Non-BPS extremal black holes:
supergravity at work

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YITP@40: Anniversary Symposium

Stony Brook, New York, 3-4-5th May, 2007

I started studying Physics at the University of Torino, where Tullio Regge and Sergio Fubini had generated a lively school…

The birth of superstrings (1975)

Ref.TH.2097-CERN

SUPERSYMMETRIC STRINGS AND COLOUR CONFINEMENT

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A B S T R A C T

The (infinite-dimensional) supersymmetry algebra in 1+1 space-time dimension is extended in order to incorporate, in a non-trivial way, an internal symmetry. It turns out that this requirement implies that the internal symmetry is realized as a local gauge symmetry. Moreover, it is possible to construct string-like models with this underlying symmetry, where colour confinement is exactly realized as a consequence of the gauge constraints.

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The Geometric approach to (super)-gravity

Group manifold approach (Regge & Ne'eman,1978)
Leonardo Castellani, Riccardo D'Auria & Pietro Fre' :

Powerful tool for constructing extended SUGRA lagrangians
as gauging of "free differential algebras" .

Uses geometric description of superspace.

Pietro Fre' and Peter van Nieuwenhuizen (1981)

Anna Ceresole, YITP@40

Castellani, D'Auria, Fre' + PvN and Alberto Lerda (2005)

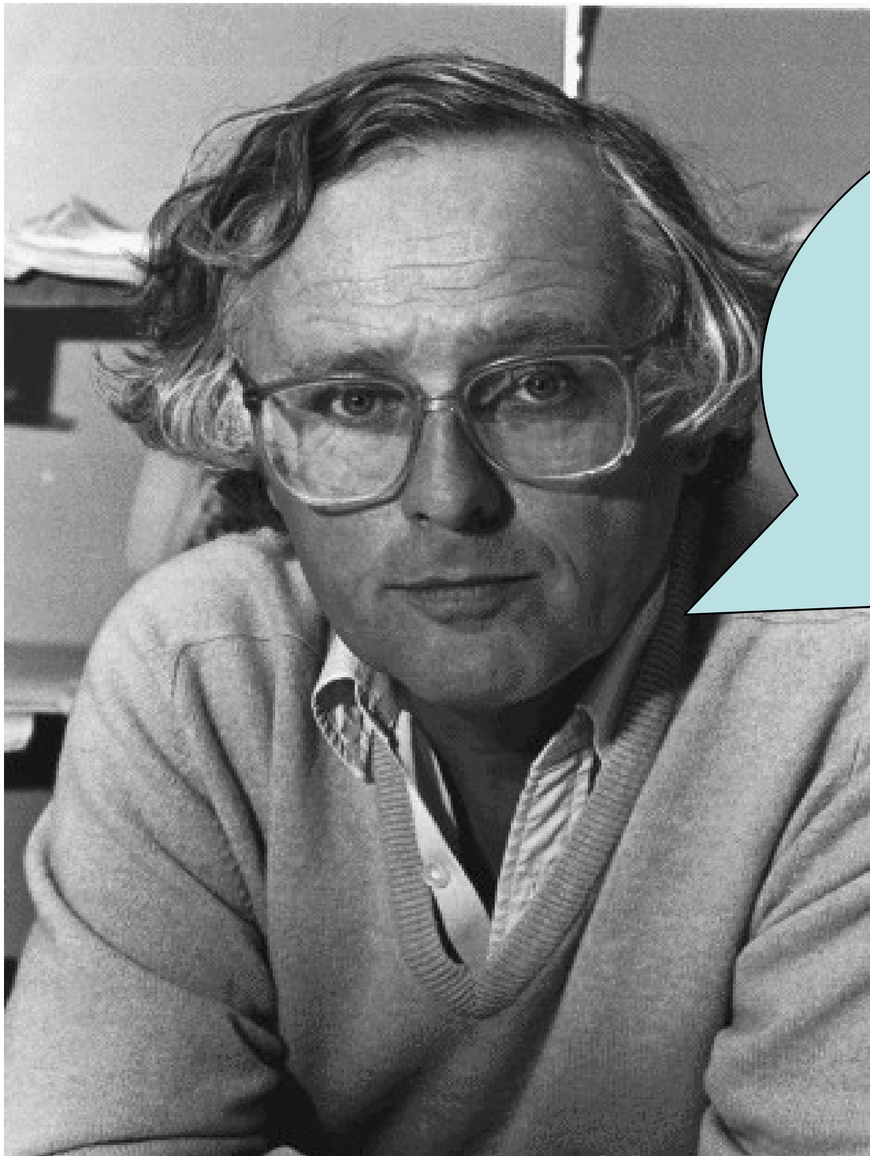
Riccardo D'Auria

(Politecnico of Torino)

Leonardo Castellani and Alberto Lerda (UPO Alessandria)

Anna Ceresole, YITP@40

The PhD at Stony Brook: 1985-1989



Anna!

