Probing a parton's transverse motion

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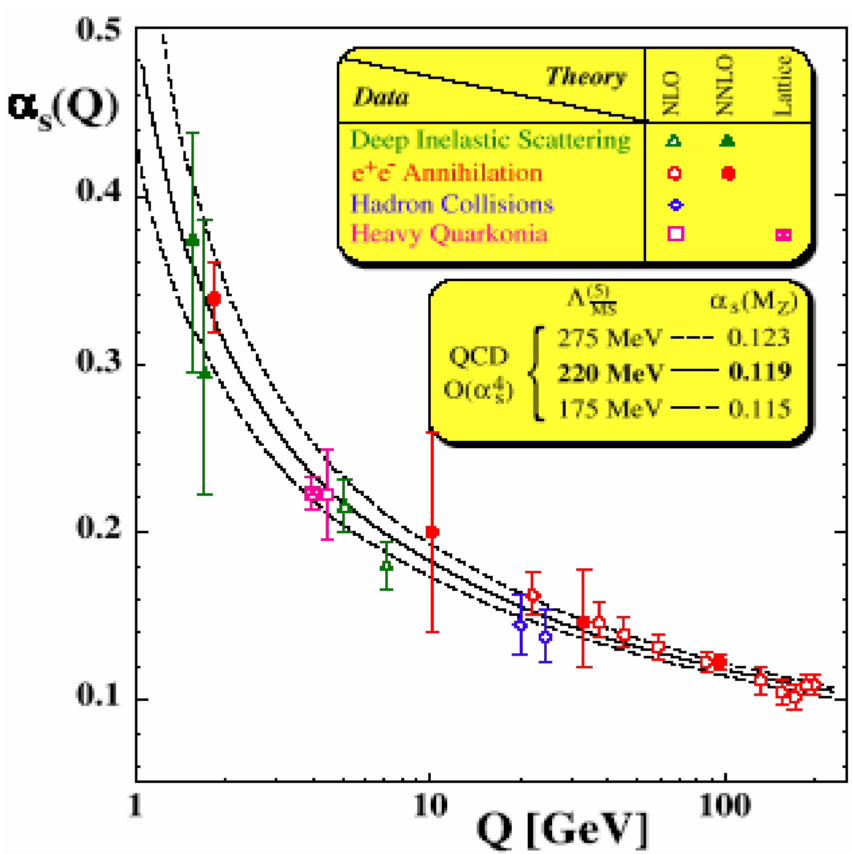
QCD and Asymptotic Freedom

Quantum Chromodynamics (QCD) is a SU(3) color gauge theory for describing the strong interaction physics



Nobel prize of 2004

$$\alpha_s(\mu_2) = \frac{\alpha_s(\mu_1)}{1 - \frac{\beta_1}{4\pi}\alpha_s(\mu_1)\ell n\left(\frac{\mu_2^2}{\mu_1^2}\right)}$$

$$\equiv \frac{4\pi}{-\beta_1 \ell n\left(\frac{\mu_2^2}{\Lambda_{QCD}^2}\right)} \Rightarrow 0 \text{ as } \mu_2 \to \infty$$

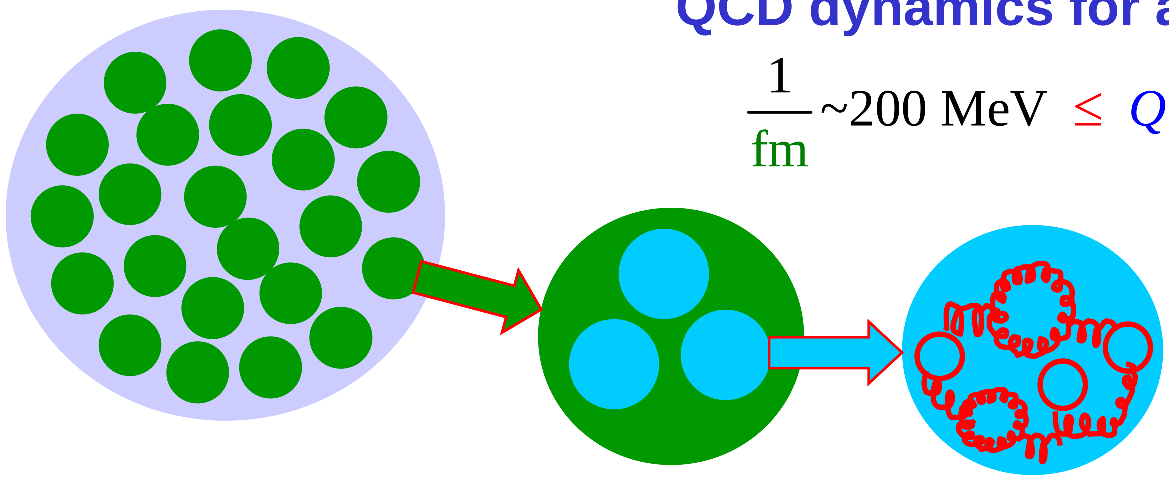
QCD perturbation theory works for calculating strong interaction dynamics at a distance scale much less than $1/\Lambda_{QCD}$

Hadronic matter

- nuclei
- hadrons
- partons

QCD dynamics for a scale

$$\frac{1}{\mathrm{fm}} \sim 200\ \mathrm{MeV} \leq Q < \infty$$



❑ Question:

How to understand hadron properties and structure in terms of partonic dynamics?

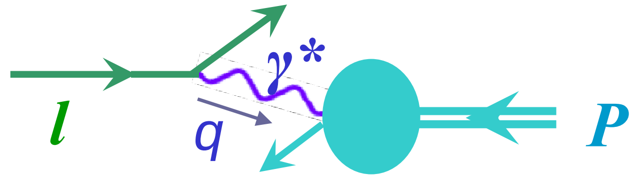
How to "see" the partonic dynamics?

