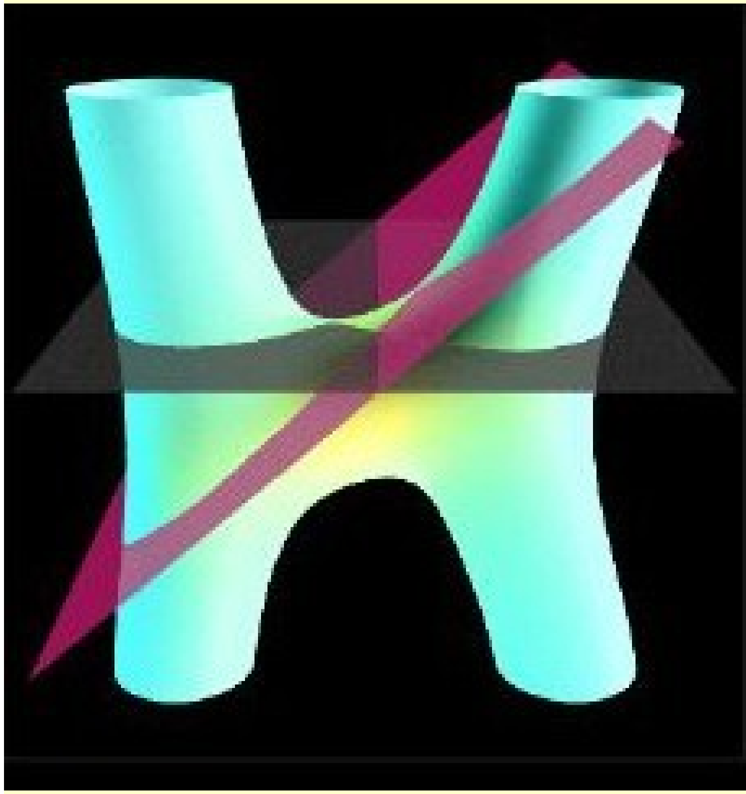
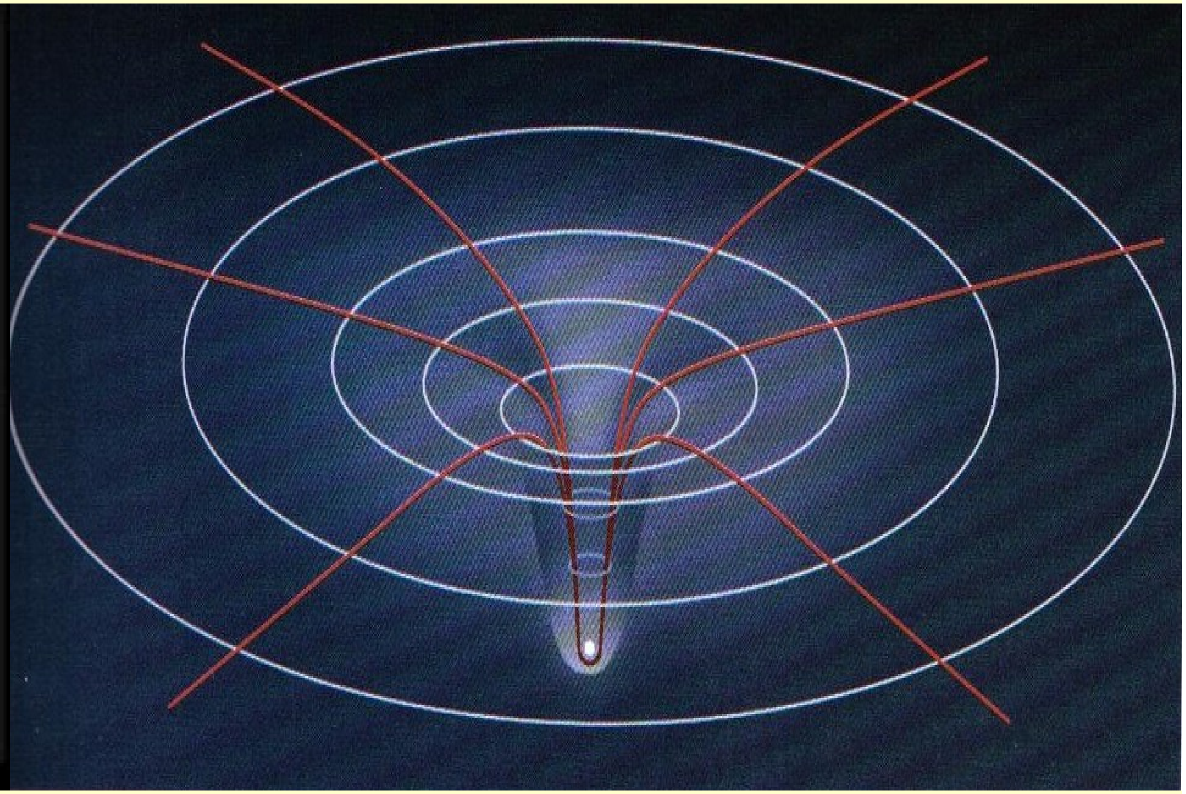
Strings and Black Holes