You want to go to Manhattan
this week-end???

We have a nice little paper to finish!

You don't work hard enough !!!!

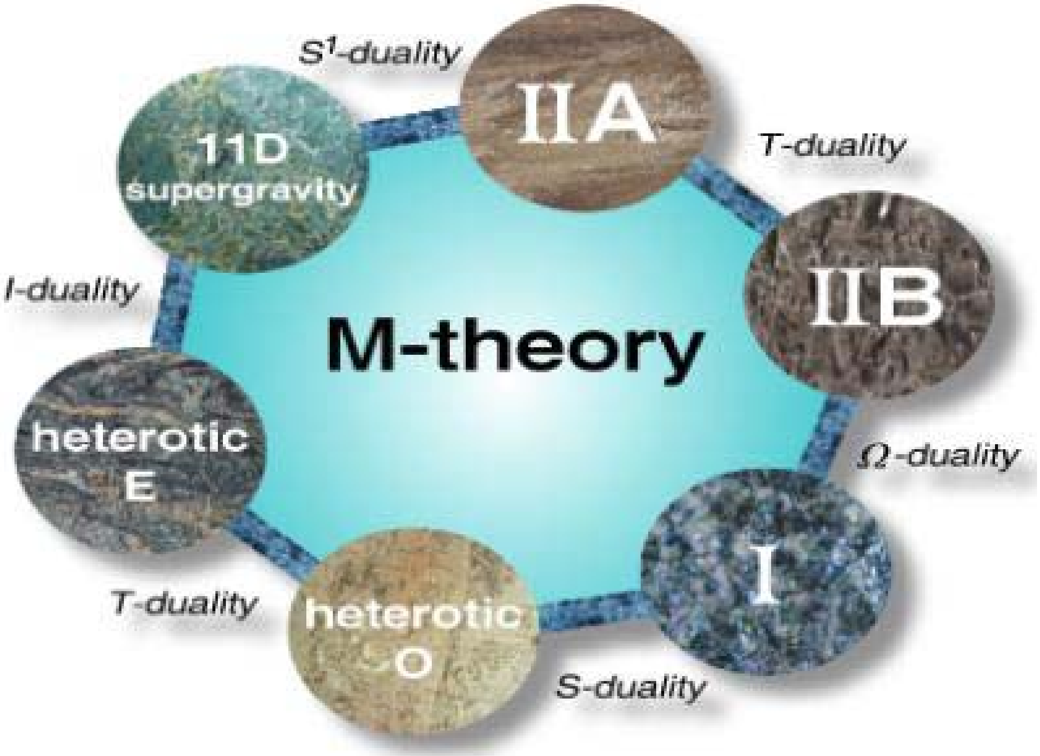
My scientific path

-P v N: “first compute, then think!” in many nice little problems

-Synergy with Mathematics:

from Yang & Mills to Superstring theory

-low energy properties dictated by geometry and topology in fundamental theory (i.e. Yukawa couplings in CY comp.)



Problems:

What to do with higher dimensions? M_4 x X

from Kaluza Klein compactification to Large Extra dim

Degeneracy: How many string/M-theory vacua consistent with Standard Model physics?

from "the space of 2d CFT's" (1987) to the Landscape (2005)

Classical and quantum symmetries?

dualities: reduce degeneracy

How to extract gauge theories and phenomenology?

from particles to strings and p-branes

How to understand cosmology and WMAP?

Evidence for small positive cosmological constant and inflation

Supergravity approximation:
effective field theory limit

(S.Ferrara,D.Freedman, P. van Nieuwenhuizen 1976)

g_s

Quantum effects

Loops…

M-theory, string field theory, quantum geometry,…

Higher curvatures..

α' string effects

Supergravities: (D,N, $\mathcal{M}(\Phi)$)

Can couple to matter multiplets in susy representations

Moduli space : manifold parametrized by scalar fields $\Phi^{\Lambda}(x)$
with sigma-model lagrangian

$$\mathcal{L} = -\frac{1}{2} G_{\Lambda\Sigma} \partial_\mu \Phi^\Lambda \partial_\nu \Phi^\Sigma g^{\mu\nu}$$

D=4,5 N=2 Vector multiplets: (very) special Kahler geometry
Hypermultiplets: quaternionic Kahler geometry

Kinetic matrices, Yukawa couplings, potential are determined by functions on moduli space dictated by geometric properties.