❑ "snap shot" of a hadron

- seen by a hard probe at a short-distance

QCD Factorization

- **Lepton-hadron deep inelastic scattering (DIS):**

$\sigma_{\ell A}\left(x_B, Q^2\right) \propto$ |  $|^2$

$Q^2 = -q^2$

$x_B = Q^2 / 2p.q$

Typical hadronic scale: 1/R ~ 1 fm^{-1} ~ Λ_{QCD}

Energy exchange in hard collisions: Q >> Λ_{QCD}

⇨ pQCD works at $\alpha_s(Q)$, but not at $\alpha_s(1/R)$

- **PQCD can be useful iff physics at different scales can be factorized:**

$$\sigma_{\text{phy}}(Q, 1/R) \sim \hat{\sigma}(Q) \otimes \varphi(1/R) + O(1/QR)$$

Measured: σ_{phy}

Short-distance: $\hat{\sigma}(Q)$

Long-distance: $\varphi(1/R)$

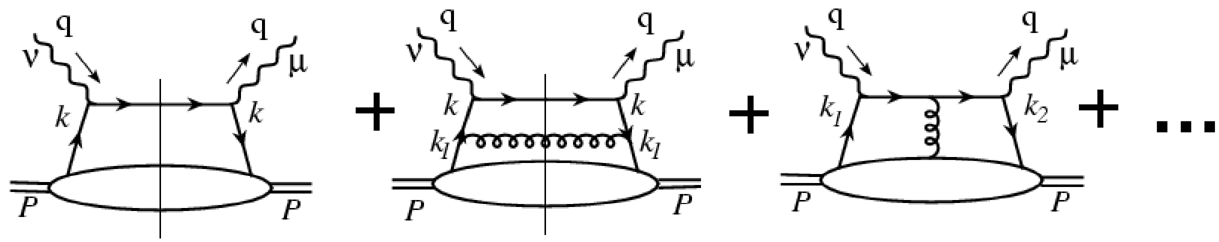
Power corrections: $O(1/QR)$

⇨ **Factorization**

"Long-lived" parton state

- **Feynman diagram representation of the scattering:**

$$\sigma_{\rm DIS} \propto W^{\mu\nu} \propto$$



- **Perturbative pinched poles:**

$$W^{\mu\nu} \propto \int d^4k \; \mathrm{H}^{\mu\nu}(Q,k)\left(\frac{1}{k^2+i\varepsilon}\right)\left(\frac{1}{k^2-i\varepsilon}\right)\mathrm{T}\left(k,\frac{1}{R}\right)$$

Dominated by a region where $k^2 \sim 0$ – "long-lived" parton state

- **Perturbative factorization:**

$$W^{\mu\nu} \approx \int \frac{dx}{x} d^2k_T \, \mathrm{H}^{\mu\nu}(Q,k^2=0) \int dk^2 \left(\frac{1}{k^2+i\varepsilon}\right)\left(\frac{1}{k^2-i\varepsilon}\right)\mathrm{T}\left(k,\frac{1}{R}\right)$$

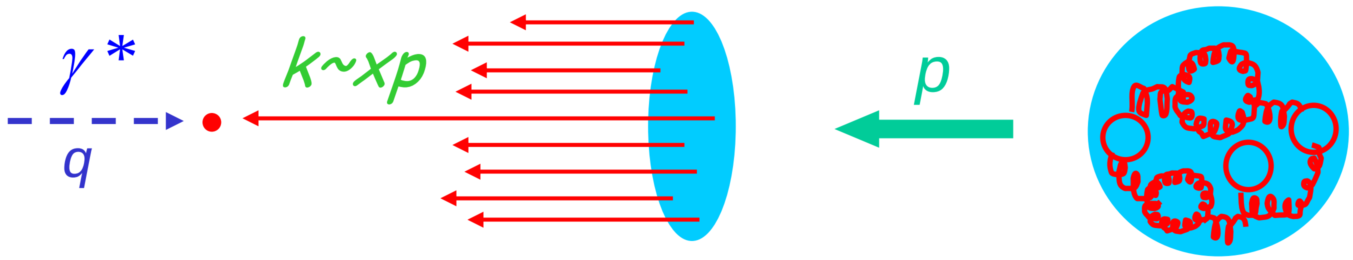
Short-distance

Nonperturbative matrix element

Collinear factorization

❑ **Collinear approximation:**

$$k^\mu \approx xp^\mu + \frac{k_T^2}{2xp\cdot n} n^\mu + k_T^\mu \approx xp \qquad \text{if } Q \sim xp\cdot n \gg k_T,\ \sqrt{k^2}$$



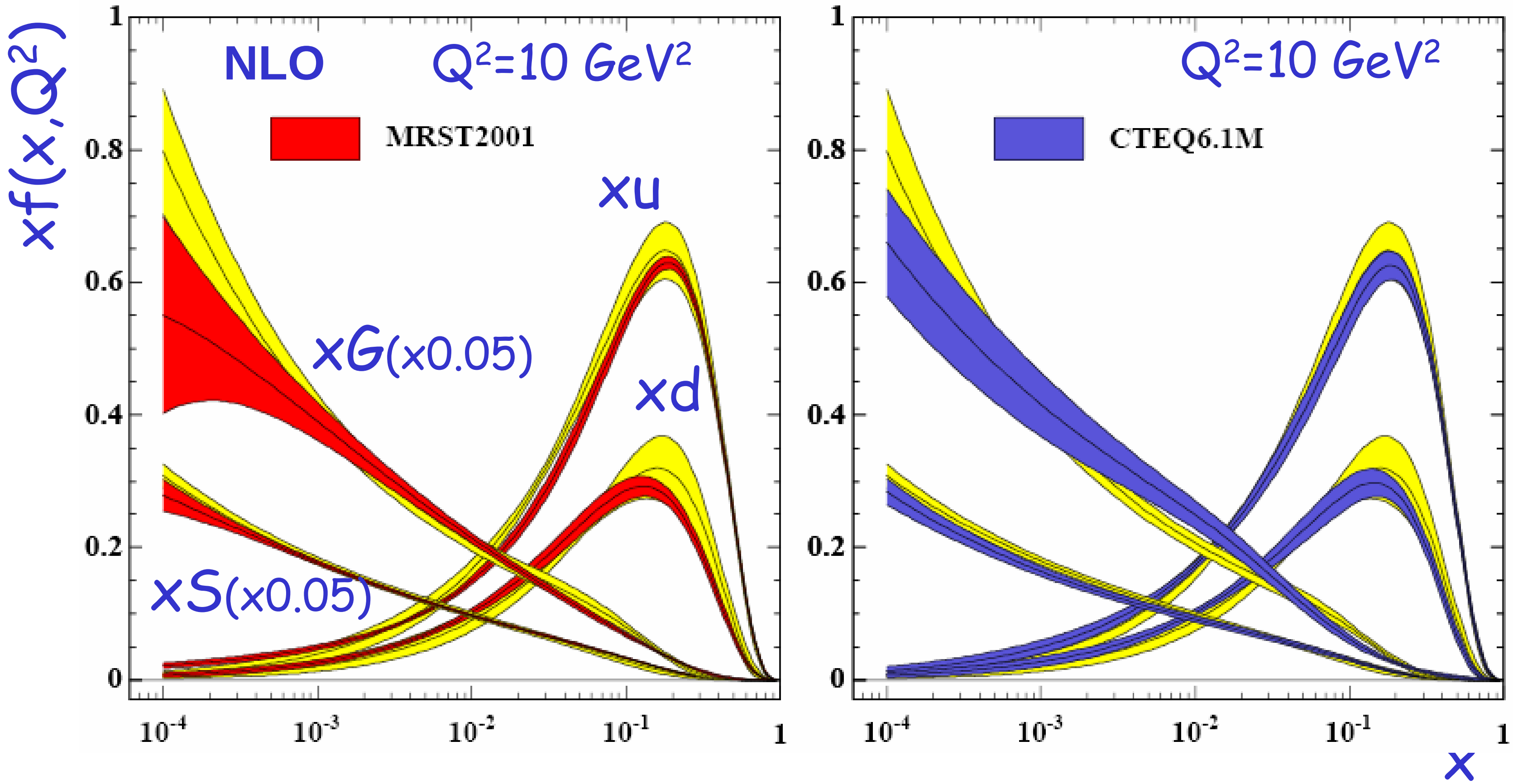
- ❖ Hadron is approximated by a beam of partons of momentum fraction x_i
- ❖ Parton's transverse motion is integrated into parton distributions: $\varphi(x)$

