

SO(10) GUT Models and Their Success in Explaining Mass and Mixing Data

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Introduction

Many models in literature attempt to explain only neutrino masses and mixings.

More ambitious attempts construct SUSY GUT models to understand both lepton and quark sectors.

My talk will be restricted to $SO(10)$ GUT models with 3 families in 4 dimensions. We'll see that several models are still quite successful in understanding the data.

SO(10) Model Structure

Essential Ingredients:

- 3 families of 16 LH q's and ℓ 's $\rightarrow 16_i, i = 1, 2, 3$
- Higgs fields in $45_H, 16_H, \overline{16}_H$ are needed to break $SO(10) \rightarrow SM$.
- 2 Higgs doublets $\rightarrow 10_H \supset 5 + \bar{5}$ of $SU(5)$
or $\rightarrow 10_H \supset (6, 1, 1) + (1, 2, 2)$ of $SU(4) \times SU(2) \times SU(2)$.
- Doublet-triplet splitting can be achieved via **Dimopoulos - Wilczek** mechanism, if $\langle 45_H \rangle$ points in $B - L$ direction.
- With only one 10_H effecting the electroweak breaking, $\tan \beta \equiv v_u/v_d \sim 55$.

Additional Higgs fields may be desirable:

- $16'_H, \overline{16}'_H$ can help to stabilize doublet-triplet splitting.
- If $\langle \bar{5}(16'_H) \rangle \neq 0$, then $H_d \sim \bar{5}(10_H) \cos \gamma + \bar{5}(16'_H) \sin \gamma$
and $\tan \beta \sim 1 - 55$ is possible.
- 126_H and $\overline{126}_H$ also possibilities.

Additional Matter Fields may be desirable:

- $16, \overline{16}$ pairs may get supermassive and can be integrated out in Froggatt-Nielsen type diagrams for the mass matrix elements.

Horizontal Flavor Symmetries

While $SO(10)$ relates q 's and ℓ 's of one family, it is necessary to invoke some horizontal flavor symmetry to avoid the bad $SU(5)$ relations: $m_d = m_e, m_s = m_\mu$. This can be done with 4 different levels of model building:

- **Level 1:** Simply impose a certain texture such as a modified Fritzsch form for the mass matrices.
- **Level 2:** Introduce an effective λ expansion for each mass matrix. The prefactors typically are not precisely determined, however.
- **Level 3:** Assign effective operators for each matrix element possibly with some flavor symmetry imposed.
- **Level 4:** Introduce a horizontal flavor symmetry which assigns flavor charges to every Higgs and matter superfield. Higgs and Yukawa superpotentials are constructed in terms of renormalizable (and possibly some non-renormalizable) terms which obey that flavor symmetry. Matrix elements then follow from **Froggatt-Nielsen** diagrams.

General Observations

- $SO(10)$ models differ by their choice of Higgs structure, horizontal flavor symmetry and flavor charge assignments (if any).
- Desirable **Georgi-Jarlskog** relations, $m_s = m_\mu/3$, $m_d = 3m_e$, can be obtained if $\langle 45_H \rangle$ points in the $B-L$ direction, or if $\langle \bar{5}(126_H) \rangle$ is involved.
- Presence of $\langle \bar{5}(16_H) \rangle$ and a flavor symmetry will typically lead to **lopsided** down quark and charged lepton mass matrices, D and L . This is useful to explain small V_{cb} and large $U_{\mu 3}$ but leads to enhanced $\tau \rightarrow \mu\gamma$ decay.
- Most early models were easily able to accommodate **SMA** solar solution, while some could accommodate **LOW** or **QVO** solution as well.
- To obtain the **LMA** solution, some fine tuning is generally required.
 - Typically models which require special features of the Dirac and right-handed Majorana mass matrices, N and M_R , to get maximal atmospheric mixing have trouble getting LMA.
 - Easier if M_R can be independently adjusted to get LMA, while N and L conspire to give maximal atmospheric mixing.

Some Selected SO(10) Models

| Model | Flavor Sym. | Texture | $\tan \beta$ | CKM | Solar | Viable |
|-------|------------------------------|----------|--------------|-----|-------|--------|
| AB | $U(1) \times Z_2 \times Z_2$ | Lopsided | ~ 5 | Yes | LMA | Yes |
| BPW | effective operators | Sym/Asym | low | Yes | LMA | Yes |
| BR | $SU(3)$ | Lopsided | 1-10 | Yes | SMA | No |
| BRT | $U(2) \times U(1)^n$ | Sym/Asym | ~ 55 | No | LMA | No |
| BW | postulated | Sym | ? | Yes | LMA | ? |
| CM | $U(2) \times (Z_2)^3$ | Sym | 10 | Yes | LOW | No |
| CW | $\Delta(48) \times U(1)$ | Sym/Asym | ~ 2 | Yes | LMA | No |
| KM | $SU(3) \times U(1)$ | Lopsided | small | ? | LMA | ? |
| M | $U(1)_A \times Z_2$ | Lopsided | small | Yes | LMA | No |
| RV-S | $SU(3)$ and Abelian | Sym/Asym | ? | Yes | LMA | Yes |

