RDB

Stony Brook

May 2007

A Small x-planation

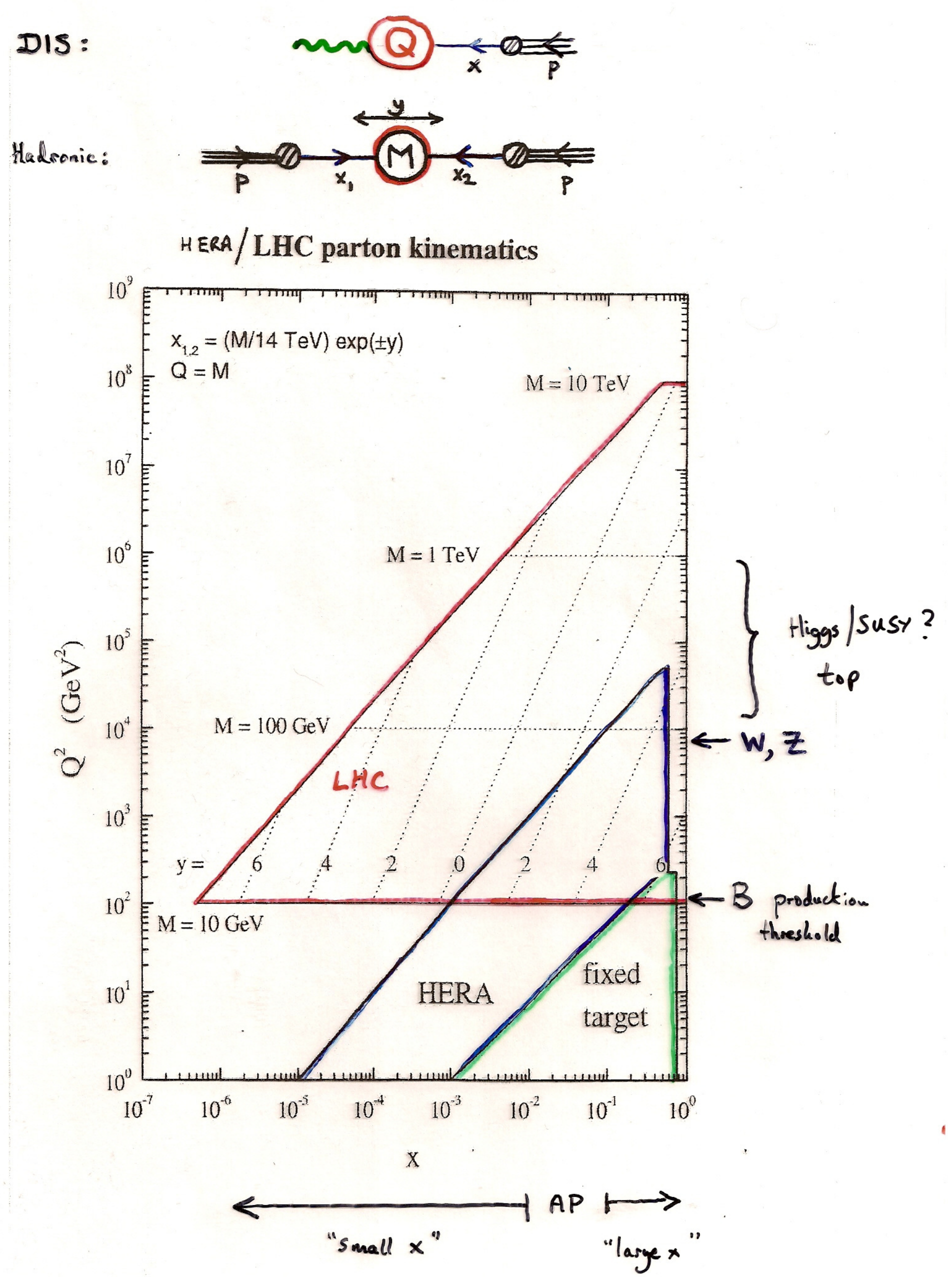
- $\log x$ vs $\log Q^2$
- AP $\longleftrightarrow$ BFKL
- HERA $\to$ Tevatron $\to$ LHC

Fadin Lipatov

Altarelli, Ball, Forte — { NPB 674 (2003) 459; NPB 742 (2006) 1; NPB 742 (2006) 158; 2 papers in preparation. }

Catani, Ciafaloni, Salam, et al.

Collins, Ellis etc.

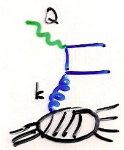

DIS :
Q
x
P
Hadronic :
y
M
P
x1
x2
P
HERA / LHC parton kinematics
x1,2 = (M/14 TeV) exp(±y)
Q = M
M = 10 TeV
M = 1 TeV
M = 100 GeV
M = 10 GeV
LHC
y = 6 4 2 0 2 4 6
HERA
fixed target
Q2 (GeV2)
x
Higgs / SUSY ?
top
W, Z
B production threshold
AP
"small x"
"large x"

Aims

- Splitting Function at LO, NLO (→ NNLO?) resummed for evolution at small x.

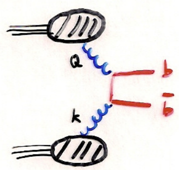
  Should be
  a) sensible! ie ~ DAS
  b) matched smoothly to AP
  c) stable (NLO corrns small)

- Consistent Extraction of Small x pdfs from HERA



Asymmetric

- Consistent Calcn of Small x hadronic xsec at LHC



Symmetric.