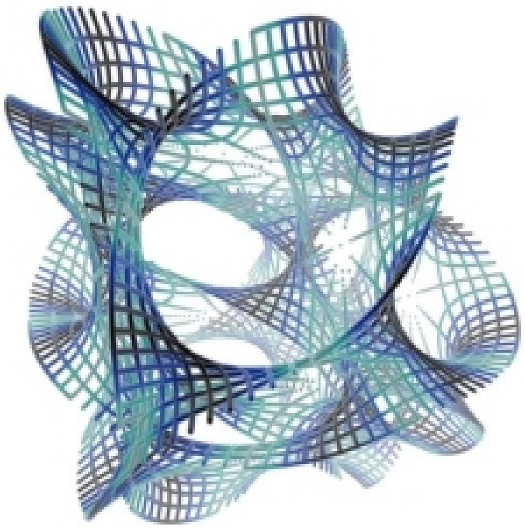
Erik Verlinde

Institute for Theoretical Physics
University of Amsterdam

General Relativity

$$R_{\mu\nu} - \tfrac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Gravity = geometry



Einstein: geometry => physics

Strings: physics => geometry

A Brief History of Black Holes

- **1916** Schwarzschild solution.

  Einstein: ``I had not expected that the exact solution to the problem could be formulated. Your analytical treatment to the problem appears to me splendid.''

- **1939** Gravitational collapse: "frozen" or "black" stars. — Oppenheimer-Volkov
- **1959~** Global structure, exact solutions. — Kruskal, Kerr, Newman
- **1967** "Black hole" singularity theorems. — Penrose-Hawking
- **1975** Black hole thermodynamics. — Bekenstein-Hawking
- **1994** Holographic principle. — 't Hooft- Susskind

. **1996** Microscopic origin of entropy. — Strominger-Vafa

A Brief History of String Theory

- **1968** **Veneziano amplitude, dual models** — Nambu-Goto
- **1971** **Bosonic and fermionic string theory** — Neveu-Schwarz-Ramond
- **1976** **Superstrings as theory of quantum gravity** — Scherk-Schwarz
- **1984** **Anomaly cancellation, heterotic strings** — Green-Schwarz
- **1995** **String dualities, D-branes** — Witten, Polchinski
- **1996** **Microscopic origin of black hole entropy** — Strominger-Vafa
- **1997** **AdS-CFT correspondence and holography.** — Maldacena

Outline

Black Holes

- **Hawking Radiation**
- **Information Paradox**
- **The Holographic Principle**