Duality transformations are symplectic transformations on moduli space

D=4 N=1 has less constraints: Kahler geometry

Matter coupled gauged supergravities N=2, D=4,5

Some of global isometries (also R-symmetries) of the matter field Lagrangean are promoted to local symmetries :

$$\partial_\mu \Phi^\Lambda \to D_\mu \Phi^\Lambda = \partial_\mu \Phi^\Lambda + g A_\mu^I K_I^\Lambda(\Phi)$$

Lagrangean is modified, susy rules must be modified, and the theory develops a potential for the scalar fields:

$$\mathcal{L} = \mathcal{L}_0 + g^2 \mathcal{V}(\Phi)$$

The form of this potential is purely dictated by geometric properties of moduli space and can be constructed by geometric methods (from susy rules of fermi fields)

D=4 N=2 and special geometry: symplectic structure (1996)

z^i Complex scalar fields in n vector multiplets (z, λ, A_μ)

$$V(z) = (X^A(z), F_A(z))$$

n+1 Symplectic sections of a Kahler-Hodge manifold

$$F_A(X) = \frac{\partial F(X)}{\partial X^A}$$

F(X) holomorphic prepotential

$$K = -\ln i(X^A \overline{F}_A - \overline{X}^A F_A)$$

Kahler potential

Curvature constraint:

$$R_{i\bar{j}l\bar{m}} = G_{i\bar{j}} G_{l\bar{m}} + G_{i\bar{m}} G_{l\bar{j}} - C_{ilp} \bar{C}_{\bar{j}\bar{m}\bar{p}} G^{p\bar{p}}$$

Black Holes

Sgr A* astro-ph/0310821

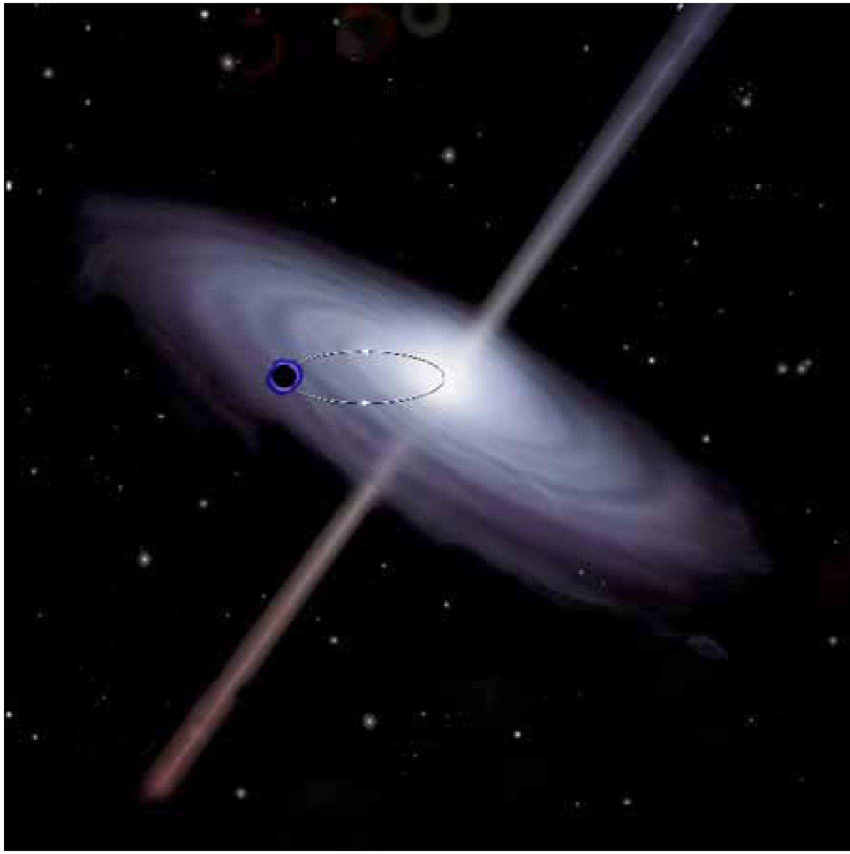
M=3,6 millions of M(sun),

Kerr (J)

D=4 !

Susy?

Extremal?



