Higher spin fields from a worldline perspective

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From Stony Brook to Bologna

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3. Permanent jobs
   - 1995-1999: Researcher at Modena U.
   - 1999-2005: Researcher at Bologna U.
   - 2005-2007: Associate Professor at Bologna U.

Outline

Stony Brook topics:

supergravity,

quantization of particles,

path integrals,

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Based on

F.B., Olindo Corradini and Emanuele Latini,
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JHEP **0702** (2007) 072 [arXiv: hep-th/0701055]

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Olindo Corradini, PhD from YITP in 2002

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- Investigate their description form a first quantized point of view $\longrightarrow$ spinning particles

Spinning particles

- Simplest model is bosonic particle ($N = 0$)

$$S[x,p] = \int d\tau \left[p_\mu \dot{x}^\mu - \frac{1}{2}\eta_{\mu\nu} p^\mu p^\nu \right]$$

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- Model is not unitary. Unitarity can be achieved gauging the translation, gauge field is the einbein e

$$S[x,p,e] = \int d\tau \left[p_\mu \dot{x}^\mu - eH \right]$$

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- In fact, canonical quantization gives a constraint

$$[\hat{x}^\mu, \hat{p}_\nu] = i\hbar\delta^\mu_\nu$$

$$\hat{H}|\phi\rangle = 0 \;, \qquad |\phi\rangle \in \text{physical Hilbert space}$$

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- Using wave functions $\phi(x) = \langle x|\phi\rangle$

$$\hat{H}|\phi\rangle = \hat{p}_\mu\hat{p}^\mu|\phi\rangle = 0 \quad \Rightarrow \quad \partial_\mu\partial^\mu\phi(x) = 0$$

recognized as the massless Klein Gordon equation (a unitary theory).

Spinning particles

- Next simplest model is $N = 1$ spinning particle

$$S[x,p,\psi] = \int d\tau \left[p_\mu \dot{x}^\mu + \frac{i}{2}\eta_{\mu\nu}\psi^\mu\dot{\psi}^\nu - \frac{1}{2}\eta_{\mu\nu}p^\mu p^\nu \right]$$

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- Gauging $N=1$ supersymmetry gives the
  $N=1$ spinning particle

$$S[x,p,\psi,e,\chi] = \int d\tau \left[p_\mu \dot{x}^\mu + \frac{i}{2}\eta_{\mu\nu}\psi^\nu \dot{\psi}^\mu - eH - i\chi Q \right]$$

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- ψ^μ realize the gamma matrix algebra, $\gamma^\mu \sim \sqrt{\frac{2}{\hbar}}\,\psi^\mu$
- Using wave functions $\psi_\alpha(x) = \langle x, \alpha|\psi\rangle$

$$\hat{Q}|\psi\rangle = \hat{\psi}^\mu \hat{p}_\mu|\psi\rangle = 0 \quad \Rightarrow \quad (\gamma^\mu)_\alpha{}^\beta\,\partial_\mu\,\psi_\beta(x) = 0$$

that is the massless Dirac equation $\gamma^\mu\partial_\mu\psi = 0$.

Spinning particles

- General case ($i = 1, .., N$)

$$S = \int dt \Big[p_\mu \dot{x}^\mu + \frac{i}{2} \eta_{\mu\nu} \psi_i{}^\mu \dot{\psi}_i^\nu - \frac{1}{2} \eta_{\mu\nu} p^\mu p^\nu \Big]$$

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- Rigid $SO(N)$ supersymmetry

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- Algebra

$$\begin{aligned}
\{Q_i, Q_j\}_{PB} &= -2i\delta_{ij} H \\
\{J_{ij}, Q_k\}_{PB} &= \delta_{jk} Q_i - \delta_{ik} Q_j \\
\{J_{ij}, J_{kl}\}_{PB} &= \delta_{jk} J_{il} - \delta_{ik} J_{jl} - \delta_{jl} J_{ik} + \delta_{il} J_{jk}
\end{aligned}$$

$SO(N)$ spinning particle

- Algebra is first class $\rightarrow$ can be gauged

$$S = \int dt \Big[p_\mu \dot{x}^\mu + \frac{i}{2}\psi_{i\mu}\dot{\psi}_i^\mu - eH - i\chi_i Q_i - \frac{1}{2}a_{ij}J_{ij} \Big]$$