Black Holes in String Theory

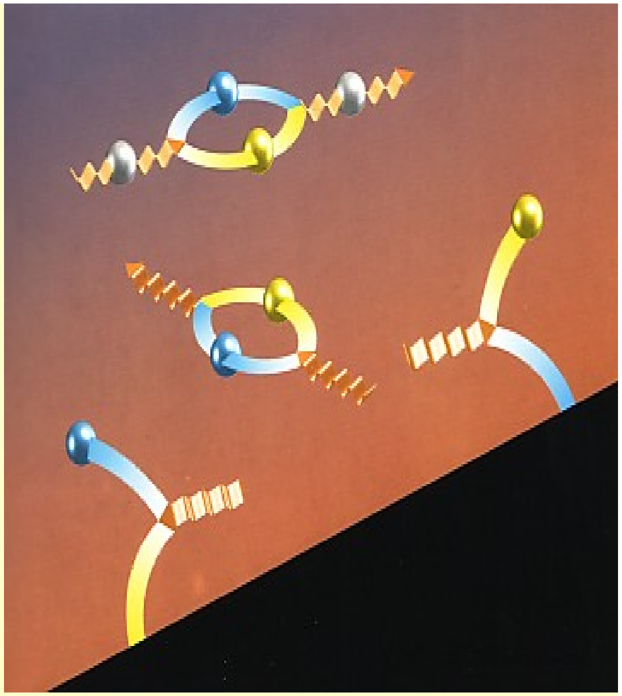
- **Strings and D-branes**
- **Microscopic Origin of Black Hole Entropy**
- **Holography in String Theory**

Hawking Radiation

Black Holes emit thermal radiation with temperature

Hawking 1975

$$T = \frac{\hbar c^3}{8\pi k G M}$$



Bekenstein-Hawking Entropy

$$S = \frac{cA}{4\hbar G}, \qquad A = 4\pi R^2$$

What is the origin of this entropy?

Black Hole Thermodynamics

Charged Black Holes: *Reisner-Nordstrom*

$$ds^2 = -H(r)dt^2 + \frac{dr^2}{H(r)} + r^2 d\Omega^2$$

$$H(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} \quad \rightarrow \left(1 - \frac{Q}{r}\right)^2$$

Inner and outer horizon

$$r_\pm = M \pm \sqrt{M^2 - Q^2}$$

Extremal Black Holes

$$M = Q$$

Electric Potential, Entropy and Temperature

$$\Phi = \frac{Q^2}{r_+^2} \qquad S = \pi r_+^2 \qquad T = \frac{1}{4\pi r_+^2}(r_+ - r_-)$$

obey the first law of thermodynamics

$$T \rightarrow 0$$

$$TdS = dM + \Phi dQ \quad + \Omega dJ$$

Einstein equation as equation of state

Ted Jacobson

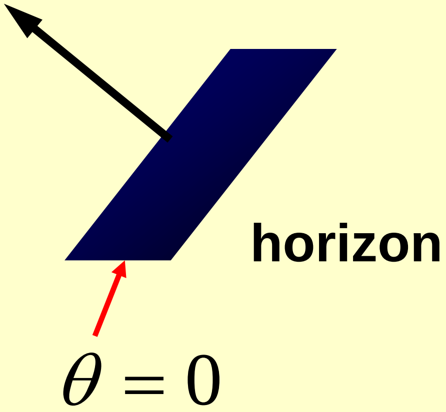
Raychaudhuri equation **(no shear or vorticity)**