BH in supergravity

Stationary, spherically symmetric, asymptotically flat, charged,
Non rotating, Dim D, susy N;
Extremal (zero temperature: stable)
BPS bound for lowest energy configurations (M=Q)
Supersymmetric solitons interpolating between the horizon, with geometry AdS2xS2 and Minkowski at infinity

ENTROPY:

- COUNTING thermodynamics
- MICROSCOPIC (Strominger, Vafa,...) Cardy's formula

- MACROSCOPIC (Ferrara, Kallosh, Strominger) Bekenstein-Hawking formula S =A/4 computed from properties of near horizon geometry AdS(2)xS2 of N=2, D=4 BH
- Sen: from variational principle based on entropy function

Attractor mechanism

no-hair theorem: Scalar fields evolve to fixed points at the BH horizon, where and the BH is completely specified by mass M and charges (p,q) $\phi \to \phi^\star(p,q;M)$

-Fixed point are extrema of BH effective potential V_{BH}, which is given in terms of the central charge of N=2 susy algebra
Stable attractors are minima of V_{BH}

$$V_{BH} = |Z|^2 + |D_i Z|^2 \qquad D_i = \partial_i + \tfrac{1}{2}\partial_i K$$

$$Z(z,\bar{z},p,q) = e^{\frac{K}{2}}(X^\Lambda(z)q_\Lambda - F_\Lambda(z)p^\Lambda)$$

Macroscopic entropy (Bekenstein-Hawking) in N=2 for susy BH:

$$D_i Z = 0 \qquad \frac{A}{4} = S = \pi V_{BH}(\phi^\star) = \pi |Z_\star|^2$$

N=2 D=4 Einstein-Maxwell theory with scalars

Gravity coupled with abelian gauge fields + neutral scalar fields in D=4

$$\mathcal{L} = -\frac{R}{2} + g_{i\bar{j}}\partial_\mu z^i \partial_\nu \bar{z}^{\bar{j}} g^{\mu\nu} + \mathrm{Im}\mathcal{N}_{\Lambda\Sigma} F^{\Lambda}_{\mu\nu} F^{\Sigma\,\mu\nu} + \mathrm{Re}\mathcal{N}_{\Lambda\Sigma} F^{\Lambda}_{\mu\nu} (*F^{\Lambda})^{\mu\nu}$$

$\mathcal{N}_{\Lambda\Sigma}(z, \bar{z})$ Vector kinetic matrix

$\int_{S^2} F^{\Lambda} = 4\pi p^{\Lambda}, \quad \int_{S^2} \mathcal{G}_{\Lambda} = 4\pi q_{\Lambda}$ E-m charges

Black-hole ansatz (static, spherically symmetric, charged, asympt. flat):

$$ds^2 = -\mathrm{e}^{2U(r)} dt^2 + \mathrm{e}^{-2U(r)} \left[c^4 \frac{dr^2}{\sinh^4(cr)} + \frac{c^2}{\sinh^2(cr)} \left(d\theta^2 + \sin^2\theta \, d\phi^2 \right) \right]$$

$c^2 = 2ST$ Extremality parameter

BH solutions

The 4D action reduces to a 1-dim effective theory of radial evolution for the warp factor U(r) and the scalar fields *z*(r) :

$$\mathcal{L} = (U'(r))^2 + g_{i\bar{\jmath}} z'^i \bar{z}'^{\bar{\jmath}} + \mathrm{e}^{2U} V_{\mathrm{BH}} - c^2$$

Hamiltonian constraint:

$$(U'(r))^2 + g_{i\bar{\jmath}} z'^i \bar{z}'^{\bar{\jmath}} = \mathrm{e}^{2U} V_{\mathrm{BH}} + c^2$$

Field equations (second order):

$$U'' = \mathrm{e}^{2U} V_{\mathrm{BH}},$$

$$z''^i + \Gamma^i_{jk} z'^j z'^k = \mathrm{e}^{2U} g^{i\bar{\jmath}} \partial_{\bar{\jmath}} V_{\mathrm{BH}},$$

For extremal BH (c=0), you can put the effective action in BPS form:

$$S = \int dr \left[\left(U' \pm e^U W\right)^2 + \left|z^{i\prime} \pm 2e^U g^{i\bar{\jmath}} \partial_{\bar{\jmath}} W\right|^2 \mp 2 \frac{d}{dr} \left(e^U W\right) \right]$$

and obtain first order "flow" equations:

$$U' = \pm e^U W,$$

$$z'^i = \pm 2e^U g^{i\bar{\jmath}} \partial_{\bar{\jmath}} W,$$

$$V_{\mathrm{BH}} = W^2 + 4 g^{i\bar{\jmath}} \partial_i W \partial_{\bar{\jmath}} W$$