❑ **Parton distributions are process independent, and QCD collinear factorization has been very successful**

Spin-averaged PDFs of a proton

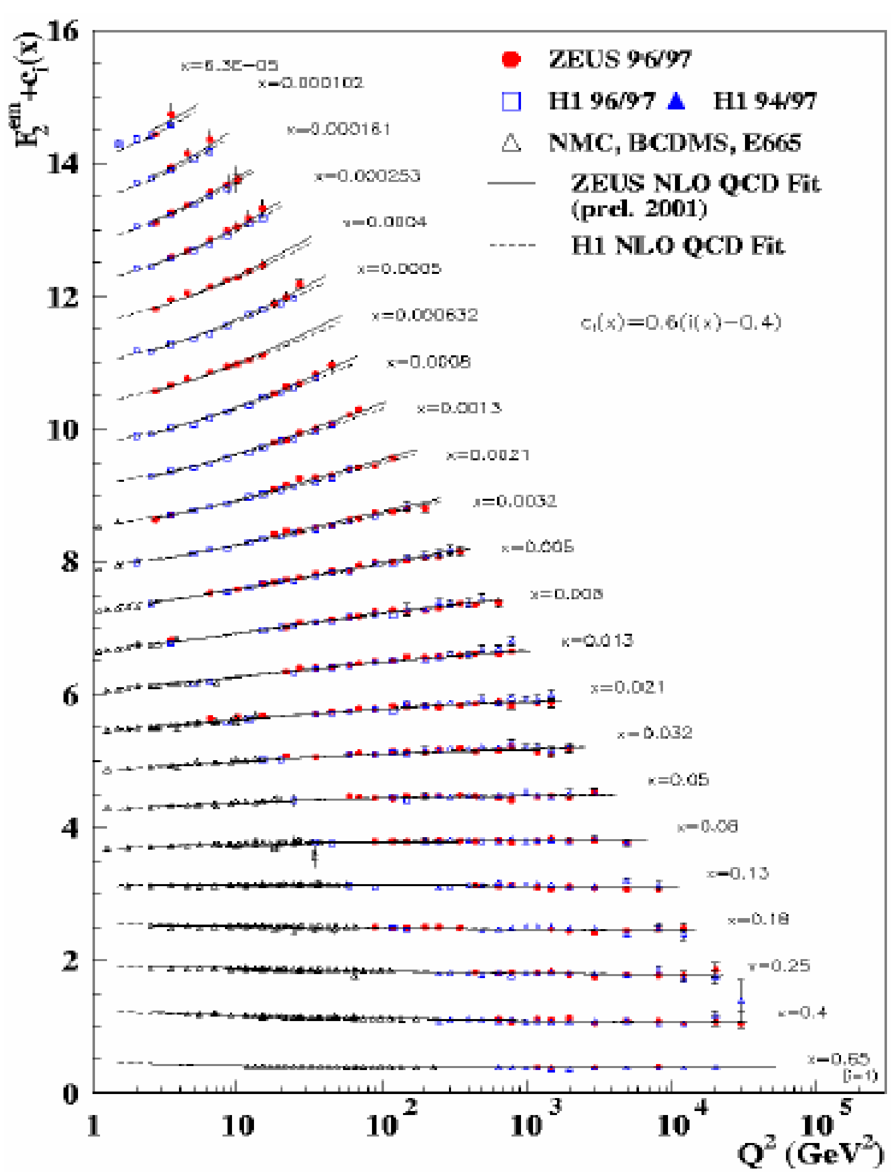
❖ **Over 20 years effort of QCD Global fits:**