| | | |
|------|-----------------------|--|
| AB | Albright, Barr | |
| BPW | Babu, Pati, Wilczek | non-seesaw contribution for M_L required |
| BR | Berezhiani, Rossi | LMA solution possible? |
| BRT | Blazey, Raby, Toby | $\rho < 0$, ν_s required |
| BW | Buchmüller, Wyler | appropriate LMA mixing? |
| CM | Chan, Manhanthappa | U_{e3} violates CHOOZ bound for LMA solution |
| CW | Chou, Wu | ν_s required to get solar LMA solution |
| KM | Kitano, Mimura | appropriate LMA mixing? |
| M | Maekawa | U_{e3} violates CHOOZ bound |
| RV-S | Ross, Velasco-Sevilla | |

Example of the LMA Solution in One Predictive SO(10) Model

Albright, Barr

Based on a $U(1) \times Z_2 \times Z_2$ flavor symmetry that solves the doublet-triplet splitting problem from which the Higgs and Yukawa superpotentials can be constructed. The mass matrices then follow from Froggatt-Nielsen diagrams involving vertex terms appearing in the superpotentials.

- Dirac Mass Matrices**

$$U = \begin{pmatrix} \eta & 0 & 0 \\ 0 & 0 & \epsilon/3 \\ 0 & -\epsilon/3 & 1 \end{pmatrix} M_U, \quad D = \begin{pmatrix} \eta & \delta & \delta' e^{i\phi} \\ \delta & 0 & \sigma + \epsilon/3 \\ \delta' e^{i\phi} & -\epsilon/3 & 1 \end{pmatrix} M_D,$$

$$N = \begin{pmatrix} \eta & 0 & 0 \\ 0 & 0 & -\epsilon \\ 0 & \epsilon & 1 \end{pmatrix} M_U, \quad L = \begin{pmatrix} \eta & \delta & \delta' e^{i\phi} \\ \delta & 0 & -\epsilon \\ \delta' e^{i\phi} & \sigma + \epsilon & 1 \end{pmatrix} M_D,$$

where

$$\begin{array}{ll} M_U \simeq 113 \text{ GeV}, & M_D \simeq 1 \text{ GeV}, \\ \sigma = 1.78, & \epsilon = 0.145, \\ \delta = 0.0086, & \delta' = 0.0079, \\ \phi = 54^\circ, & \eta = 8 \times 10^{-6} \end{array}$$

are input parameters defined at the GUT scale. All 9 quark and charged lepton masses plus the 3 CKM angles and 1 phase are well-fitted with these 8 input parameters (after evolution from the GUT scale).

- **Right-Handed Majorana Matrix**

The type of $\nu_e \leftrightarrow \nu_\mu, \nu_\tau$ solar neutrino mixing is determined by the texture of M_R , since the solar and atmospheric mixings are essentially decoupled in this model.

The LMA solution requires a nearly hierarchical texture which can be understood with **Froggatt-Nielsen** diagrams:

$$M_R = \begin{pmatrix} b^2\eta^2 & -b\epsilon\eta & a\eta \\ -b\epsilon\eta & \epsilon^2 & -\epsilon \\ a\eta & -\epsilon & 1 \end{pmatrix} \Lambda_R$$

The pre-SNO allowed LMA region could be covered with

$$1.0 \lesssim a \lesssim 2.5, \quad 1.8 \lesssim b \lesssim 5.2 \quad \text{Albright, Geer}$$

With the recent SNO results, nearly half of the allowed parameter region is eliminated, as

$$\tan^2 \theta_{12} \gtrsim 0.30, \quad \sin^2 2\theta_{12} \gtrsim 0.71 \Rightarrow a \lesssim 1.8$$

- As an interesting special case, we note that with

$$a = 1, \quad b = 2 \quad \text{and} \quad \Lambda_R = 2.72 \times 10^{14} \text{ GeV}$$