$$\frac{d\theta}{d\lambda} = -\frac{1}{2}\theta^2 - R_{\mu\nu}k^\mu k^\nu$$

$$\theta = k^\mu{}_{;\mu}$$

$$k^\mu k_\mu = 0$$

can be integrated using the Einstein equation

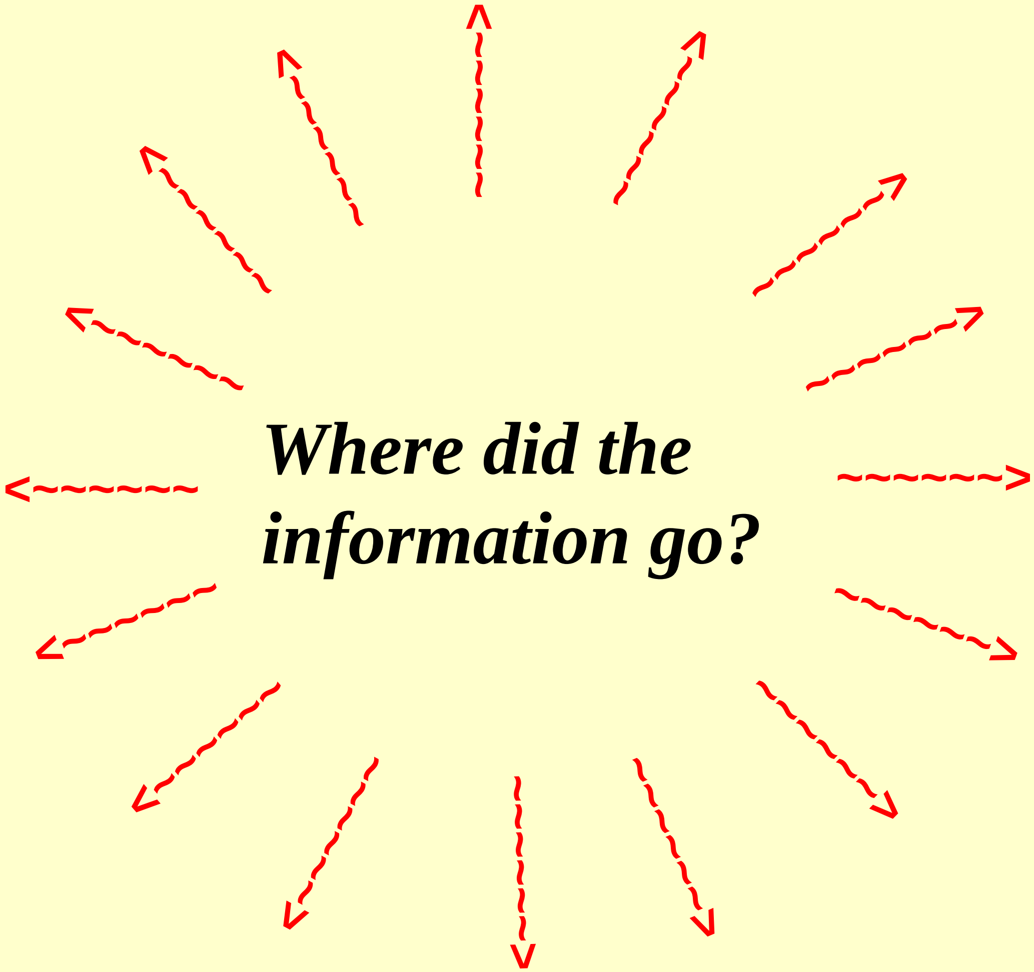


$$\frac{1}{8\pi G}\theta = \lambda T_{\mu\nu}k^\mu k^\nu + O(\lambda^2)$$

= local version of 1st law of thermodynamics

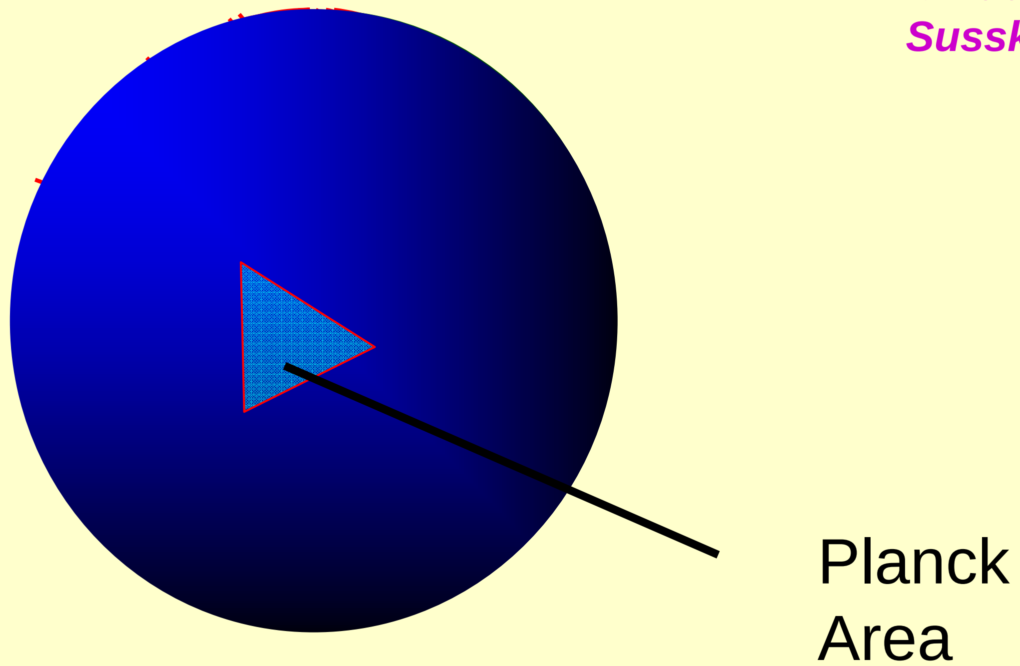
$$Tds = \rho$$

The Information Paradox (early '90's)