→ **Modern sets of PDFs with uncertainties**

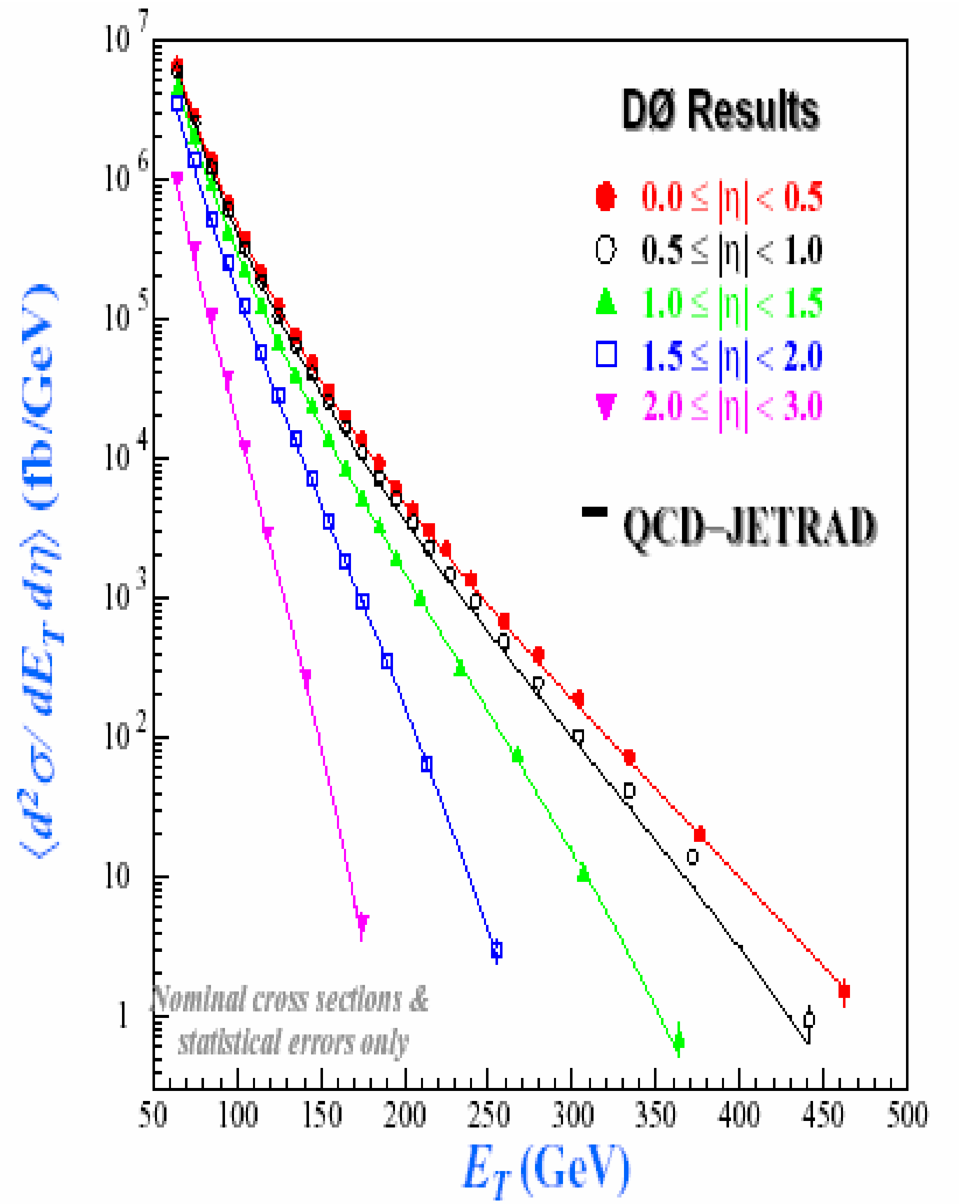


Consistently fit almost all data with Q > 2GeV

DIS structure function



Inclusive jet at Tevatron



Questions

What is the effect of a parton's transverse motion?

How to directly probe a parton's transverse motion?

…, etc.

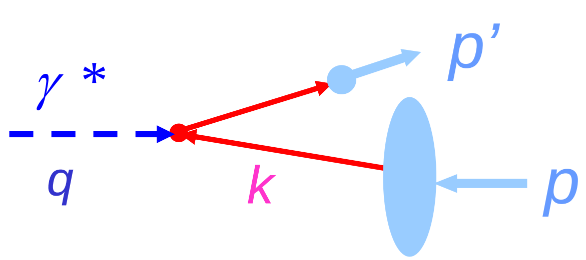
k_T – factorization

- Momentum of the "long-lived" parton is not necessary collinear to the hadron momentum

$$k^2 \approx 0 \;\Rightarrow\; k^\mu \approx xp^\mu + \frac{k_T^2}{2xp\cdot n} n^\mu + k_T^\mu$$

For cross sections with a single hard scale Q,
dk_T leads to power corrections: $\left(k_T^2/Q^2\right)^N$

- Physical processes with two observed scales: Q and q with a large Q to ensure QCD factorization, while $q \sim k_T$ probes a parton's transverse momentum



Both p and p' are observed
p' probes the parton's k_T

Effect of k_T is not suppressed by Q

$\Rightarrow$ $\varphi(x) \;\Rightarrow\; \varphi(x, k_T^2)$ = TMD parton distributions

TMD parton distributions

- Transverse momentum dependent (TMD) parton distributions:

Belitsky, Ji, Yuan, 2003

$$\mathcal{M}_a = \int \frac{P^+ d\xi^-}{\pi} \frac{d^2\xi_\perp}{(2\pi)^2} e^{-ix\xi^- P^+ + i\xi_\perp \cdot k_\perp} \langle PS|\overline{\psi}_a(\xi)\mathcal{L}_v^\dagger(\infty;\xi)\mathcal{L}_v(\infty;0)\psi_a(0)|PS\rangle$$
$$= \frac{1}{2}\left[\underbrace{f_a^{\rm SIDIS}(x,k_\perp)}_{\text{Spin-averaged}}\gamma_\mu P^\mu + \frac{1}{M_P}\underbrace{q_{Ta}^{\rm SIDIS}(x,k_\perp)}_{\text{Spin-dependent - Sivers function}}\epsilon_{\mu\nu\alpha\beta}\gamma^\mu P^\nu k^\alpha S^\beta + \dots \right]$$

- Connection to normal parton distributions

$$q_a(x) = \int d^2k_T\, f_a^{\rm SIDIS}(x, k_T) + {\rm UVCT}$$

- Spin-dependent TMD parton distributions

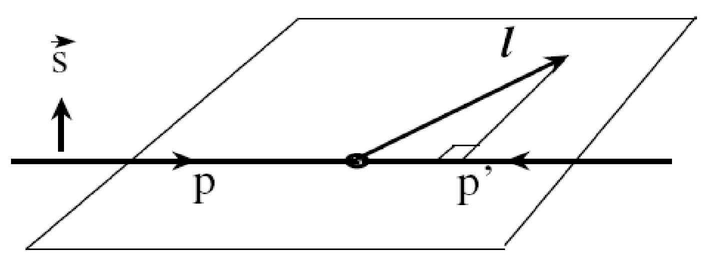
→ Parton's orbital motion and
Non-vanish single transverse-spin asymmetries

