm to classify Hermitian matrix definiteness”
Ref.: Gómez, Andrés, and Javier Mas. "Hybrid Quantum algorithm to classify Hermitian matrix definiteness." arXiv preprint arXiv:2009.04117 (2020).

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Group 3: “Can the ‘WaveFunctionCollapse’ algorithm run on an actual quantum computer?”
Ref: paper by Karth and Smith, In Proceedings of FDG’17