The Holographic Principle (1994)

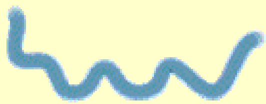
't Hooft
Susskind



$$\#\,\text{States} = \exp \frac{A}{4G}$$

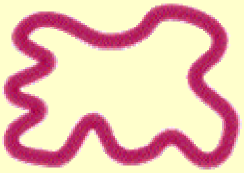
Strings, D-branes and Gravity

Open strings: gauge interactions

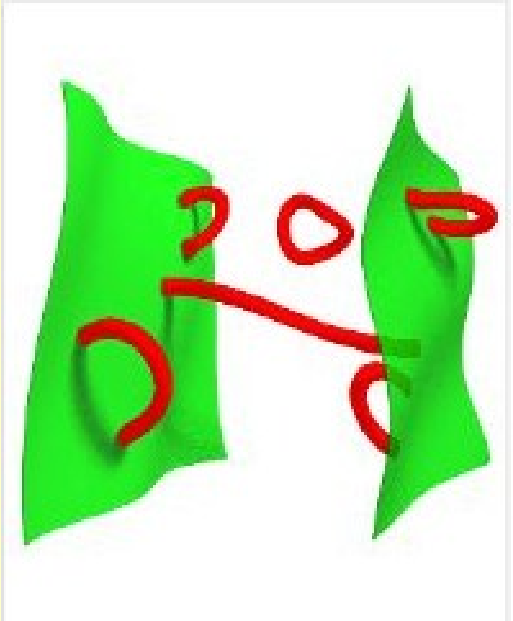


$A_\mu \, , X^I$

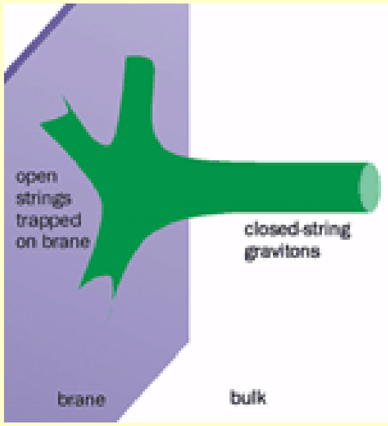
Closed strings: gravity



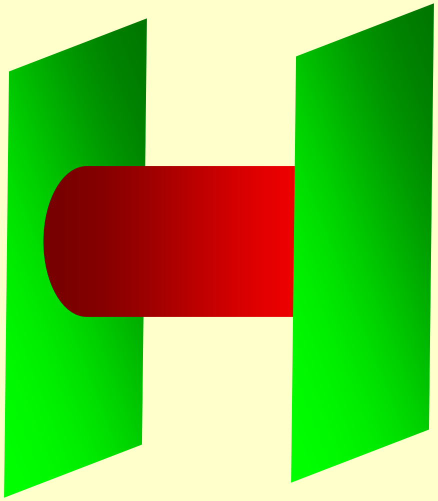
$g_{\mu\nu} \, , B_{\mu\nu} \, , \Phi$



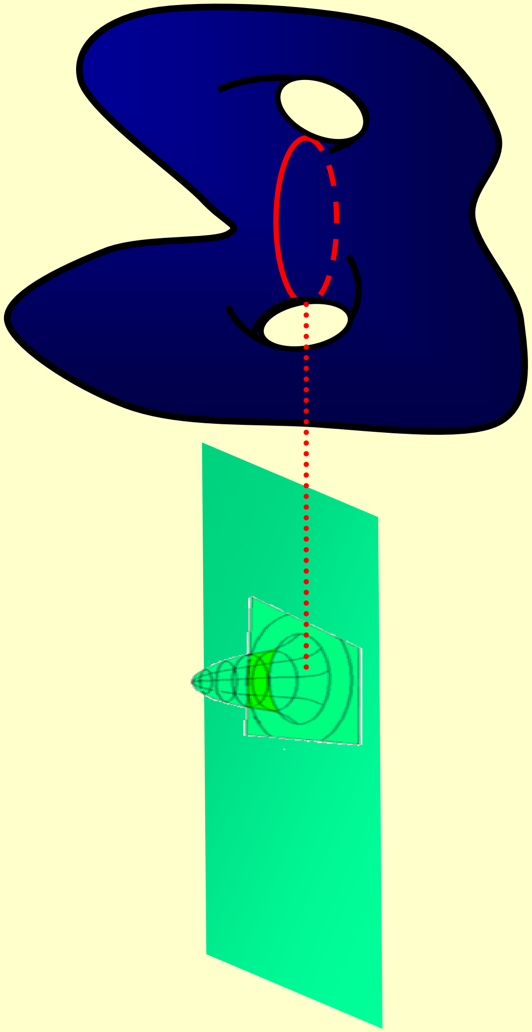
D-branes: gauge theory on worldvolume.
source for gravity:



gravity induced by open string loops



D-branes as Black Holes



D-branes wrap certain “cycles” inside compactification manifold. They become charged massive objects: (extremal) black holes.

Their world-volume theory gives a microcopic description of the states associated with the black holes.

$$ds^2 = -\frac{1}{H(r)}dt^2 + H(r)\left(dr^2 + r^2 d\Omega^2\right)$$

$$H(r) = \left(1 + \frac{Q}{r}\right)^2 \qquad Q = \phi^i Q_i$$

Microscopic Origin of Entropy

Excited string states have high degeneracy

$$M^2 = N\ell_s^{-2} \qquad \# \text{states} \approx \exp \pi\sqrt{cN}$$

But not enough to explain black hole entropy

$$\exp \pi M \ell_s \ll \exp \pi M^2 \ell_p^2$$

For extremal black holes

$$\# \text{states} \approx \exp \pi Q^2$$

D-brane described microscopically by "gas of strings" with

$$c = Q^2 \qquad N = Q^2$$

Exact counting

Extremal "dyonic" black holes:

$$S(Q,P)=\frac{A}{4}=\pi\sqrt{P^2Q^2} \qquad S(Q,P)=\log D(Q^2,P^2)$$

Generating function:

$$\sum_{N,M} D(N,M)e^{-tN-sM}=\prod_{n,m}\left(1-e^{-nt-ms}\right)^{-c(nm)}$$

with:

