Deformation of Dijkgraaf-Vafa Relation via Spontaneously Broken N=2 Supersymmetry

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with K. Fujiwara & M. Sakaguchi

- The model
- Spontaneous Partial Breaking of $\mathcal{N} = 2$ Supersymmetry
- Mass Spectrum $\cdots$
- Computation & Structure of $W_{\rm eff}$

The Model and Limiting Cases

$\mathcal{N}=1$ large	interpolates by e, m, ξ	small $\mathcal{N}=2$
$S_W^{\mathcal{N}=1}$	$S_{\rm FIS}^{\mathcal{N}=2}$	$S_{\rm SYM}^{\mathcal{N}=2}$

μ_0

μ $\mathcal{F}^{(\rm eff)}_{\rm m.m.}(S_i)$, $W_{\rm eff}$

DV relation

R.S. & matrix model description

DV, CIV, CDSW, H.I.-Morozov '02

$\cdots$

$\mu_{\rm IR}$

$\mathcal{F}^{\rm eff}_{\rm SW}(\phi_i)$ SW

R.S. description

Gorsky et.al., Martinec-Warner, H.I.-Morozov '95 $\cdots$

$$S_{\rm FIS}^{\mathcal{N}=2} = \int d^4x d^4\theta \left[-\frac{i}{2}{\rm Tr}\left(\bar{\Phi}e^{adV}\frac{\partial\mathcal{F}(\Phi)}{\partial\Phi} - h.c.\right) + \xi V^0\right] + \left[\int d^4x d^2\theta \left(-\frac{i}{4}\frac{\partial^2\mathcal{F}(\Phi)}{\partial\Phi^a\partial\Phi^b}\mathcal{W}^a\mathcal{W}^b + e\Phi^0 + m\frac{\partial\mathcal{F}(\Phi)}{\partial\Phi^0}\right) + h.c.\right]$$

cf. $S_W^{\mathcal{N}=1} = \int d^4x d^4\theta {\rm Tr}\bar{\Phi}e^{adV}\Phi + \left[\int d^4x d^2\theta {\rm Tr}\left(i\tau\mathcal{W}\mathcal{W} + W(\Phi)\right) + h.c.\right]$

$\mathcal{N}=2$ Supersymmetry with (Bare) Superpotential

- Strategy to get $\mathcal{N}=2$:

$$\boldsymbol{\lambda}_i^a = \begin{pmatrix} \lambda^a \\ \psi^a \end{pmatrix} \rightarrow \boldsymbol{\lambda}^{ia} = \begin{pmatrix} \psi^a \\ -\lambda^a \end{pmatrix} = R\boldsymbol{\lambda}_i^a R^{-1}$$

$R\delta^{(1,\xi)}_{\eta_1=\theta} R^{-1} \equiv \delta^{(2,-\xi)}_{\eta_2=\theta}$ so that $0 = \delta^{(2,\xi)}_{\eta_2=\theta} S(\xi)$ follows from $R\delta^{(1,\xi)}_{\eta_1=\theta} S(\xi) R^{-1} = 0$

- Take a generic superpotential and a gauge kinetic function and impose R invariance:

  The solution $W = eA^0 + m\mathcal{F}_0, \quad \tau_{ab} = \mathcal{F}_{ab}$

- Transformation laws:

$$\boldsymbol{\delta\lambda}_J{}^a = i(\boldsymbol{\tau} \cdot \boldsymbol{D}^a)_J{}^K \boldsymbol{\eta}_K + \cdots \ .$$

$$\boldsymbol{D}^a = \hat{\boldsymbol{D}}^a - \sqrt{2} g^{ab^*} \partial_{b^*} \left(\boldsymbol{\mathcal{E}} A^{*0} + \boldsymbol{\mathcal{M}} \mathcal{F}_0^* \right) .$$

fermion bilinears

$$\boldsymbol{\mathcal{E}} = (0,\ -e,\ \xi)\ , \quad \boldsymbol{\mathcal{M}} = (0,\ -m,\ 0),$$

Spontaneous Partial Breaking of $\mathcal{N}=2$ Supersymmetry

• basic mechanism: $\left\{\bar{Q}^j_{\dot\alpha}, \mathcal{S}^m_{\alpha i}(x)\right\} = 2(\sigma^n)_{\alpha\dot\alpha}\delta_i{}^j T^m_n(x) + (\sigma^m)_{\alpha\dot\alpha}C_i{}^j$

· $C_i{}^j$: not a VEV but follows simply from the algebra.

The model predicts:

• $C_i{}^j = 4m\xi\tau_1 \xrightarrow{90^{\mathrm{o}}\mathrm{rot.}} 4m\xi\tau_3$ The scalar ptl VEV $\langle\langle\mathcal{V}\rangle\rangle = \mp 2m\xi = 2|m\xi|$

• $\therefore$ Half of the supercharges annihilates the vacuum while the remaining half takes $\infty \sim |m\xi| \int d^4x$ matrix elements.