Single transverse-spin asymmetry

$$A(\ell,\vec{s}) \equiv \frac{\Delta\sigma(\ell,\vec{s})}{\sigma(\ell)} = \frac{\sigma(\ell,\vec{s}) - \sigma(\ell,-\vec{s})}{\sigma(\ell,\vec{s}) + \sigma(\ell,-\vec{s})}$$

❑ **Hadronic** $p\uparrow + p \to \pi(l)X$:

$$A_N = \frac{1}{P_{\rm beam}} \frac{N^{\pi}_{\rm left} - N^{\pi}_{\rm right}}{N^{\pi}_{\rm left} + N^{\pi}_{\rm right}}$$



If partons are collinear, $A_N \propto \alpha_s m_q$, to be small

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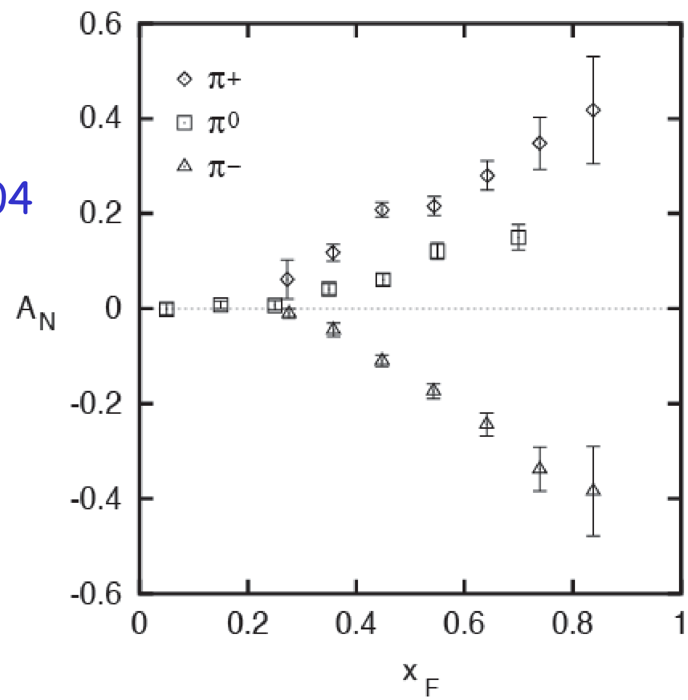
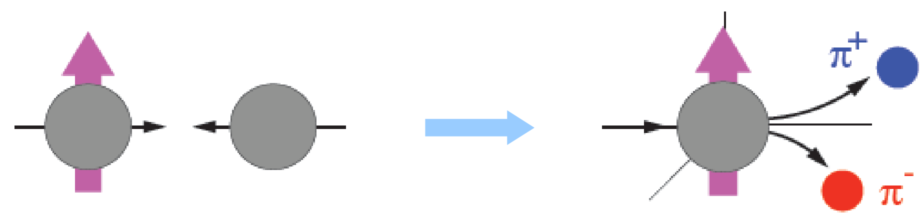




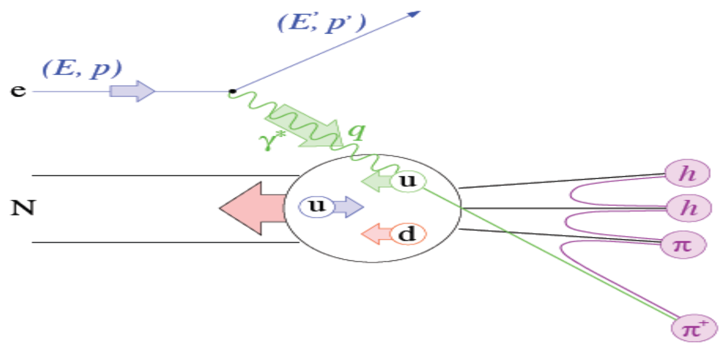
$$A_N \propto i\,\vec{s}_p \cdot (\vec{p} \times \vec{\ell})$$

– the phase "i" is required by time-reversal invariance

– covariant form: $A_N \propto i\epsilon^{\mu\nu\alpha\beta}\, p_\mu s_\nu \ell_\alpha p'_\beta$

Parton's orbital motion plays a key role

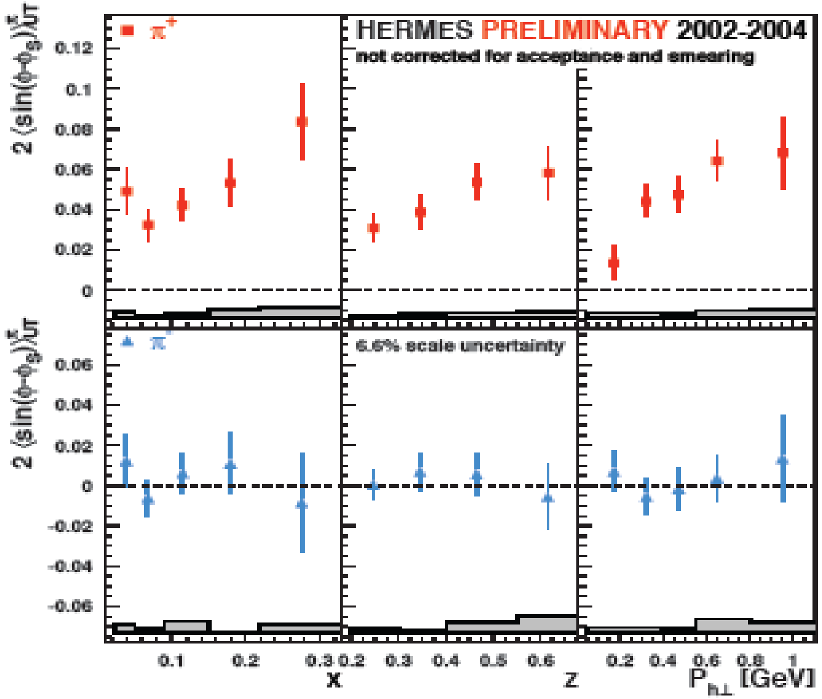
Semi-inclusive DIS (SIDIS)