$$\sum_{n} c(N)q^N=\prod_{n}\left(\frac{1+q^n}{1-q^n}\right)^4$$

AdS/CFT Correspondence

Anti-de Sitter- Conformal FieldTheory

Near horizon geometry of a D3-brane

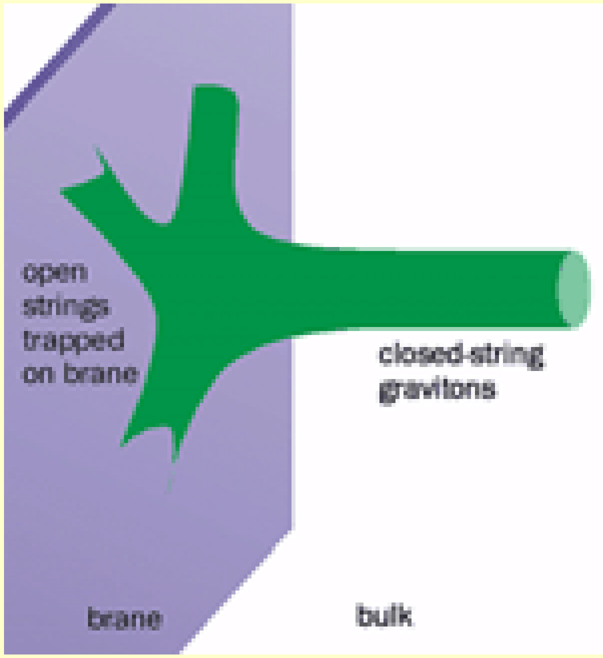
AdS black hole = thermal CFT

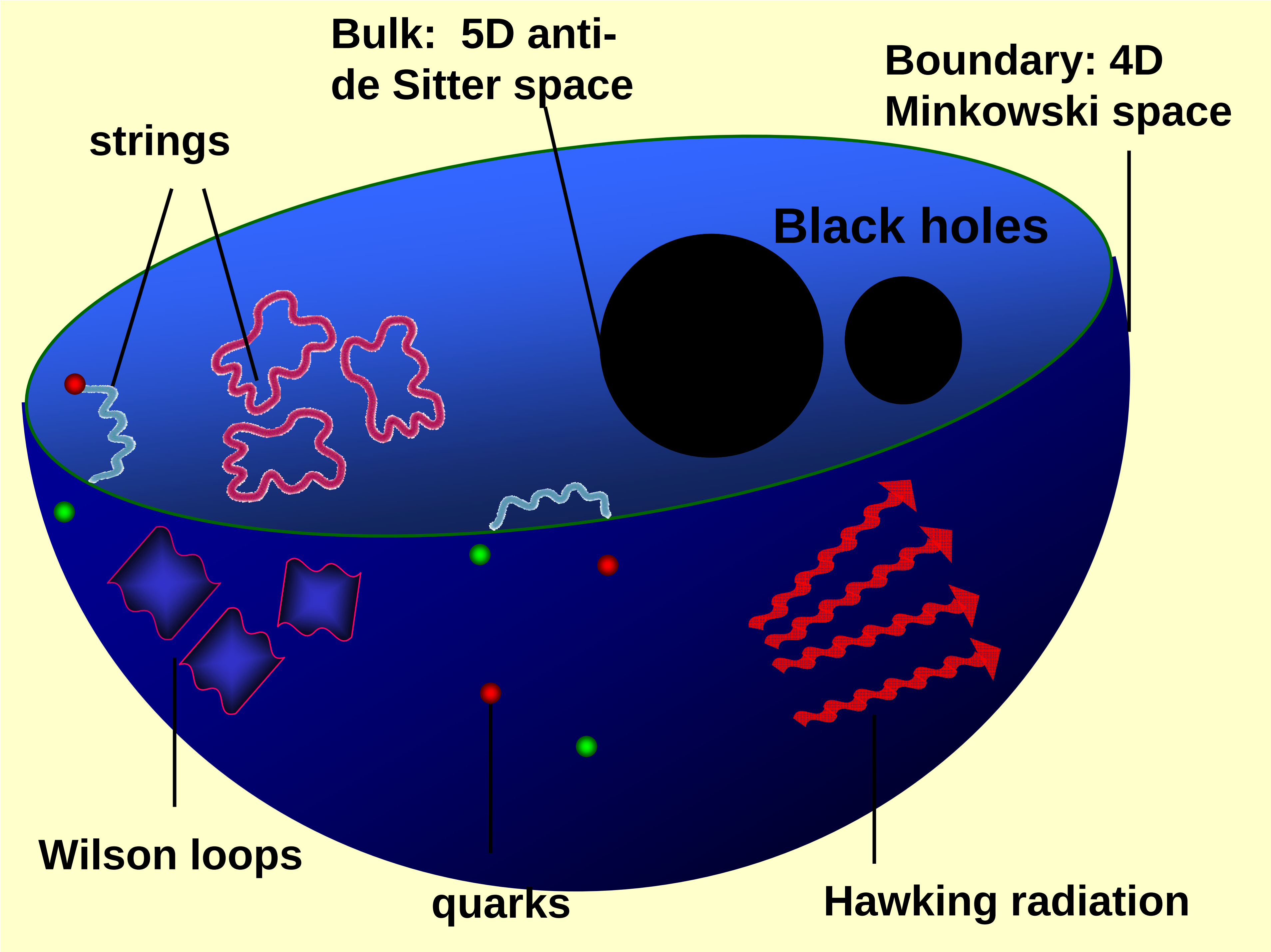
$$ds^2 = -H(r)dt^2 + \frac{dr^2}{H(r)} + r^2 d\Omega^2$$