Basic Ingredients:

Mellin wrt $\ln Q^2$ → $M\, G(N,M) = G_0(N) + \gamma(\alpha,N)\, G(N,M)$ "AP eq"

$N\, G(N,M) = \bar{G}_0(M) + \chi(\alpha,M)\, G(N,M)$ "BFKL eq"

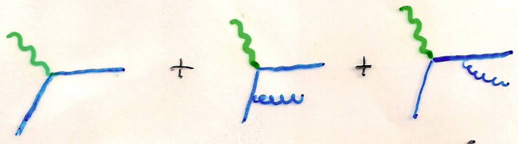
↑ Mellin wrt $\ln 1/x$

Resummation

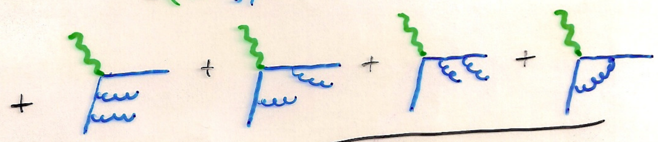
Calculate xsecs in parton theory:

$\alpha \ll \alpha_s$

eg



$O(\alpha)$ $\qquad$ $O(\alpha\,\alpha_s)$



$O(\alpha\,\alpha_s^2)$ $\qquad$ etc.

But $\exists$ large (kinematic) logs which spoil this: in general

$$\sigma = \sum_0^\infty (\alpha_s L)^n + \alpha_s \sum_0^\infty (\alpha_s L)^n + \ldots\ldots$$

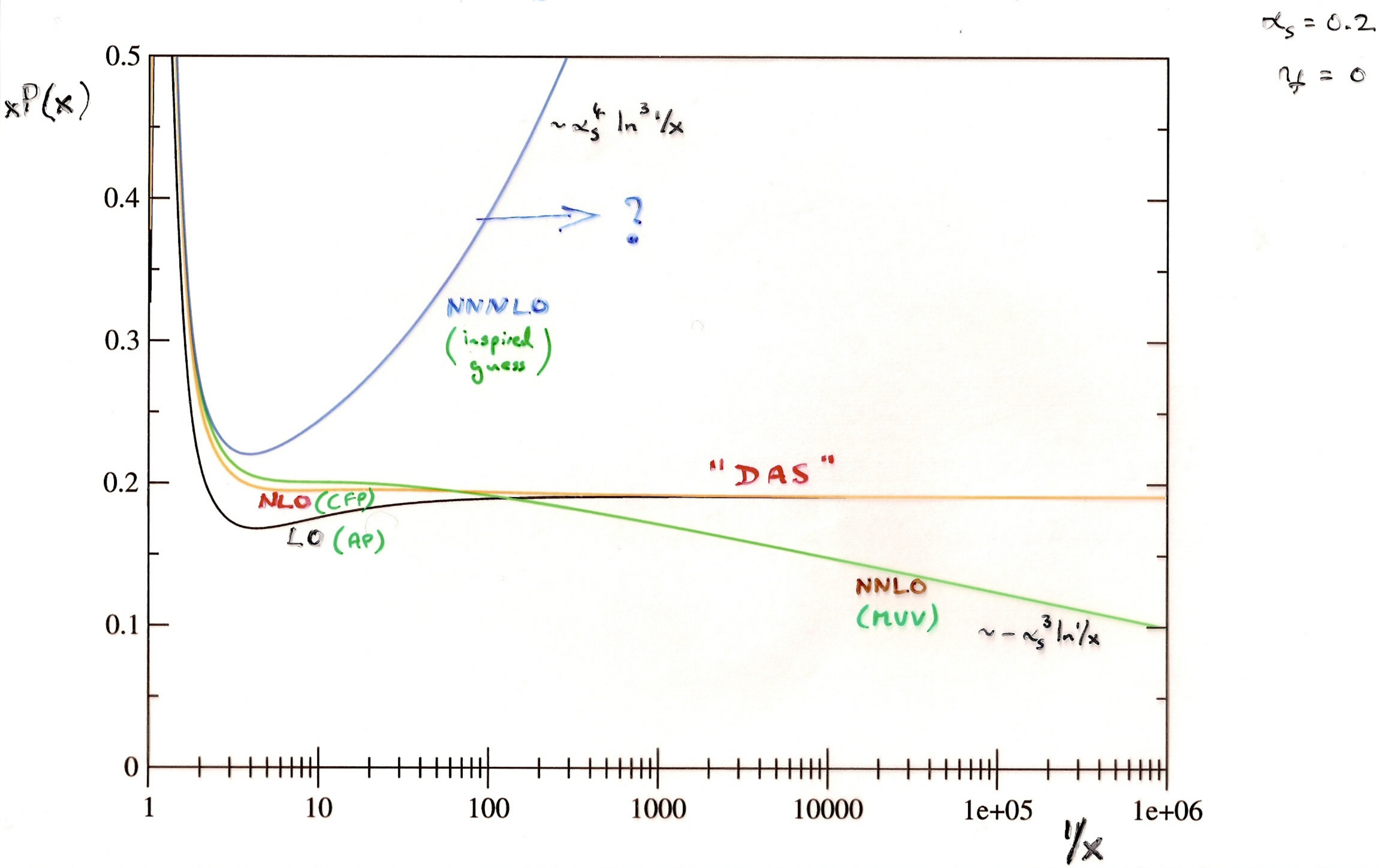
↑ partonic xsec $\qquad$ LL $\qquad$ NLL

- Large Q^2: $\quad L = \ln Q^2/\Lambda^2$ $\qquad$ AP evoln. (collinear logs)
- Large x: threshold resummation $\quad L = \ln(1-x)$ $\qquad$ Sudakov ('soft' logs)

  Important for top / Higgs / SUSY prod.
- Small x: 'BFKL resummation' $\quad L = \ln {}^1/x$ $\qquad$ 'BFKL logs' (high energy).

  Important for high energy prod. (eg B at LHC)

Splitting Function in Fixed Order (AP) pertb. th.