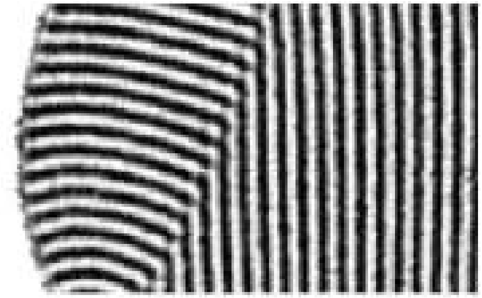
BH effective potential given in terms of superpotential W.

For an N=2 susy theory, W=Z and the flow equations derive from vanishing of fermionic susy transformation rules:

$\delta\psi=0$, $\delta\lambda=0$

Extremality= preserved susy : Killing spinor equations!

Domain Wall solutions



- d-dim objects separating (d+1)-dim spacetime in two regions corresponding to different vacua

- AdS(5)/CFT(4) correspondence

RG flows of deformations of YM theories are dual to SG domain wall solutions (DW/QFT correspondence)

Freedman, Gubser, Pilch, Warner hep-th/9904017

- Brane world cosmological scenarios: Randall Sundrum models for gravity trapping. Can they be accomodated into a susy theory? To answer must study the scalar potential

- Study of the Landscape of string vacua

A.C., Dall'Agata, Giryavets, Kallosh and Linde hep-th/0605266

$$ds^2 = \mathrm{e}^{2U(r)}\hat{g}_{ij}dx^i dx^j + \mathrm{e}^{pU(r)}dr^2$$

$$\begin{aligned}
dS_3 \quad \hat{g}_{ij}dx^i dx^j &= -dt^2 + \mathrm{e}^{2\sqrt{\Lambda}t}(dx_1^2 + dx_2^2) \qquad \Lambda > 0, \\
AdS_3 \quad \hat{g}_{ij}dx^i dx^j &= d\tau^2 + \mathrm{e}^{-2\sqrt{-\Lambda}\tau}(-dt^2 + dx^2) \qquad \Lambda < 0, \\
M_3 \quad \hat{g}_{ij}dx^i dx^j &= -dt^2 + dx_1^2 + dx_2^2, \qquad \Lambda = 0.
\end{aligned}$$

$$\mathcal{L} = \mathrm{e}^{2U(r)}\left[(U'(r))^2 - g_{i\bar{\jmath}}z'^i \bar{z}'^{\bar{\jmath}} - \mathrm{e}^{2U}V_{\mathrm{DW}} + \Lambda\right]$$

$$U'' = -e^{2U} V_{\rm DW} - \frac{1}{3} g_{i\bar{\jmath}} z'^i \bar{z}'^{\bar{\jmath}},$$

$$z''^i + \Gamma^i_{jk} z'^j z'^k = U' z'^i + e^{2U} g^{i\bar{\jmath}} \partial_{\bar{\jmath}} V_{\rm DW}.$$

$$V_{\rm DW} = -W^2 + \frac{4}{3} \frac{1}{\gamma^2} g^{i\bar{\jmath}} \partial_i W \partial_{\bar{\jmath}} W,$$

$$\gamma \equiv \sqrt{1 + e^{-2U} \frac{\Lambda}{W^2}}.$$

$$U' = \pm e^U \gamma(r) W,$$

$$z'^i = \mp e^U \frac{2}{\gamma^2} g^{i\bar{\jmath}} \partial_{\bar{\jmath}} W,$$

$$c^2 = \Lambda \geq 0$$

- However: attractor mechanism is more general. It has been observed also for NON-BPS BH solutions (no susy preserved):

Where do the first order flow equations come from???

QUESTION:

Can I still have first order equations for extremal (T=0)

but non susy BH (no Killing spinors), which would be related to the attractor equations???

ANSWER: use knowledge from DOMAIN WALL solutions of (super)gravity theories since also DW can be reduced to 1-dim effective theory of radial evolution very similar to BH

(DW curvature related to extremality parameter)

Use (Skenderis & Townsend) on DW stability and…

Fake Supergravity

initiated by Freedman et al

hep-th/0410126 A. Celi, A.C. G. Dall'Agata, A. Van Proeyen and M. Zagermann

Gravitational theories in d-dimensions that are not susy, but contain some "fake BPS equations" for the metric and scalar fields that are of first order and solve the ordinary Einstein and scalar equations of motion.