Strings on $AdS_5 \times S^5$

= dual to a CFT: N=4 SuperYang-Mills

$$Z_{CFT}(g,\phi) = Z_{string}(g,\phi)$$




Bulk: 5D anti-de Sitter space
Boundary: 4D Minkowski space
strings
Black holes
Wilson loops
quarks
Hawking radiation

4D Attractor Black Holes and Entropy

Semiclassical entropy

$$S(Q,P) = \left[X^{\Lambda} \overline{F}_{,\Lambda} - \overline{X}^{\Lambda} F_{,\Lambda} \right]_{Q,P}$$

$$\mathrm{Re}\left(X^{\Lambda}\right) = Q^{\Lambda}$$

$$\mathrm{Re}\left(F_{,\Lambda}\right) = P_{\Lambda}$$

Ooguri, Strominger, Vafa

$$\Omega(P,Q) = \int d\Phi \overline{\Psi}_{P,Q}(\Phi) \Psi_{P,Q}(\Phi)$$

$$\Psi_{P,Q}(\Phi) = e^{i\Phi P} \Psi(Q+\Phi)$$

$$F\left(Q,\Phi\right) = 2\,\mathrm{Im}\,F(Q + \tfrac{i}{2}\Phi)$$

$$S(Q,P) = \log D(Q,P)$$

Mixed partition function factorizes as

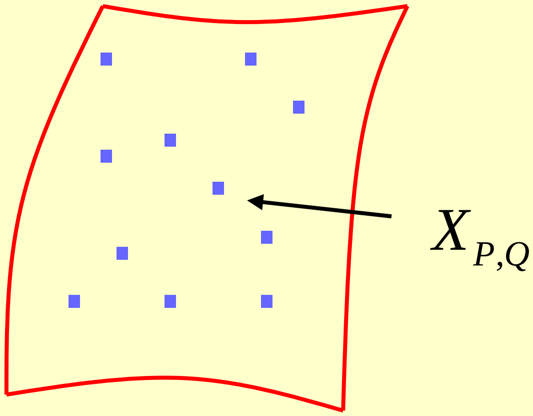
$$\sum_{p} \Omega\left(Q,P\right) e^{-P\Phi} = \left|\Psi\left(Q + i\Phi\right)\right|^{2}$$

$$\Psi\left(X\right) = \exp iF\left(X\right)$$

The Entropic Principle

Flux Vacua

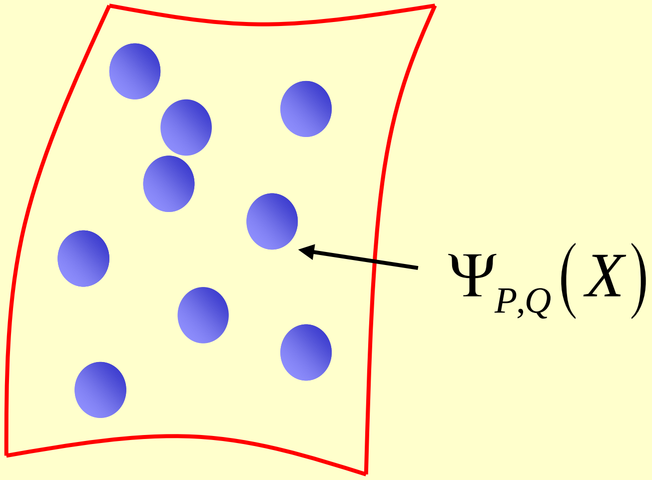
- **Moduli fixed by fluxes : discrete points.**



Flux Wave Functions

- **Flux vacua as wave functions on moduli space**
- **Relative probability determined by entropy**

$$\left\langle \bar{\Psi}_{P,Q} \middle| \Psi_{P,Q} \right\rangle \approx \exp S(P,Q)$$



Entropic Principle

- **Nature is (most likely) described by state of maximal entropy**
- **Constructive way to select vacua (in contrast with "Anthropic Principle")**

[redacted]