Mellins

Fixed coupling

AP eqn:

$$\frac{\partial}{\partial \ln Q^2} G(x,Q^2) = \int_x^1 \frac{dy}{y} P\left(\frac{x}{y},\alpha\right) G(y,Q^2)$$

$$\updownarrow \qquad\qquad \updownarrow$$

$$M\,G(N,M) - G_0(N) = \gamma(N,\alpha)\, G(N,M)$$

Soln:

$$G(N,M) = \frac{G_0(N)}{M-\gamma(N,\alpha)}$$

BFKL eqn:

$$\frac{\partial}{\partial \ln 1/x} G(x,Q^2) = \int \frac{dk^2}{k^2} K\left(\frac{Q^2}{k^2},\alpha\right) G(x,k^2)$$

$$\updownarrow \qquad\qquad \updownarrow$$

$$N\,G(N,M) - \bar{G}_0(M) = \chi(M,\alpha)\, G(N,M)$$

Soln:

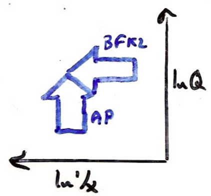
$$G(N,M) = \frac{\bar{G}_0(M)}{N-\chi(M,\alpha)}$$

Altarelli, Ball, Forte, (2000)

Duality

Fixed coupling

Assume that (at small x and large Q^2)
AP and BFKL true simultaneously



$$G(N,M) = \frac{G_0(N)}{M - \gamma(\alpha,N)} = \frac{\bar{G}_0(M)}{N - \chi(\alpha,M)}$$

AP soln. = BFKL soln

Only possible if

$$M = \gamma(\alpha,N) \Leftrightarrow N = \chi(\alpha,M)$$

so

$$M = \gamma(\alpha, \chi(\alpha,M))$$
$$N = \chi(\alpha, \gamma(\alpha,N))$$

"Duality"
$M \leftrightarrow N$
$\gamma \leftrightarrow \chi$

i.e. $\gamma = \chi^{-1}$ and $\chi = \gamma^{-1}$.

→ γ, χ figs

eg $\gamma(\alpha,N) = \frac{\bar{\alpha}}{N}$ $\Leftrightarrow$ $\chi(\alpha,M) = \frac{\bar{\alpha}}{M}$ DLL

[x figs]

Also need to match b.c. :

$$G_0(N) = \frac{\bar{G}_0(\gamma(\alpha,N))}{-\chi'(\alpha,\gamma(\alpha,N))}, \quad \bar{G}_0(M) = \frac{G_0(\chi(\alpha,M))}{-\gamma'(\alpha,\chi(\alpha,M))}$$

N.B.
$1 = \gamma'\chi'$

$\chi(\alpha,M)$: the facts

- Fixed order pertbn th:

$$\chi(\alpha,M) = \alpha\,\chi_0(M) + \alpha^2\chi_1(M) + \cdots\cdots$$

BFKL (1976) FL (1998)

- Collinear Resummation (near $M=0$):

$$\chi(\alpha,M) = \chi_s\left(\frac{\alpha}{M}\right) + \alpha\,\chi_{ss}\left(\frac{\alpha}{M}\right) + \cdots\cdots$$

AP CFP + duality

- Momentum Conservation:

$$\chi(\alpha,0) = 1 \quad \therefore \quad \gamma(\alpha,1) = 0$$

to all orders in pertbn th. ABF (2000)

- Symmetry : in symmetric variables (eg $1/x = s/Qk$)

cf. Ciafaloni (1988) $\chi_{sym}(\alpha,M) = \chi_{sym}(\alpha,1-M)$

at each order in pertbn. th.

- Minimum: χ_{sym} has a minimum at $M = 1/2$

$\therefore$ χ_{asym} has the same minimum

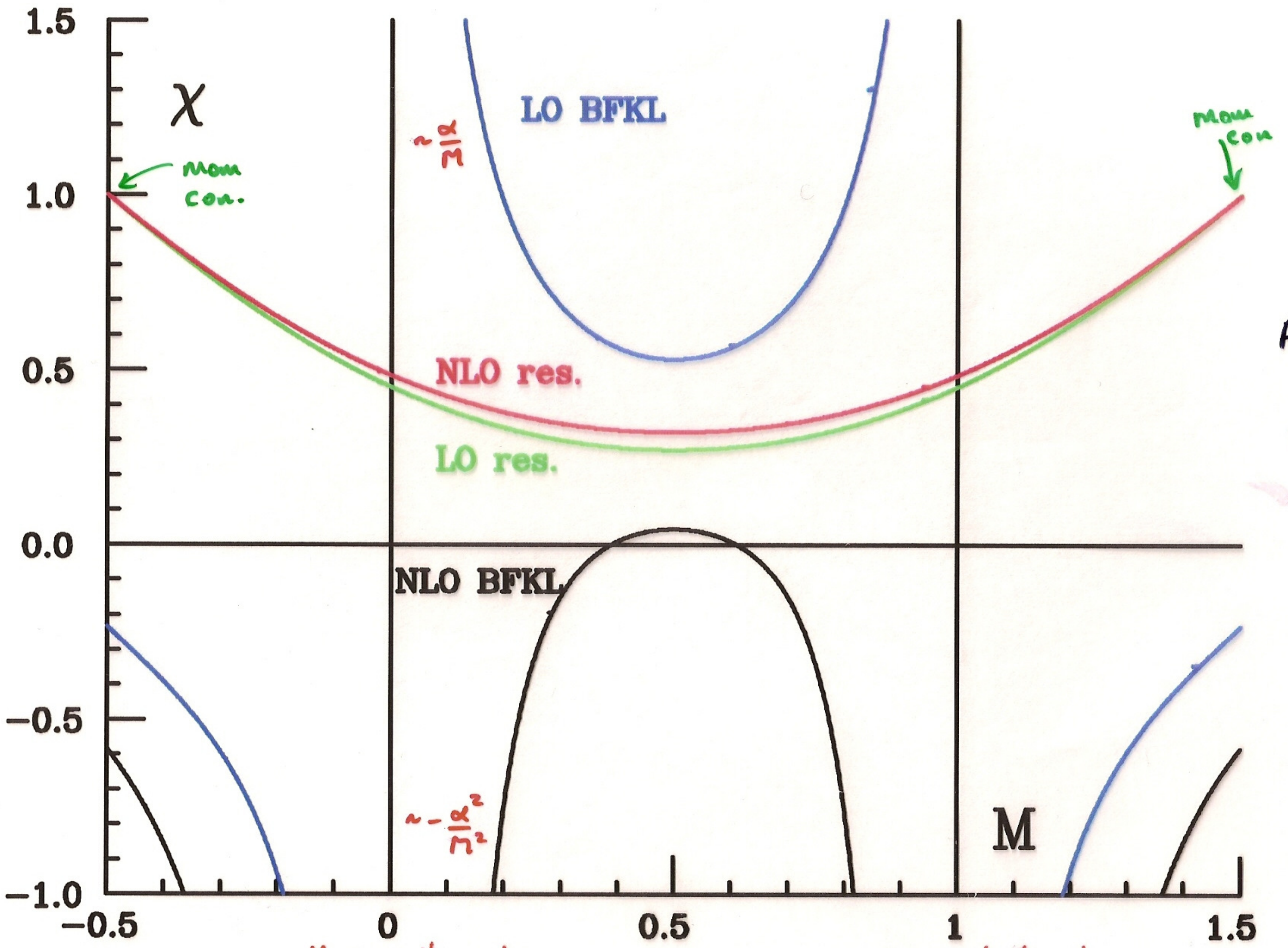
- Running Coupling Resummation around minimum