Azimuthal distribution of pions

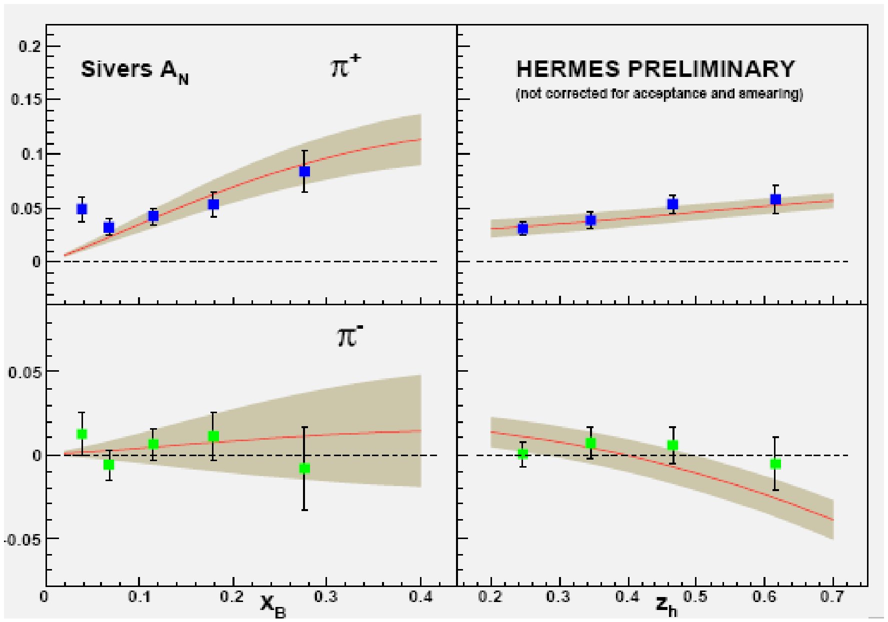
(Hermes and Compass)

$$A_N \propto \sum_q q_T^{\text{SIDIS}}\left(x_B, l_T^2\right) \otimes D_q(z) + \ldots$$



Global fits

Vogelsang & Yuan PRD72, 054028



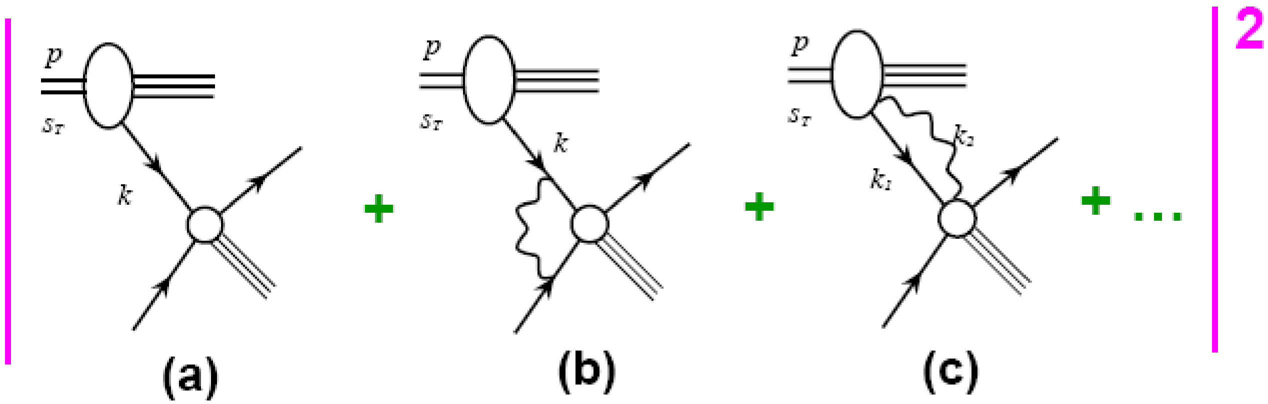
Nonvanish Sivers function → Nonvanish orbital motion

Asymmetry in collinear factorization

Efremov, Teryaev, 1982, Qiu, Sterman, 1991

When both scales are much larger than Λ_{QCD}, or there is only one large scale, collinear factorization should work

$\sigma(s_T)$ ~

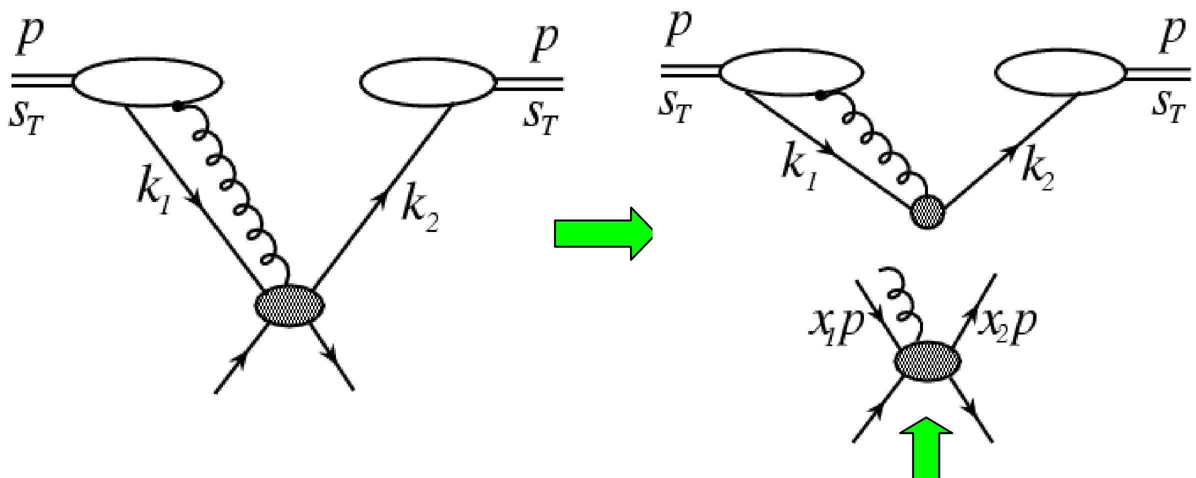


- Leading spin dependent part of the cross section
  - ➡ Interference between amplitudes (a) and (b) or (c)
- The hadronic phase – the "i"
  - ➡ $\mathrm{Re}[(a)]$ interferes with $\mathrm{Im}[(b)]$ or $\mathrm{Im}[(c)]$
- $\mathrm{Re}[(a)] \times \mathrm{Im}[(b)] \propto m_Q \ \delta q(s_\perp)$