$\therefore$ Partial Breaking of Extended SUSY is a Reality.

A Few Tree Properties

$$\langle\langle \mathcal{F}_{\underline{jj}} \rangle\rangle = -2(\frac{e}{m} \mp i\frac{\xi}{m}) = -2\zeta \text{ ; the vac. condition}$$

$$\langle\langle g_{\underline{jj}} \rangle\rangle = \mp 2\frac{\xi}{m},$$

$$\langle\langle \boldsymbol{D}^{\underline{j}} \rangle\rangle = \frac{m}{\sqrt{N}} \begin{pmatrix} 0 \\ -i \\ \pm 1 \end{pmatrix}$$

- NG fermion $\frac{1}{\sqrt{2}}(\lambda^o + \psi^0)$ resides in the overall $U(1)$ part but not decoupled
- Breaking pattern of gauge symmetry: $\deg \mathcal{F} = n + 2$

$$U(N) \to \prod_{i=1}^{n} U(N_i) \quad \text{with} \quad \sum_{i=1}^{n} N_i = N$$

cf. partition of N eigenvalues

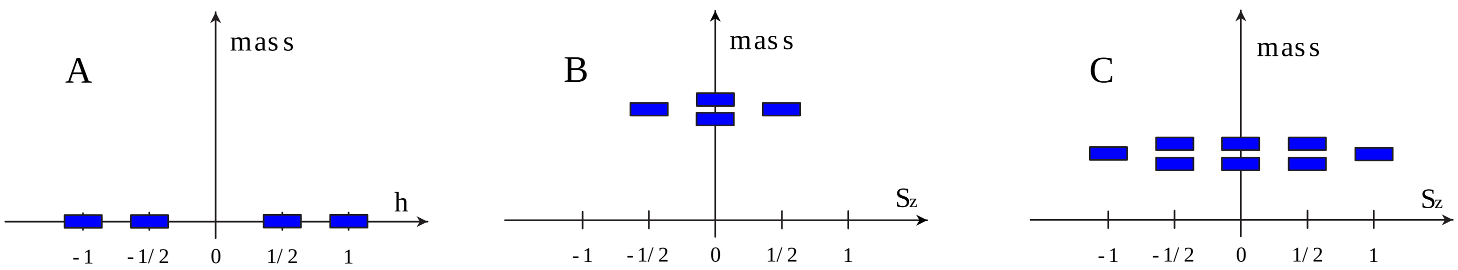
Mass Spectrum

$$\text{index labelling} \quad a, b, \ \cdots = \begin{cases} \alpha, \beta, \cdots \text{ for unbroken generators} \\ \mu, \nu, \cdots \text{ for broken generators} \end{cases}$$

- the table

field	mass	label	# of polarization states
v_m^α	0	A	$2d_u(d_u \equiv \dim \prod_i U(N_i))$
v_m^μ	$\lvert \frac{1}{\sqrt{2}} f_{\mu \underline{i}}^{\nu} \lambda^{\underline{i}} \rvert$	C	$3(N^2 - d_u)$
$\frac{1}{\sqrt{2}}(\lambda^\alpha \pm \psi^\alpha)$	0	A	$2d_u$
$\frac{1}{\sqrt{2}}(\lambda^\alpha \mp \psi^\alpha)$	$\lvert m \langle\langle g^{\alpha\alpha} \rangle\rangle \langle\langle \mathcal{F}_{0\alpha\alpha} \rangle\rangle \rvert$	B	$2d_u$
$\boldsymbol{\lambda}_I^\mu$	$\lvert \frac{1}{\sqrt{2}} f_{\mu \underline{i}}^{\nu} \lambda^{\underline{i}} \rvert$	C	$4(N^2 - d_u)$
A^α	$\lvert m \langle\langle g^{\alpha\alpha} \rangle\rangle \langle\langle \mathcal{F}_{0\alpha\alpha} \rangle\rangle \rvert$	B	$2d_u$
$\mathcal{P}_\mu^{\tilde{\mu}} A^\mu$	$\lvert \frac{1}{\sqrt{2}} f_{\mu \underline{i}}^{\nu} \lambda^{\underline{i}} \rvert$	C	$N^2 - d_u$

- $\mathcal{N} = 1$ supermultiplet



Fermionic Shift Symmetry of $S_W^{\mathcal{N}=1}$ and W_{eff}

$$S \equiv -\frac{1}{32\pi^2}\text{Tr}\mathcal{W}^\alpha\mathcal{W}_\alpha \quad \ni \text{Tr}\lambda^\alpha\lambda_\alpha \qquad \text{gluino condensate variables} \qquad \text{DV, CDSW}$$

$$w^\alpha \equiv \frac{1}{4\pi}\text{Tr}\mathcal{W}^\alpha \qquad\qquad U(N) \text{ unbroken for simplicity}$$