Shifts cut in $\gamma(\alpha,N)$ to a pole (Airy)

cf. Lipatov (1986)

χ in Symmetrical Variables

$\alpha_s = 0.2$



After resummation NLO > LO! But NLO corrections

M~0: collinear logs

M~1: anticollinear logs.

Let $\chi_0 \to \chi_{sym}$ in Small x MC (? notice quadratic approx is excellent....)

Momentum Conservation

$$\Rightarrow \quad \boxed{\begin{aligned} \gamma(\alpha, 1) &= 0 \\ \chi(\alpha, 0) &= 1 \end{aligned}} \qquad \begin{aligned} &\text{to all orders } [\partial^\mu T_{\mu\nu} = 0] \\ &\text{by duality} \end{aligned}$$

Exact all order result for γ & χ.

e.g.

$$\gamma(\alpha, N) \sim \frac{\bar{\alpha}}{N} - \bar{\alpha}$$

→ γ fig

$$M = \frac{\bar{\alpha}}{N} - \bar{\alpha} \quad \Rightarrow \quad N = \frac{\bar{\alpha}}{M + \bar{\alpha}}$$

so

$$\chi(\alpha, M) \sim \frac{\bar{\alpha}}{M + \bar{\alpha}} = \frac{\bar{\alpha}}{M} - \left(\frac{\bar{\alpha}}{M}\right)^2 + \left(\frac{\bar{\alpha}}{M}\right)^3 - \cdots\cdots$$

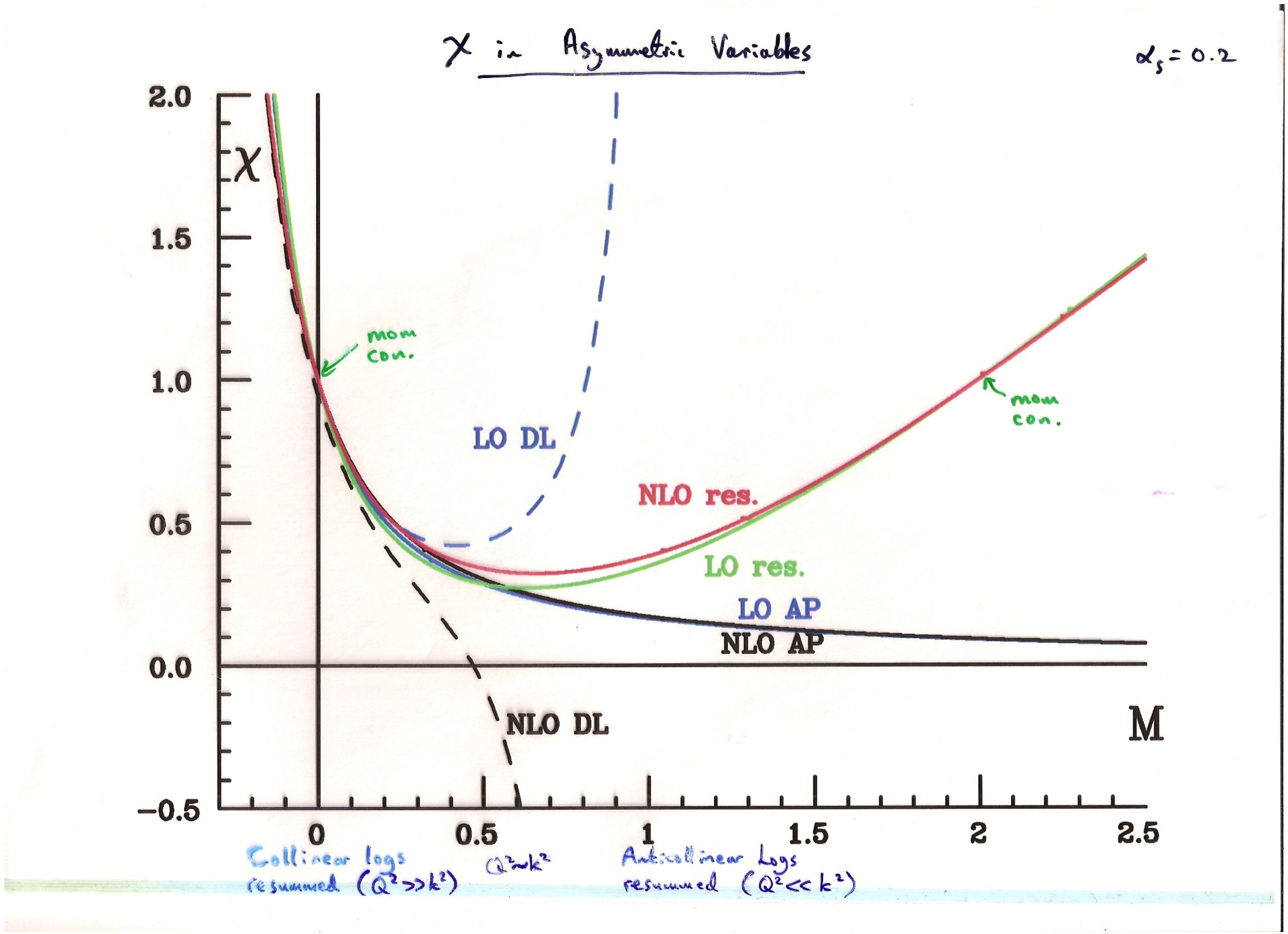
$\chi_0 \sim \frac{1}{M}$ ↑ $\quad \chi_1 \sim -\frac{1}{M^2}$ ↑ $\quad \chi_2 \sim \frac{1}{M^3}$ ↑