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- Gauge transformations on supergravity multiplet

$$\begin{aligned}
\delta e &= \dot{\xi} + 2i\chi_i \epsilon_i \\
\delta \chi_i &= \dot{\epsilon}_i - a_{ij}\epsilon_j + \alpha_{ij}\chi_j \\
\delta a_{ij} &= \dot{\alpha}_{ij} + \alpha_{im} a_{mj} + \alpha_{jm} a_{im} \ .
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- Eliminating momenta p_μ

$$S = \int dt \Big[\frac{1}{2} e^{-1} (\dot{x}^\mu - i\chi_i \psi_i^\mu)^2 + \frac{i}{2}\psi_{i\mu}\dot{\psi}_i^\mu - \frac{i}{2} a_{ij} \psi_{i\mu}\psi_j^\mu \Big]$$

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  - $D = 4 \;\Rightarrow$ spin $N/2$ particle (Bargmann-Wigner eq.)
    wave function $\Psi_{\alpha_1 \cdots \alpha_N}(x) \quad \to \quad \partial_{\alpha_i}{}^{\tilde{\alpha}_i} \Psi_{..\tilde{\alpha}_i..}(x) = 0$

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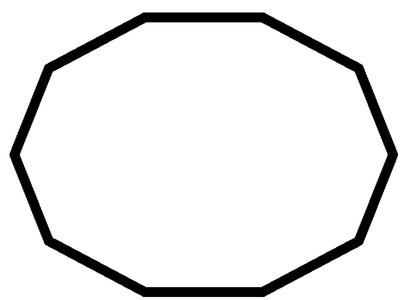

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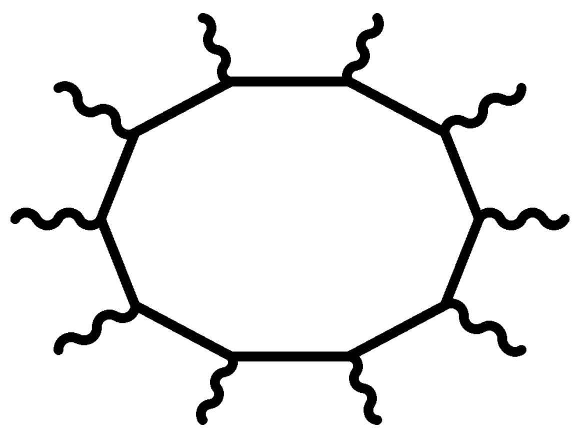

Path integral quantization

- Configuration space (Wick rotated) action

$$S[X,G] = \int_0^1 d\tau \, \frac{1}{2}\Big(e^{-1}(\dot{x}^\mu - \chi_i \psi_i^\mu)^2 + \psi_i^\mu(\delta_{ij}\partial_\tau - a_{ij})\psi_{j\mu}\Big)$$

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- One-loop partition function (PBC for bosons, ABC for fermions)

$$Z = \int_{S^1} \frac{\mathcal{D}X \, \mathcal{D}G}{\mathrm{Vol}\,(\mathrm{Gauge})} \, e^{-S[X,G]}$$

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- gauge fixing $\rightarrow$ FP determinants + modular parameters

$$\begin{aligned} e &\rightarrow \beta && \text{(proper time)} \\ \chi_i &\rightarrow 0 \\ a_{ij} &\rightarrow \hat{a}_{ij}(\theta_k) && k = 1,..,r; \quad r = \mathrm{rank}\; SO(N) \text{ angles} \end{aligned}$$

Path integral quantization

$$Z = -\frac{1}{2}\int_0^\infty \frac{d\beta}{\beta} \int \frac{d^D x}{(2\pi\beta)^{\frac{D}{2}}}$$

$$K_N \prod_{k=1}^{r} \int_0^{2\pi} \frac{d\theta_k}{2\pi} \left(\mathrm{Det}_{ABC}(\partial_\tau - \hat{a}_{vec}) \right)^{\frac{D}{2}-1} \mathrm{Det}'_{PBC}(\partial_\tau - \hat{a}_{adj})$$

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fermionic determinants + susy ghosts

gauge symmetry ghosts

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$Dof(D,N)$ normalized such that $Dof(D,0) = 1$

Path integral quantization

- $N = 2r$ $\quad K_{2r} = \dfrac{2}{2^r r!}$ reduces to a fundamental domain

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- Determinants look like this