The scalar potential can formally be written in terms of a superpotential (matrix)

Applications: curved DW in SUGRA, cosmological solutions (Skenderis Townsend), add vectors, Black holes, superstars…

Naturally incorporates 1st order flow equations and stability form of potential

Caution when you have many scalar and in particular hypermultiplets

"AdS_d–sliced" Domain Walls

Action:

$$S = \int d^{d+1}x\sqrt{-g}\left[\frac{1}{2\kappa^2}R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right]$$

Geometry:

$$\mathrm{d}s^2 = \mathrm{e}^{2U(r)} g_{mn}(x)\,\mathrm{d}x^m\,\mathrm{d}x^n + \mathrm{d}r^2$$

Potential:

$$V(\phi) = \frac{2(d-1)^2}{\kappa^2}\left(\frac{1}{2}\,\mathrm{Tr}\right)\left[\frac{1}{\kappa^2}(\partial_\phi \mathbf{W})^2 - \frac{d}{d-1}\mathbf{W}^2\right]$$

(fake) BPS equations

$$
\begin{aligned}
\left[\nabla_\mu + \gamma_\mu \mathbf{W}\right] \epsilon &= 0 \\
\left[\gamma^\mu \nabla_\mu \phi - \frac{2(d-1)}{\kappa^2} \partial_\phi \mathbf{W}\right] \epsilon &= 0
\end{aligned}
$$

$$\nabla_\mu \epsilon = \left(\partial_\mu + \tfrac{1}{4}\, \omega_\mu{}^{\nu\rho} \gamma_{\nu\rho}\right) \epsilon$$

Flow Equations for Non-BPS extremal black holes

with Gianguido Dall'Agata, JHEP03 (2007)110

Aim: understand origin of attractor equations for non susy black holes

Use similarities between domain wall and black hole solutions of (super) gravity theories, in particular D=4, N=2

Construct an interesting class of stable extremal non-BPS BH : scalar fields satisfy first order flow equations (like in BPS case!)

Fake Superpotential W rather than central charge Z appears in BH potential V(phi) and drives flow equations

Give general procedure for finding such BH and identifying their (fake) superpotential W

Outlook

Conference "30 years of Supergravity" (Paris, October 2006):

H. Ooguri's summary talk: Much of what has been done in these 30 years in string theory can be classified as supergravity

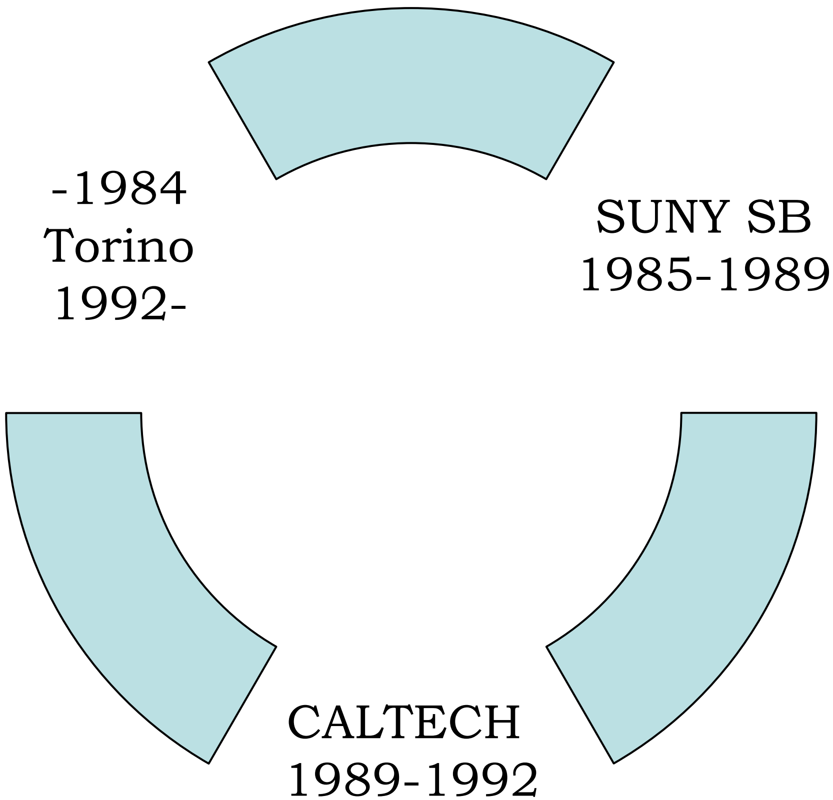
1) Construction: D=4,5 N=2 + matter and various type of solutions

2) Properties: special geometry, quaternionic geometry; Calabi-Yau mirror symmetry; attractor mechanism

3) Applications: M-theory, phenomenology, cosmology, topological field theories,

also quantum information (Duff, Kallosh & Linde, Ferrara)?