Collinear poles (ie LQ)

→ χ fig.

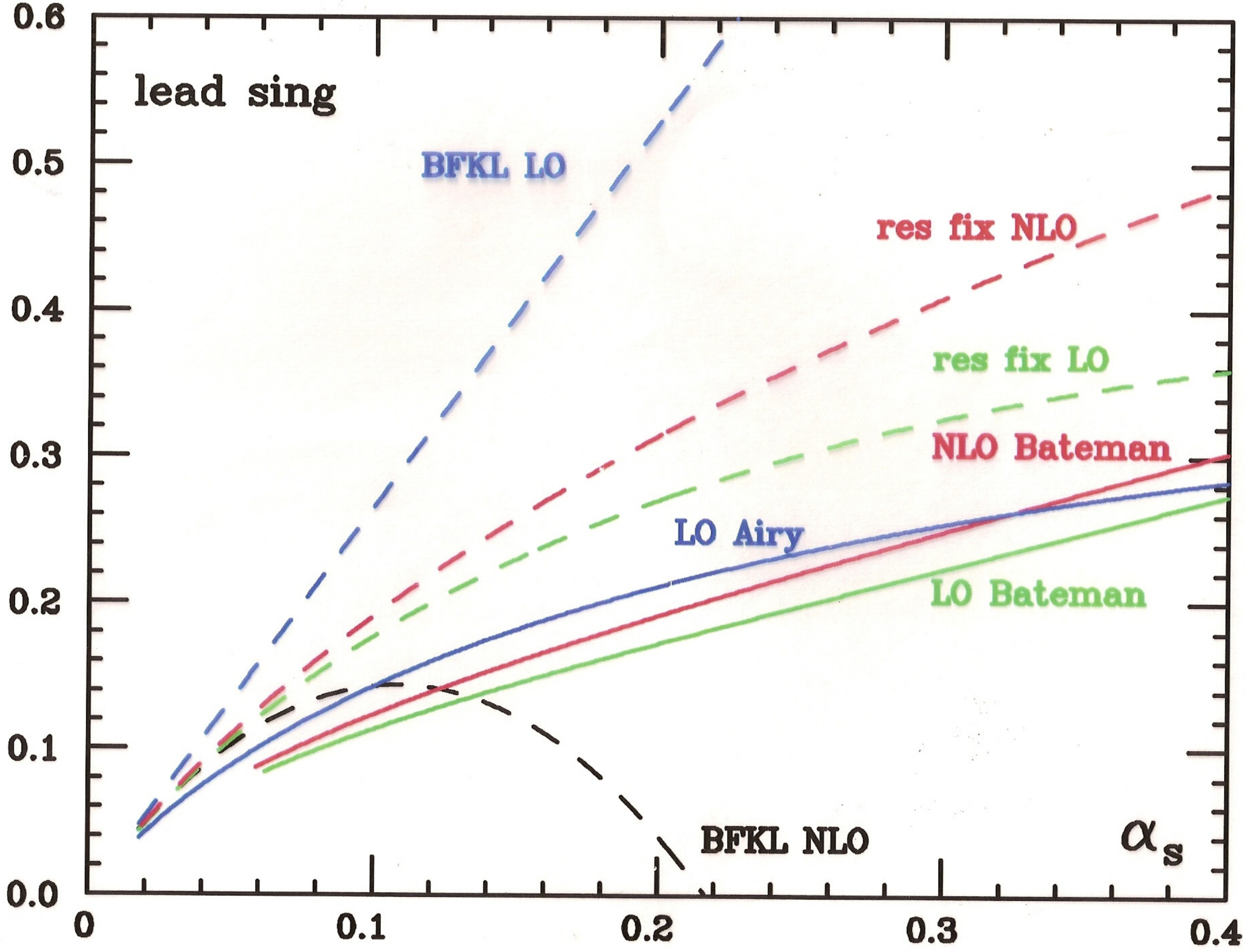
- Fixed order calcns of χ very poor near $M = 0$ $(Q^2 \gg k^2)$
- Momentum conservation resums the collinear poles in $\chi(\alpha, M)$.