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\begin{aligned}
\mathrm{Det}_{_{ABC}}(\partial_\tau - \hat{a}_{vec}) &= \prod_{k=1}^{r} \mathrm{Det}\,(\partial_\tau + i\theta_r)\,\mathrm{Det}\,(\partial_\tau - i\theta_r) \\
&= \prod_{k=1}^{r} \left(2\cos\frac{\theta_k}{2}\right)^2
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- Similarly for $\mathrm{Det}'_{PBC}(\partial_\tau - \hat{a}_{adj})$

Path integral quantization

- Get the formula (case of even $N = 2r$)

$$
\begin{aligned}
Dof(D,N) \;&=\; \frac{2}{2^r r!}\prod_{k=1}^{r}\int_0^{2\pi}\frac{d\theta_k}{2\pi}\left(2\cos\frac{\theta_k}{2}\right)^{D-2} \\
&\times\;\prod_{k<l}\left[\left(2\cos\frac{\theta_k}{2}\right)^2-\left(2\cos\frac{\theta_l}{2}\right)^2\right]^2
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$$\times \prod_{k<l}\left[\left(2\cos\frac{\theta_k}{2}\right)^2 - \left(2\cos\frac{\theta_l}{2}\right)^2\right]^2$$

- It vanishes for odd dimensions $D > 1$

$$Dof(2d+1,N) = 0$$

Path integral quantization

- Change of variables $x_k = \sin^2 \frac{\theta_k}{2}$

$$
\begin{aligned}
Dof(2d, 2r) \;&=\; \frac{2^{2(d-1)r+(r-1)(2r-1)}}{\pi^r r!} \\
&\times\; \prod_{k=1}^{r} \int_0^1 dx_k \; x_k^{-1/2} (1 - x_k)^{d-3/2} \prod_{k<l} (x_l - x_k)^2
\end{aligned}
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(Van der Monde determinant)^2

Degrees of freedom

- Even dimension, even rank

$$Dof(2d, 2r) = 2^{r-1} \frac{(2d-2)!}{[(d-1)!]^2} \prod_{k=1}^{r-1} \frac{k\,(2k-1)!\,(2k+2d-3)!}{(2k+d-2)!\,(2k+d-1)!}$$

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- A similar formula for odd rank case: $Dof(2d, 2r+1)$

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Special cases

- $Dof(2,N) = 1\ , \qquad \forall N$
- $Dof(4,N) = 2\ , \qquad \forall N$
- $Dof(2d,2) = \frac{(2d-2)!}{[(d-1)!]^2}$

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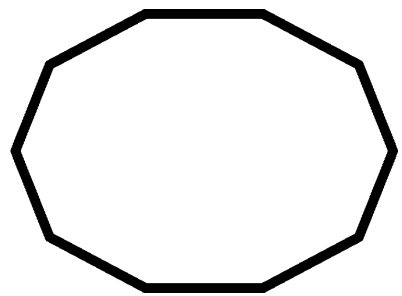
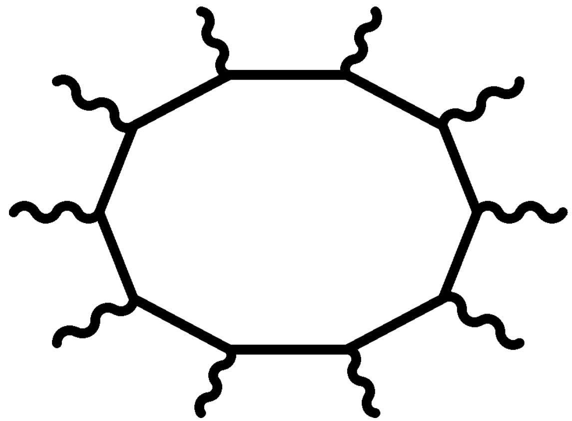
- Calculated dimensions of certain $SO(N)$ irreps.
- Higher spin fields propagate in the loop



Outlook

- Couple spinning particles to external gravity



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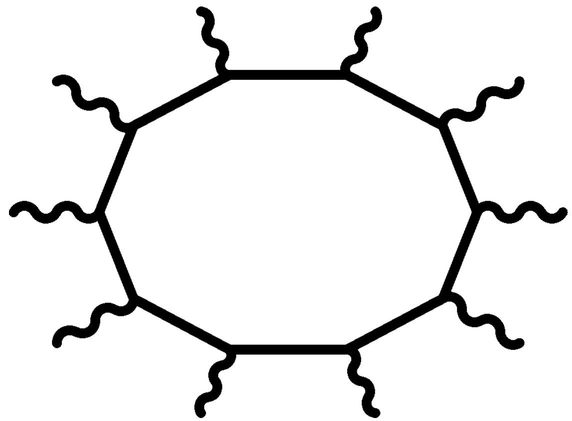