Summary



Fre', Nicolai, Castellani, D'Auria, Ferrara, Van Nieuwenhuizen, Lerda, Pizzochero, Frau, Pilch, Bouwknegt, Mc Carthy, Lerche, Regge, Rasetti, Andrianopoli, Trigiante, Dall'Agata, Zagermann, Celi, Van Proeyen, Kallosh, Linde …

X (nice little) papers
Y collaborators
Z conferences
W students
and…

Maria (1996), Clara & Margherita (2001)

…I hope my students (or my children) will go to Stony Brook!

- **Thanks Peter van N !!!**
- **Thanks YITP folks!**
- **Happy 40th birthday YITP!!!**

Summary of AdS(d)-sliced DW

$$S = \int d^{d+1}x\sqrt{-g}\left[\frac{1}{2\kappa^2}R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right],$$

$$\mathrm{d}s^2 = \mathrm{e}^{2U(r)}g_{mn}(x)\mathrm{d}x^m\mathrm{d}x^n + \mathrm{d}r^2$$

$$V(\phi) = \frac{2(d-1)^2}{\kappa^2}\left(\frac{1}{2}\mathrm{Tr}\right)\left[\frac{1}{\kappa^2}(\partial_\phi\mathbf{W})^2 - \frac{d}{d-1}\mathbf{W}^2\right].$$

$$[\nabla_\mu + \gamma_\mu\mathbf{W}]\epsilon = 0,$$

$$\left[\gamma^\mu\nabla_\mu\phi - \frac{2(d-1)}{\kappa^2}\partial_\phi\mathbf{W}\right]\epsilon = 0,$$

where $\nabla_\mu\epsilon = (\partial_\mu + \frac{1}{4}\omega_\mu{}^{\nu\rho}\gamma_{\nu\rho})\epsilon.$

Domain walls in the landscape (N=1,D=4)

A.C., Dall'Agata, Giryavets, Kallosh and Linde hep-th/0605266

Counting problem: how many vacua are there in CY compactifications of IIB strings with fluxes?

$N_{vac} \sim 10^{500}$

1) Must study extrema of scalar field potential V.

2) Study also the shape of mountain ridges (interpolating walls)

??? Cosmology: (Linde,1982) inflation may divide our universe into many domains corresponding to different metastable vacua (multiverse)
How do DW form?
What are the properties of domain walls separating different vacua?
How large a fraction of the universe do they contain?

hep-th/0104056 A.C., G. Dall'Agata, R. Kallosh and A. Van Proeyen

Curved BPS domain walls in matter coupled 5D, N=2 SUGRA

Curved walls need hypermultiplets

Fake SG is the effective description of true SG when restricting to one flowing scalar supporting a given BPS domain wall

Fake SG good description only locally upon chosing 1)set of adapted coordinates on scalar manifold to reduce problem to 1 flowing scalar

2) Gauge fixing of SU(2) connection for hypers

D=5, N=2 with vector and hyper-multiplets

$$e^{-1}\mathcal{L} = -\frac{1}{2}R - \frac{1}{2}g_{xy}\partial_\mu\varphi^x\partial^\mu\varphi^y - \frac{1}{2}g_{XY}\partial_\mu q^X\partial^\mu q^Y - g^2\mathcal{V}(\varphi, q),$$

$$\mathcal{V}(\phi, q) = g^2\{2W^{\tilde{a}}W^{\tilde{a}} - \left[2P_{ij}P^{ij} - P_{\tilde{a}ij}P_{\tilde{a}}^{ij}\right] + 2\mathcal{N}_{iA}\mathcal{N}^{iA}\}$$

$$\begin{aligned}
\delta_\varepsilon\psi_{i\mu} &= \mathcal{D}_\mu\varepsilon_i + \frac{i}{\sqrt{6}}g\,\gamma_\mu\varepsilon^j P_{ij},\\
\delta_\varepsilon\lambda_i^{\tilde{a}} &= -\frac{i}{2}f_{\tilde{x}}^{\tilde{a}}\gamma^\mu\varepsilon_i\,\mathcal{D}_\mu\phi^{\tilde{x}} + g\varepsilon^j P_{ij}^{\tilde{a}} + gW^{\tilde{a}}\varepsilon_i,\\
\delta_\varepsilon\zeta^A &= -\frac{i}{2}f_{iX}^A\gamma^\mu\varepsilon^i\mathcal{D}_\mu q^X + g\varepsilon^i\mathcal{N}_i^A.
\end{aligned}$$