A_N from polarized twist-3 correlations

Qiu, Sterman, 1991, 1998

❖ Factorization:



$$T_F(x_1, x_2) \propto \langle \bar{\psi}\gamma^+ F^{+\perp} \psi \rangle$$

$$T_D(x_1, x_2) \propto \langle \bar{\psi}\gamma^+ D_\perp \psi \rangle$$

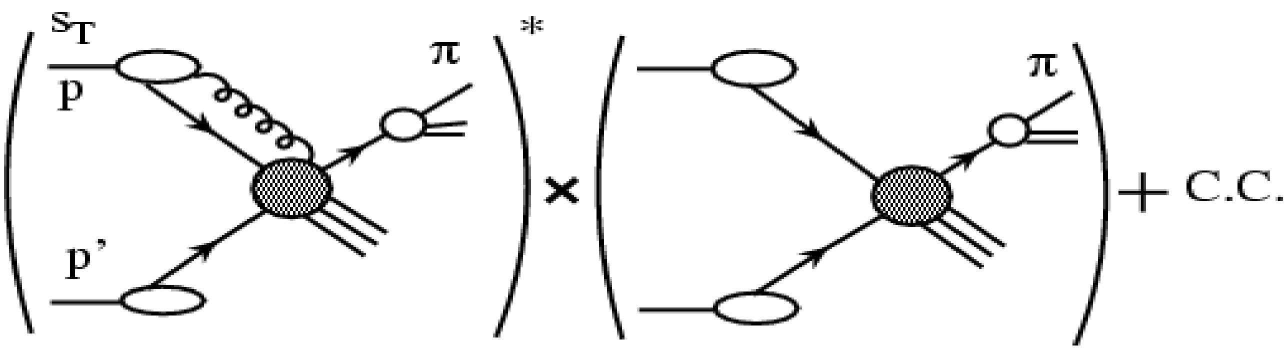
Normal twist-2 distributions

❖ Twist-3 correlation functions:

- ❑ $T_F(x_1, x_2)$ and $T_D(x_1, x_2)$ **have different properties under the P and T transformation**
- ❑ $T_D(x_1, x_2)$ **does not contribute to the A_N**
- ❑ $T_F(x_1, x_2)$ **is universal, $x_1 = x_2$ for A_N due to the pole**

Leading twist-3 contribution to A_N

❖ **Minimal approach (within the collinear factorization):**



❖ Leading $(\partial/\partial x)T_F(x,x)$ contribution to the asymmetries

$$E\frac{d\Delta\sigma}{d^3\ell} \propto \epsilon^{\ell_T s_T n \bar{n}}\, D_{c\to\pi}(z) \otimes \left[-x\frac{\partial}{\partial x}T_F(x,x)\right]$$

$$\otimes \frac{1}{-\hat{u}}\left[G(x')\otimes \Delta\hat{\sigma}_{qg\to c} + \sum_{q'} q'(x')\otimes \Delta\hat{\sigma}_{qq'\to c}\right]$$

❖ $$A_N \propto \left(\frac{\ell_\perp}{-\hat{u}}\right)\frac{n}{1-x} \quad \text{if} \quad T_F(x,x)\propto q(x)\propto (1-x)^n$$

What is the $T^{(3)}(x)$?

- Twist-3 correlation $T_F(x,x)$:

$$T_F(x,x) = \int \frac{dy_1^-}{4\pi} \mathrm{e}^{ixP^+y_1^-}$$

$$\times \langle P, \vec{s}_T | \bar{\psi}_a(0) \gamma^+ \left[\int dy_2^- \, \epsilon^{s_T \sigma n \bar{n}} \, F_\sigma^{\ +}(y_2^-) \right] \psi_a(y_1^-) | P, \vec{s}_T \rangle$$

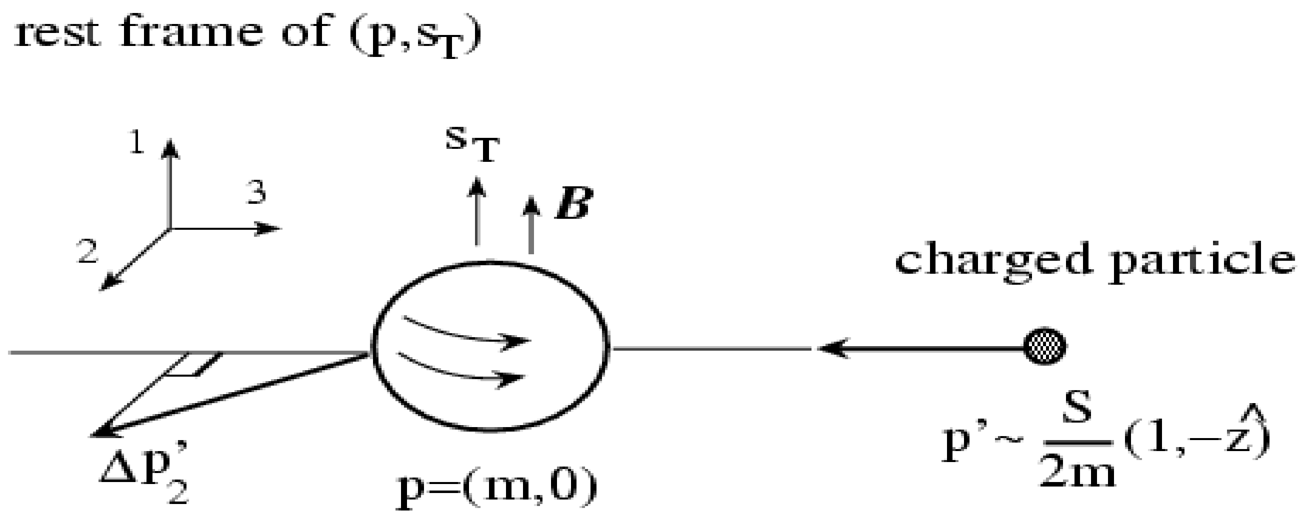
- Twist-2 quark distribution:

$$q(x) = \int \frac{dy_1^-}{4\pi} \mathrm{e}^{ixP^+y_1^-} \langle P, \vec{s}_T | \bar{\psi}_a(0) \gamma^+ \psi_a(y_1^-) | P, \vec{s}_T \rangle$$

T_F **Represents a fundamental quantum correlation between quark and gluon inside a hadron**

What the $T^{(3)}(x)$ tries to tells us?