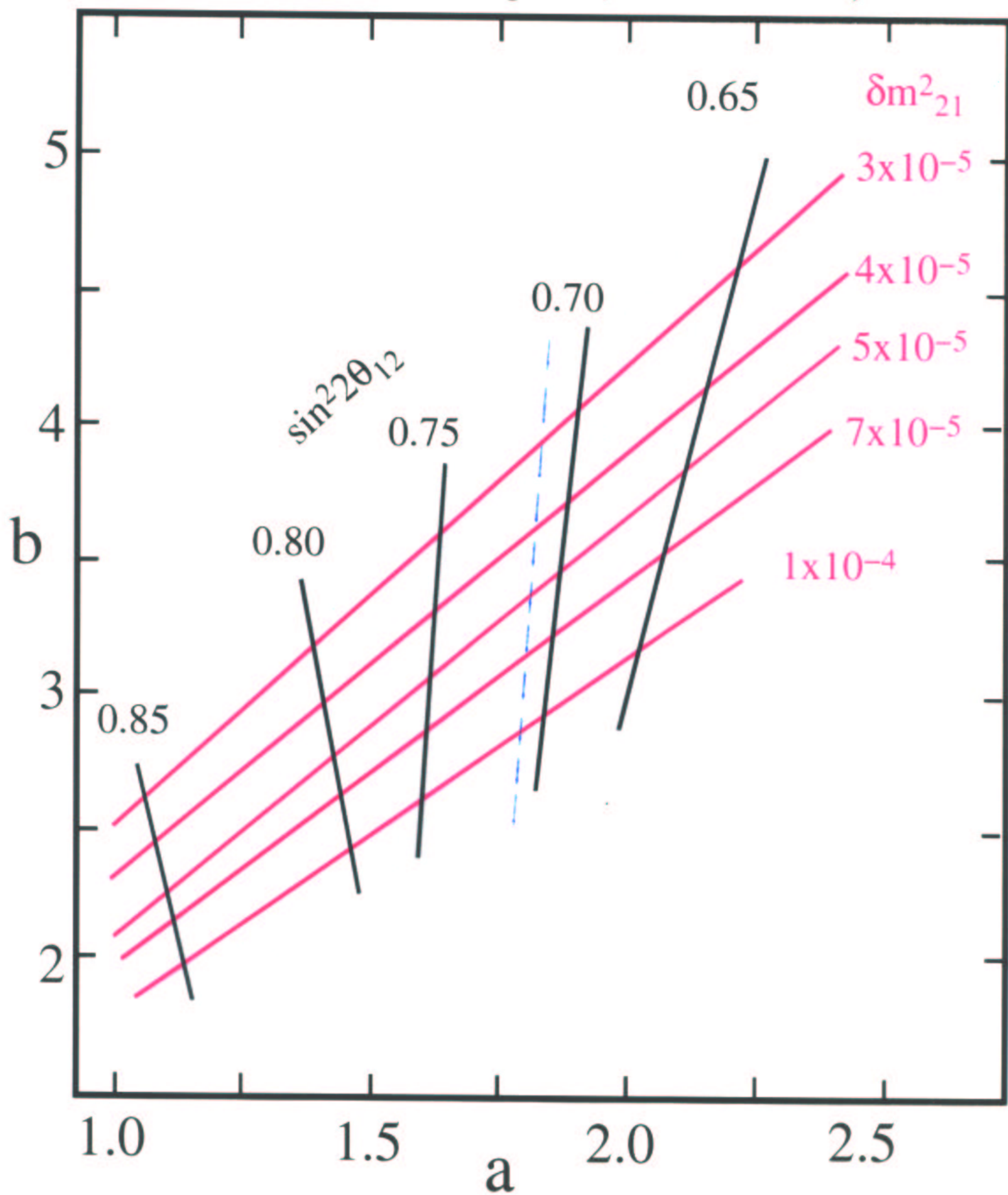
the seesaw mechanism leads to

$$M_\nu = \begin{pmatrix} 0 & -\epsilon & 0 \\ -\epsilon & 0 & 2\epsilon \\ 0 & 2\epsilon & 1 \end{pmatrix} M_U^2 / \Lambda_R$$