→ $\gamma + \chi$ fig

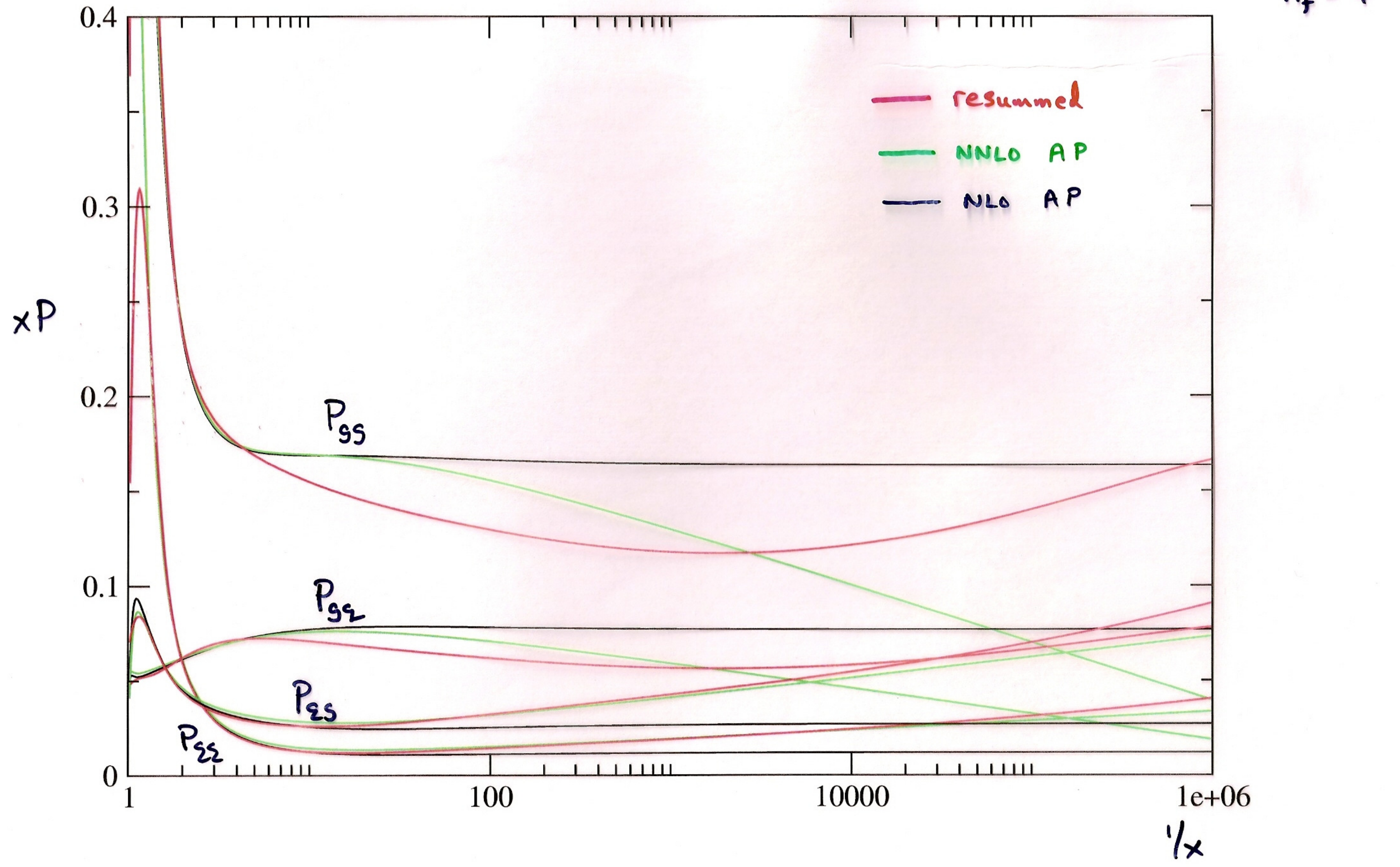


χ in Asymmetric Variables
$\alpha_s = 0.2$
2.0
1.5
1.0
0.5
0.0
−0.5
χ
mom con.
LO DL
NLO res.
LO res.
LO AP
NLO AP
NLO DL
M
0
0.5
1
1.5
2
2.5
Collinear logs resummed ($Q^2 \gg k^2$)
$Q^2 \sim k^2$
Anticollinear Logs resummed ($Q^2 \ll k^2$)

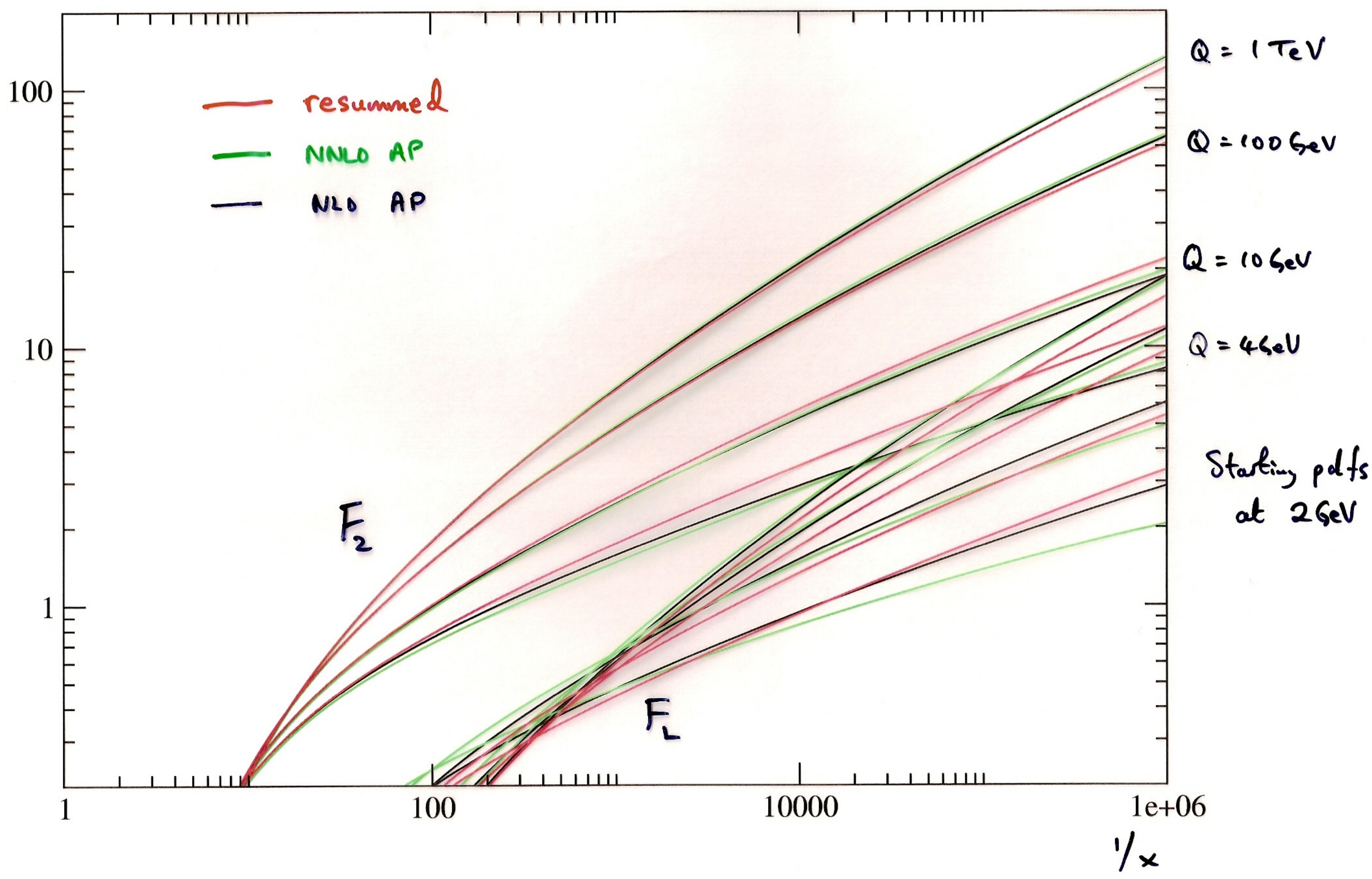
Cuts and Poles vs α_s.