- Introduce "grassmann coordinates" ψ^α

$$\begin{aligned}\hat{\mathcal{S}} &= -\frac{1}{2}\text{Tr}\left(\frac{1}{4\pi}\mathcal{W}^\alpha - \psi^\alpha\mathbf{1}\right)\left(\frac{1}{4\pi}\mathcal{W}_\alpha - \psi_\alpha\mathbf{1}\right)\\ &= S + \psi w - \frac{1}{2}\psi\psi N\end{aligned}$$

- The fermionic shift symmetry $\quad\rightsquigarrow\quad$ decoupling of overall $U(1)$

  acts as $\quad \delta\hat{\mathcal{S}} = \epsilon\frac{d}{d\psi}\hat{\mathcal{S}}$

- $^\exists\mathcal{F}$ s.t.

$$W_{\text{eff}} = \int d^2\psi\mathcal{F}(\hat{\mathcal{S}}) = N\frac{\partial\mathcal{F}(S)}{\partial S} + \frac{\partial^2\mathcal{F}(S)}{\partial S^2}ww \qquad \text{DV relation}$$

- Remnant of the 2nd supersymmetry of $S_{\text{FIS}}^{\mathcal{N}=2}$

$W_{\rm eff}$ of $S_{\rm FIS}^{\mathcal{N}=2}$; Deformation of DV Formula

So far the matter induced part only

- summary of our understanding;

$$W_{\rm eff}^{(h-1)} = N\frac{\partial F^{(h-1)}}{\partial S} + \frac{\partial^2 F^{(h-1)}}{\partial S^2} w^\alpha w_\alpha - \frac{16\pi^2 i m g_3}{m g_2}\left(\frac{\partial F^{(h-1)}}{\partial S}\right)\frac{S}{m} + W_2^{(h-1)}$$

h : # of index loops
$F^{(h-1)}$; the $(h-1)$ loop contribution to the planar free energy of the matrix model
$W_2^{(h-1)}$; replace one coupling constant mg_ℓ by $\frac{16\pi^2 i g_{\ell+1} S}{Nh}$, for $\ell \geq 3$ in the 1st term

- basis of our argument;
  - · integrate $\bar{\Phi}$ out
  - · propagator

$$\Delta(p,\pi) = \int_0^\infty ds e^{-s(p^2+m'+\frac{1}{2}ad\mathcal{W}^\alpha\pi_\alpha - ig_3' M)}$$

$$M_{abcd} = (\mathcal{W}\mathcal{W})_{da}\delta_{bc} + (\mathcal{W}\mathcal{W})_{bc}\delta_{da} + \mathcal{W}_{da}\mathcal{W}_{bc}$$

  - · vertices

type I. $m\frac{g_k a^k}{k!}\mathrm{Tr}\Phi^k, \quad k = 3,\ldots,n+1$

type II. $-\frac{i}{4}\sum_{s=0}^{k-1}\frac{g_k a^{k-1}}{k!}\mathrm{Tr}(\mathcal{W}\Phi^s\mathcal{W}\Phi^{k-1-s}),$
$k = 4,\ldots,n+1$

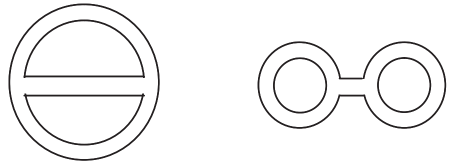
cf. Grisaru et. al.
· universal to every $(h-1)$-loop planar diagram up to c.c. & symmetric factors

⋮

· sample diagram :

$$\int \frac{d^4p_1}{(2\pi)^4}\frac{d^4p_2}{(2\pi)^4} d^2\pi_1 d^2\pi_2 \Delta(p_1,\pi_1)\Delta(p_2,\pi_2)\Delta(-p_1-p_2,-\pi_1-\pi_2)$$
$$= \int ds_1 ds_2 ds_3 e^{-(\sum_i s_i)m'}\{3NS^2 + 6Sw^\alpha w_\alpha - ig_3'(\sum_i s_i)S^3\}$$

insert two more $\mathcal{W}$ $\times\times$



Generalized Konishi Anomaly Equation

$$R(z) = -\frac{1}{64\pi^2}\left\langle \mathrm{Tr}\mathcal{W}^\alpha\mathcal{W}_\alpha \frac{1}{z-\Phi}\right\rangle_\Phi, \qquad T(z) = \left\langle \mathrm{Tr}\frac{1}{z-\Phi}\right\rangle_\Phi$$

$$R(z)^2 = W'(z)R(z) + \frac{1}{4}f(z),$$
$$2R(z)T(z) = W'(z)T(z) + \frac{1}{4}c(z) + 16\pi^2 i\mathcal{F}'''(z)R(z) + \frac{1}{4}\tilde{c}(z)$$

$f(z)$ and $c(z)$ are polynomials of degree $n-1$ in z and $\tilde{c}(z)$ is a polynomial of degree $n-2$.


String Theory