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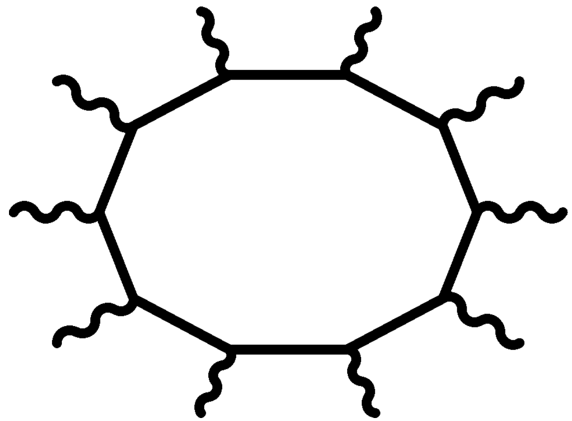

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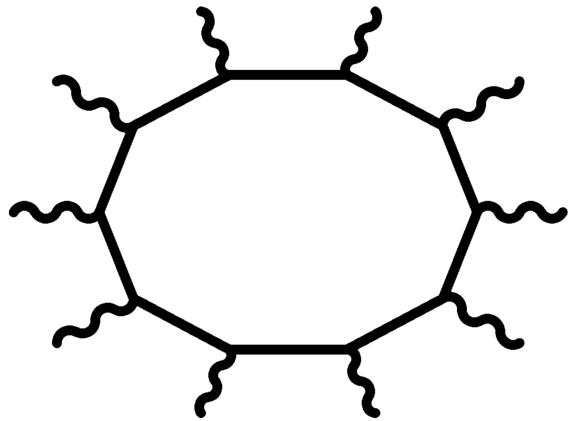

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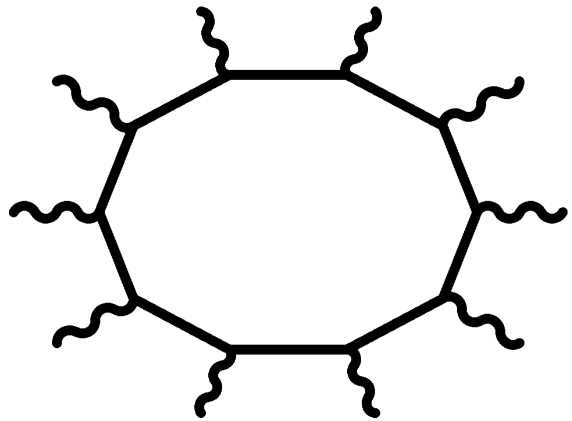

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  - Higher N: tricky, presumably feasible on AdS and dS spaces (gauge algebra becomes nonlinear)

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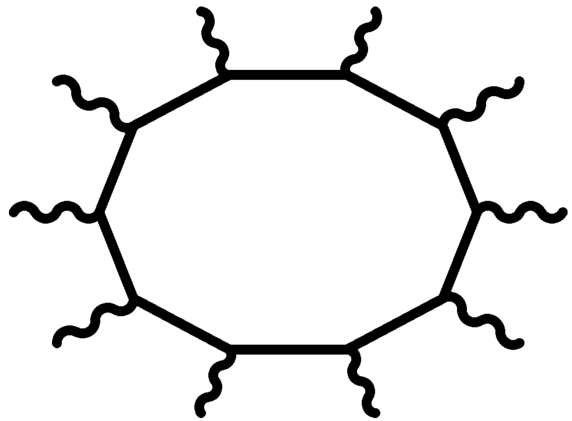

  - Tool: F.B. and P. van Nieuwenhuizen, *"Path Integrals and Anomalies in Curved Space"* (CUP 2006)
  - N=0,1 (with Zirotti, Corradini)
  - N=2 (with Benincasa, Giombi: all differential forms)
  - Higher N: tricky, presumably feasible on AdS and dS spaces (gauge algebra becomes nonlinear)
- More general couplings?

Thank you YITP!!