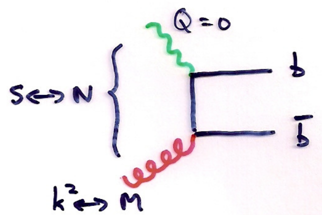
Splitting Functions



Structure Functions



Photoproduction of B

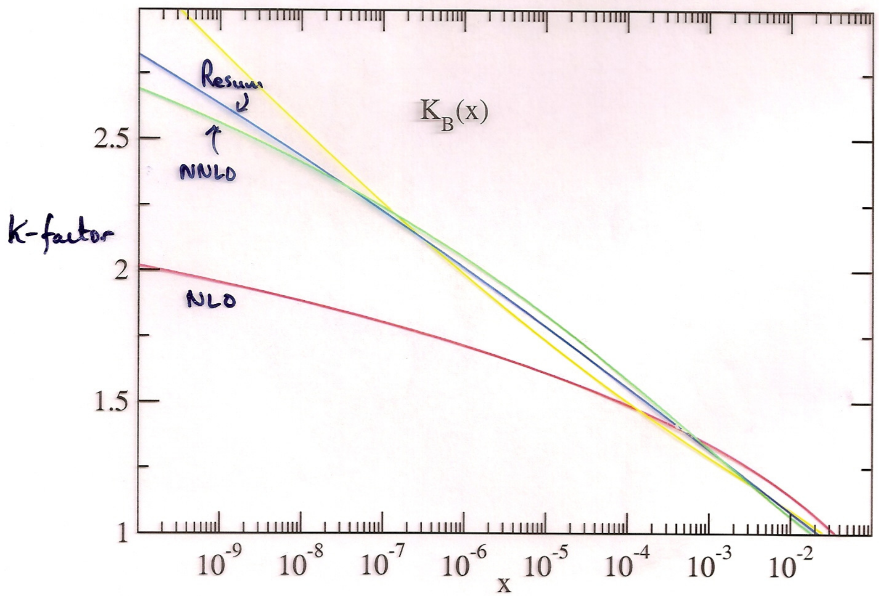


Hard xsec $\quad C(M,N)$: regular at $\begin{cases} M = 0 & \text{no coll. sings} \\ N = 0 & \text{no small x sings.} \end{cases}$

Infrared pole at $M = 1$

$$\frac{1}{1-M} \sim 1 + M + O(M^2)$$

↑ NLO coeff fn $\qquad$ ↑ NNLO



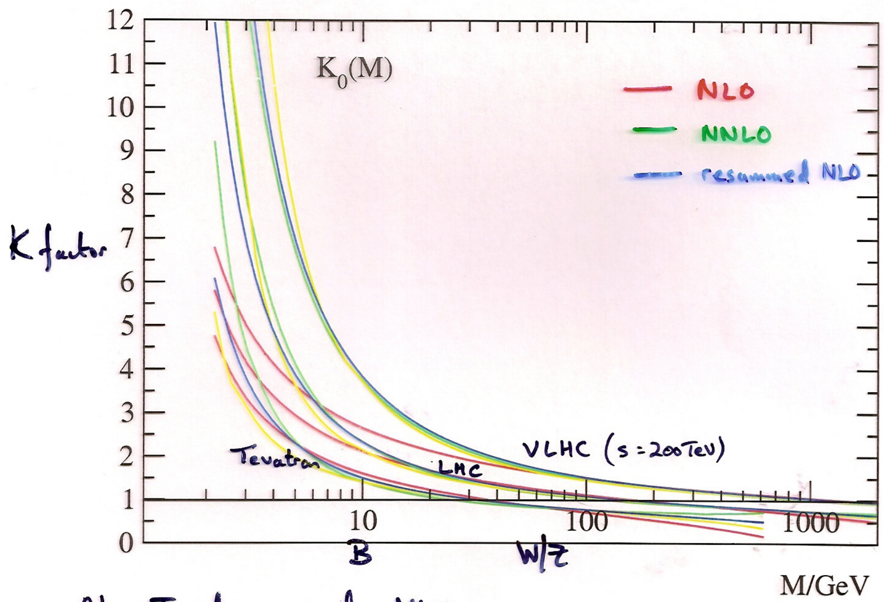
Hadroproduction (gluonic prod. only)

eg DY :

M : invariant mass of dilepton pair.