❖ Consider a classical (Abelian) situation:



– change of transverse momentum

$$\frac{d}{dt}p_2' = e(\vec{v}' \times \vec{B})_2 = -ev_3 B_1 = ev_3 F_{23}$$

– in the c.m. frame

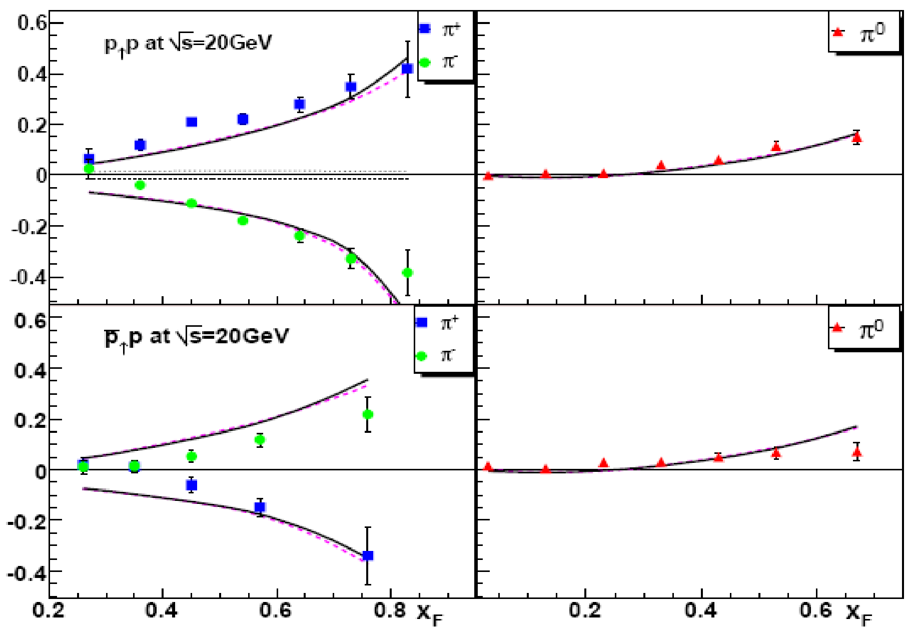
$$(m, \vec{0}) \to \bar{n} = (1, 0, 0_T), \quad (1, -\hat{z}) \to n = (0, 1, 0_T)$$

$$\Longrightarrow \quad \frac{d}{dt}p_2' = e\,\epsilon^{s_T \sigma n \bar{n}}\, F_\sigma^{\ +}$$

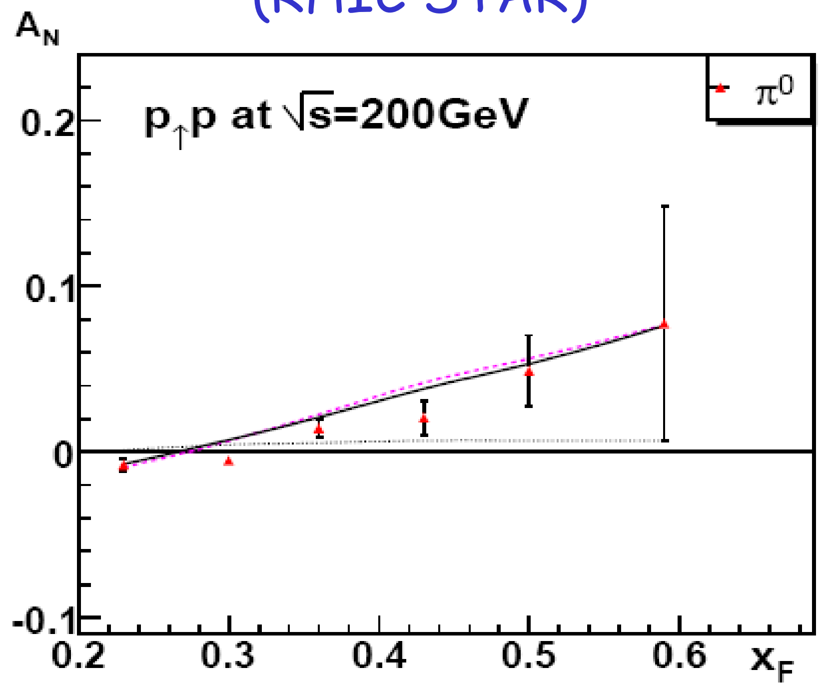
– total change: $\Delta p_2' = e \int dy^- \epsilon^{s_T \sigma n \bar{n}}\, F_\sigma^{\ +}(y^-)$

Asymmetries from the $T^{(3)}(x)$

(FermiLab E704)



(RHIC STAR)



Kouvaris, Qiu, Vogelsang, Yuan, 2006

Nonvanish twist-3 function ⟹ Nonvanish transverse motion

The twist-3 collinear factorization approach and
The k_T-factorization are not independent

$$\frac{1}{M_P}\int d^2\vec{k}_\perp \vec{k}_\perp^2 q_T(x,k_\perp) = T_F(x,x)$$

Summary

- Single transverse-spin asymmetry is a good observable to probe a parton's transverse motion, because the leading power collinear contribution is strongly suppressed

- Twist-3 collinear factorization approach works when all observed momentum scales are much larger than $\Lambda_{QCD.}$ It measures the averaged transverse motion

- The k_T-factorization approach, when it is applicable, measures directly a parton's transverse momentum

- Information on a parton's transverse motion is valuable for understanding the structure of a hadron