$$P_{ij} \equiv h^I P_{I\,ij}, \qquad P_{ij}^{\tilde{a}} \equiv h^{\tilde{a}I}P_{I\,ij}, \qquad W = \sqrt{\tfrac{1}{3}P_{ij}P^{ij}}$$

$$W^{\tilde{a}} = \frac{\sqrt{6}}{4}h^I K_I^{\tilde{x}} f_{\tilde{x}}^{\tilde{a}}, \qquad \mathcal{N}^{iA} = \frac{\sqrt{6}}{4}h^I K_I^X f_X^{Ai}.$$

Stability form of potential and BPS equations

$$V = -6W^2 + \frac{9}{2} g^{\Lambda\Sigma} \partial_\Lambda W \partial_\Sigma W \, ,$$

$$gW = \left| \frac{a'}{a} \right| = \pm \frac{a'}{a} \, .$$

$$\phi^{\Lambda'} = \mp 3g \, g^{\Lambda\Sigma} \partial_\Sigma W \, .$$

$$E = \int_{-\infty}^{+\infty} dx^5 \, a^4 \left[\frac{1}{2} \left(\phi^{\Lambda'} \mp 3g \partial^\Lambda W \right)^2 - 6 \left(\frac{a'}{a} \mp gW \right)^2 \right]$$
$$\mp 3g \int_{-\infty}^{+\infty} dx^5 \frac{\partial}{\partial x^5} \left(a^4 W \right) + 4 \int_{-\infty}^{+\infty} dx^5 \frac{\partial}{\partial x^5} \left(a^3 a' \right) \, .$$

Domain Walls in N=1, D=4 Supergravity with chiral multiplets

Bose action:

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{2} R - g_{i\bar{\jmath}} \partial_\mu z^i \partial^\mu \bar{z}^{\bar{\jmath}} - V(z, \bar{z}) \right)$$

$$V = e^K \left(g^{i\bar{\jmath}} D_i W \overline{D_j W} - 3|W|^2 \right)$$

Potential "stability form"

Boucher, Skenderis Townsend

$W(z)$ superpotential K Kahler potential

$$D_i = \partial_i + \partial_i K$$

We consider the flat spacelike DW ansatz:

$$ds^2 = a^2(r)(-dt^2 + dx^2 + dy^2) + dr^2$$

- The susy transformations give rise to the BPS conditions

$$\begin{aligned}
\delta\psi_{rL} &= \partial_r\epsilon_L + \frac{i}{2}A^B_r\epsilon_L + \frac{1}{2}e^{K/2}W\gamma_r\epsilon_R \\
\delta\psi_{iL} &= \frac{a'}{2}\gamma_{\underline{i}}\gamma_{\underline{r}}\epsilon_L + \frac{1}{2}e^{K/2}W\gamma_i\epsilon_R \\
\delta\chi_L &= \frac{1}{2}\gamma^r z^{i'}(r)\epsilon_R - \frac{1}{2}e^{K/2}g^{i\bar{j}}\overline{D_j W}\epsilon_L
\end{aligned}$$

$$A^B_\mu \equiv \frac{i}{2}\left(\partial_\mu \bar{z}^{\bar{\imath}}\partial_{\bar{\imath}}K - \partial_\mu z^i\partial_i K\right) \qquad Z = e^{K/2}W$$

BPS equations

$$\frac{a'}{a} = \pm|\mathcal{Z}| \qquad z^{i\prime} = \mp 2 g^{i\bar{\jmath}} \partial_{\bar{\jmath}} |\mathcal{Z}|$$

Coincide with old result (Cvetic, Griffies, Rey) for one scalar field

These are gradient flow equations for warp factor and scalar fields encoding the geometry of domain wall solutions

All properties driven by holomorphic superpotential Z=exp(K/2)W

Summary of Results

- Static BPS domain wall solutions of N=1, D=4 SG connecting minima, maxima and saddle points of scalar field potential
- In particular AdS or Mink vacua with unbroken susy
- General gradient flow equations in generic models with arbitrary potential, Kahler potential and superpotential and number of moduli
- By uplifting , obtain metastable de Sitter vacua and then study vacuum decay
- Sinks in the landscape are channels of irreversible vacuum decay that may swallow probabilities and lead to new view of string cosmology