$$\sigma(s,M) = \int dx_1 dx_2 \; g(x_1) g(x_2) \sum(\hat{s}, \eta)$$

$\hat{s} = x_1 x_2 s$ $\qquad \eta = \frac{1}{2} \ln(x_1/x_2)$ (rapidity)



At Tevatron, need NLO

At LHC, need NNLO or resummed

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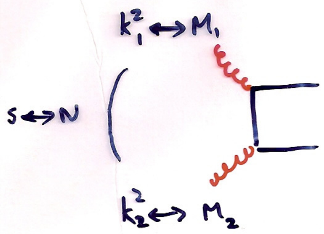
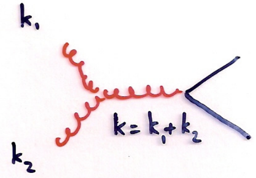
The Hadronic Singularity

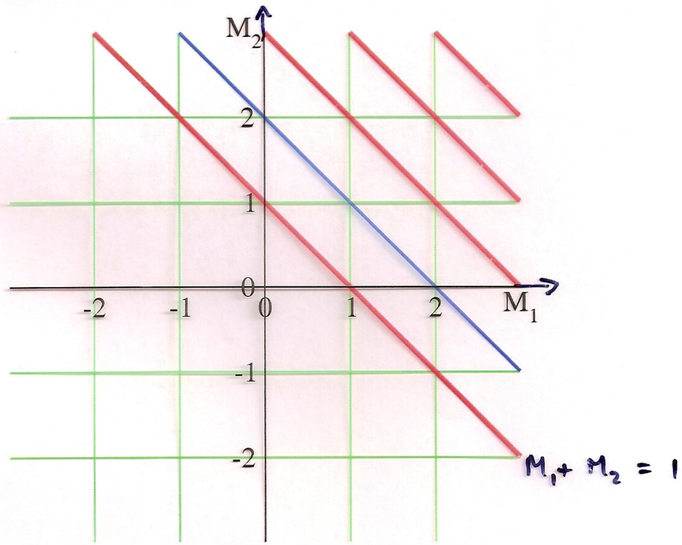




$$\Sigma(M_1, M_2, N) \underset{N\sim 0}{\sim} \alpha_s^2 \frac{1}{(1-M_1-M_2)^3}$$

hard xsec

line of singularity

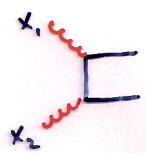


Strong enhancement

(RDB + RKE, 2001)

$M_1 + M_2 = 1 \quad \longleftrightarrow \quad k^2 = (k_1 + k_2)^2 \ll k_1^2, k_2^2$

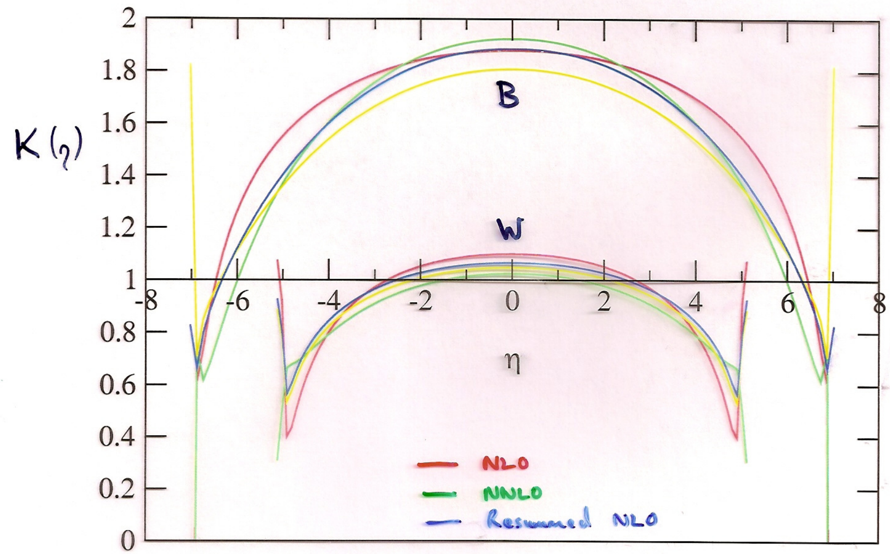
Rapidity Distbns



$$\eta = \frac{1}{2} \ln x_1/x_2 \qquad \text{"rapidity"}$$

So when $|\eta| \gg 1$, $x_1 \ll x_2$ or $x_2 \ll x_1$

$\Rightarrow$ resummation important



Uncertainty of $\sim 5\%$ in W xsec from small x effects.

Summary

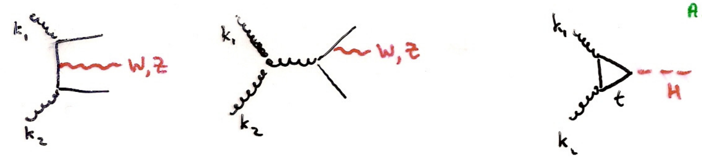
- Reliable resummed evolution at small x
  - Naive expectation : BFKL LO $\gg$ BFKL NLO $\gg$ AP

    Reality: Res. LO $<$ Res NLO $<$ AP for HERA/LHC ($10^{-1} \lesssim x \lesssim 10^{-6}$)
  - No rise until $x \lesssim 10^{-6}$, then very mild — so less need for shadowing.
- Determination of resummed pdfs from HERA data — NNPDF — soon!
- Predictions for B prod. etc at LHC — patience....

  DY, Higgs, ...

  $x \sim 10^{-3}$ – 10^{-4} $x \sim 10^{-2}$ – 10^{-3}

$m_t \to \infty$
Hautmann
A. Lipatov



Where do we need small x corrections to hard xsecs?

$\ln Q^2 \gg \ln 1/x \quad : \quad M \ll N$

$\ln Q^2 \sim \ln 1/x \quad : \quad M \sim N$

$\ln Q^2 \ll \ln 1/x \quad : \quad M \gg N$

Small x logs only dominate at very low scales: