Einstein's Path to GR

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4 Stages:

Dec. 1907 – June 1911
Gravity does not fit naturally into SR
Heuristic consider's, no theory;
Gravity ~ acceleration. $m_{in} = m_{grav}$

Febr. – March 1912
Scalar theory of gravity, $c \to c(\vec{x})$ static
~ gravit. potential, law of free fall.

Aug. 1912 — Autum 1915
$g_{\alpha\beta}(x^\gamma)$ basic field – Outline theory –
hole argument – gravit. field equ:
degree of covariance?, action princ?
conservation laws?

Nov. 1915: Towards the field equ.
1916: Hilbert-Einstein action princ.
1917: Λ 1918: Conservation of total energy-momentum

Stage 1.

Jahrbuch paper 1907. (Before Minkowski)

External homog. grav. fields indistinguishable from acceleration of frame [Newt. Corr. 6 in mech.], holds generally.

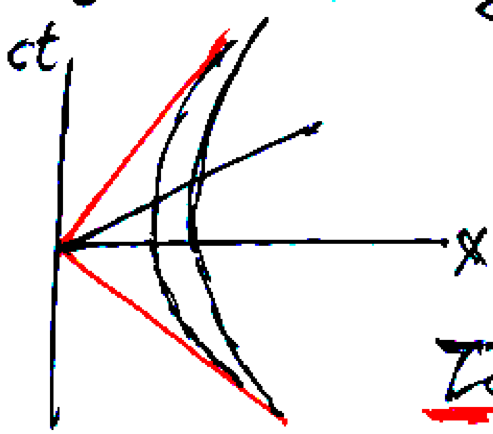
Extension of principle of relativity to some accelerated frames?

Transl. accel. frame Σ:

$$(ds^2 =)\ d\xi^2 + d\eta^2 + d\zeta^2 - \left(1 + \frac{g\xi}{c^2}\right)^2 c^2 d\tau^2$$

$\sigma = \left(1 + \frac{g\xi}{c^2}\right)\tau$ "local time" = proper time at ξ,

$\tau = \sigma$ at origin only, τ def. simultaneity.



Boosted inertial frames coincide instantaneously with Σ. Accel. g.

Time dilation, Sun, $\frac{\Delta\lambda}{\lambda} = 2\cdot 10^{-6}$,

Light rays curved, $n \approx 1 - \frac{g\xi}{c^2}$.

Tentative generalization: $g\xi \to \phi$ = grav. potential

Heuristic value, no theory (yet).

June 1911. Universality of free fall should play fundamental role; Eötvös.

Freely falling radiation then requires

$\frac{E}{c^2}$ = inertial and (passive) gravit. mass.

Energy argument for falling "photons";

$\frac{\Delta\lambda}{\lambda}$, gravit. time dilation } in ext. stationary fields

Curvature of light paths }

Bending angle for solar deflection of light $\alpha = \frac{2GM}{c^2 R}$, observable?

Equivalence principle, box argument.

c variable

Stage 2, 1912.

Scalar theory of static gravity

$c(\vec{x}) \sim$ gravit potential, $\Delta c = G c \rho$,

Space Euclidean, non-rotating coord.-system.

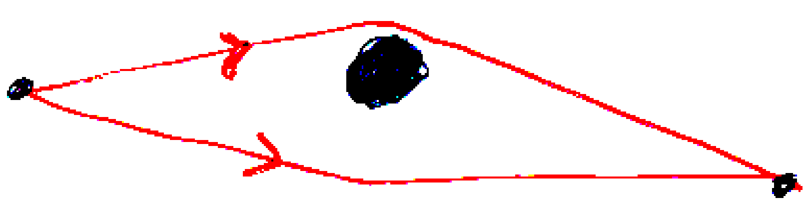
Eq. of motion of a material point :

$$\delta \int \{ c(\vec{x})^2 dt^2 - d\vec{x}^2 \}^{1/2} = 0$$

(c = const. : Planck 1906)

Lorentz trfs. not valid. (Abraham controv.)

Note book : Gravit. lensing



Not observable. 1936, Zwicky,

... 1979 observed, ...

Stage 3

August 1912: $g_{\alpha\beta}(x^\sigma)$ basic structure of spacetime. SR preserved as local approximation.

Einstein-Grossmann, Outline Theory (1913)

(Outline of a generalized theory of relativity and a theory of gravitation.)

$m_{in} = m_{grav}$, Eötvös; radioactive decay $\rightarrow \Delta m_{in}$; if $\Delta m_{in} \neq \Delta m_{grav.} \Rightarrow$ observable.

Free fall: $\delta \int |g_{\alpha\beta}\, dx^\alpha dx^\beta|^{1/2} = 0$,

$g_{\alpha\beta}$ Lorentzian.

ds invariant. x^α in general not directly related to measurable lengths or durations.

Stress energy of "incoherent" matter:

$T^{\alpha\beta} = \rho_0 U^\alpha U^\beta \quad \left(U^\alpha = \frac{dx^\alpha}{ds}\right)$,

$$\Rightarrow \mathfrak{T}^\beta_{\alpha,\beta} = \tfrac{1}{2}\mathfrak{T}^{\beta\gamma} g_{\beta\gamma,\alpha} \quad (\Leftrightarrow T^{\alpha\beta}{}_{;\beta} = 0)$$

Energy balance gravitational field $\leftrightarrow$ any kind of matter.

Always kept as basic law.

Field eq. (general. of $\Delta\phi = 4\pi G\rho$)?

Ansatz: $V^{\alpha\beta}(g_{..}, \partial g_{..}, \partial^2 g_{..}) = \kappa T^{\alpha\beta}$

↳ tensor.

- A law of this kind which reduces reasonably to Newton-Poisson, does not exist;

⇒ abstain from general invariance.

- $\Delta\phi = \nabla^\alpha \nabla_\alpha \phi$, but this cannot be "imitated" since $\nabla_\alpha g_{\beta\gamma} = 0$.

⇒ Smaller, unknown covariance group which should contain the linear trfs.

Die gesuchte (mindestens) lineare Feldgl. soll a) mit der Energiebilanz verträglich sein, b) eine Erhaltungsgleichung

$$\partial_\beta (T_\alpha^\beta + t_\alpha^\beta) = 0$$ implizieren

Probieren ergibt (eine) Lösung:

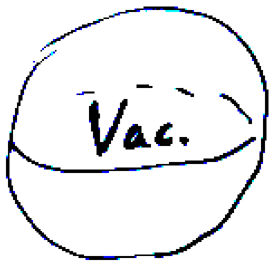
$$\partial_\lambda (\sqrt{g}\, g^{\lambda\mu} g_{\alpha\nu} \partial_\mu g^{\beta\nu}) = \kappa (T_\alpha^\beta + t_\alpha^\beta)$$

(nichtlin. Wellengl.)

Maxwellgln. wie üblich verallgemeinern.

Einstein - Besso : Perihelion motion, ⊖.

Hole argument (Jan. 1914) against generally cov. grav. field eq.



$T^{\mu\nu}$

(Mach):

$T^{\mu\nu} \longmapsto g_{\mu\nu}$,

but incompatible with general covariance.

(Coordinates identify events)

Nov. 1914. Additions to "outline th.":

Perfect fluids, electrodyn. in matter.

Field eq. changed: action principle,

$$\delta\left\{\int \sqrt{-g}\, H(g^{\cdot\cdot}, \partial g^{\cdot\cdot}) + \text{matter term}\right\} = 0$$

$$\Rightarrow \qquad \mathcal{E}_{\alpha\beta} = \kappa\, T_{\alpha\beta}$$

↑ Euler-Lagr. deriv. of $\mathcal{H}$.

⇒ Total cons. law.

Choice of H permits compatibility with matter/field balance equation, provided a coordinate condition is added.

[H not generally covariant up to a divergence.]

Wrong theory, but Newtonian approximation achieved.

Stage 4. November 1915

Nov. 4. Field equ. invariant w.r.t. unimodular trfs., $\det\left(\frac{\partial x^{\alpha'}}{\partial x^{\beta}}\right) = 1$.

Action principle, $L = g^{\sigma\tau}\Gamma^{\alpha}_{\sigma\beta}\Gamma^{\beta}_{\tau\alpha}$.

$T^{\alpha\beta}{}_{;\beta} = 0$ not implied by field eq.,
but combined with f. eq. a total energy-momentum law $\partial_\beta(T^\beta_\alpha + t^\beta_\alpha) = 0$ follows.

Newtonian approx. ✓ Accel. Rotat. frames ✓

But $\sqrt{-g}$ scalar, $\partial_\alpha\sqrt{-g}$ covector, $\neq 0$,
$\Rightarrow$ anisotropy, no local inertial coordinate systems exist.

Nov 11. General invariance achieved,
$R^{\alpha\beta} = \kappa T^{\alpha\beta}$, but
matter restricted by $T \equiv 0$.
($\sqrt{-g} = 1$ admissible)

Nov. 18. Perihelion advance,
double light deflection,
space curved
1PNA, spherically symmetric static
field (→ Schwarzschild 1916)

Nov. 25. The final field eq.

$$R_{\alpha\beta} = \varkappa \left(T_{\alpha\beta} - \tfrac{1}{2} g_{\alpha\beta} T \right)$$

All requirements satisfied:

$\Rightarrow T^{\alpha\beta}{}_{;\beta} = 0$, $\partial_\beta \left(T_\alpha^\beta + t_\alpha^\beta \right) = 0$,

Newtonian approximation.

Resolution(s) of hole argument.

Action principle: Hilbert (20.11.15, but...)
31.3.16)
Einstein (20.3.16)

Meaning of "conservation law": Einst 1918

Λ (Oversight in 1916 review and in 1922)