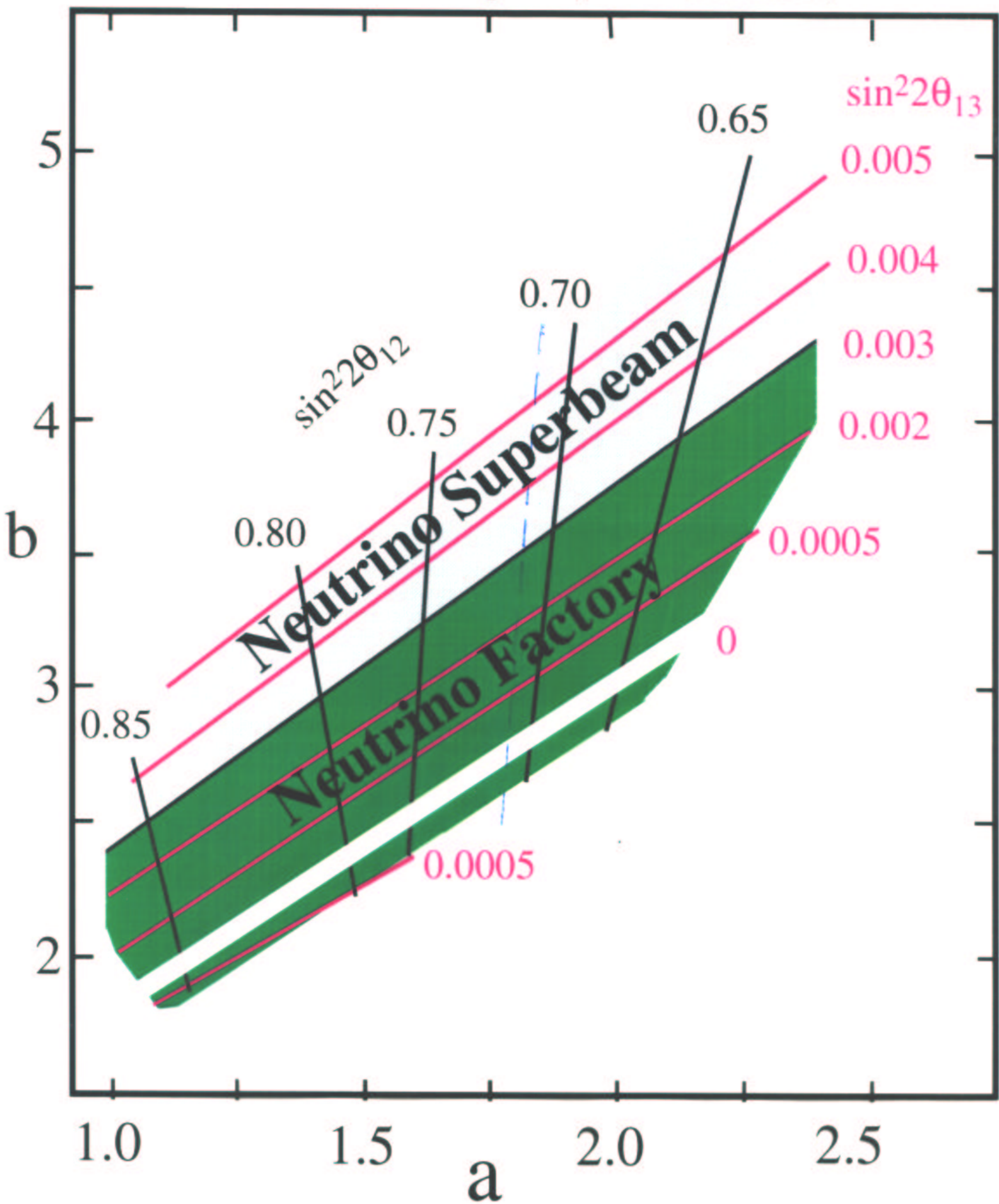
$$\begin{aligned} M_1 &= 3.2 \times 10^8, \quad M_2 = 3.6 \times 10^8, \quad M_3 = 2.8 \times 10^{14} \text{ GeV}, \\ \Delta m_{32}^2 &= 2.5 \times 10^{-3} \text{ eV}^2, \quad \sin^2 2\theta_{\text{atm}} = 0.994, \\ \Delta m_{21}^2 &= 5.1 \times 10^{-5} \text{ eV}^2, \quad \sin^2 2\theta_{\text{sol}} = 0.88, \quad \tan^2 \theta_{\text{sol}} = 0.49, \\ U_{e3} &= -0.014, \quad \sin^2 2\theta_{\text{reac}} = 0.0008 \end{aligned}$$

Whereas one might have expected an inverted hierarchy with M_1 and M_2 so close and much smaller than M_3 , the resultant form of M_ν leads to a normal hierarchy.

Viable Parameter Space (LMA Solution)



Viable Parameter Space (LMA Solution)



Future Tests of SO(10) Models

Of the models listed, many are already nearly ruled out by the more accurate quark and lepton mixing data, but several still survive. Of course I have assumed there are no sterile neutrinos and that the LMA solution is the correct one. In addition, some models may be revived by their authors with further tweaking.

Critical tests to be made in the future with Superbeams and possibly Neutrino Factories:

- **Normal** vs. **inverted** hierarchy.
- Value of θ_{13} . Presently the CHOOZ bound is
$$|U_{e3}| = \sin \theta_{13} < 0.16 \text{ or } \sin^2 2\theta_{13} < 0.10$$
- Test of **CP violation** in the leptonic sector involving the Dirac phase δ and the two Majorana phases χ_1, χ_2 .

For the three models which clearly appear to be still viable, the predictions are as follows.

| Model | Hierarchy | $ U_{e3} $ | $\sin^2 2\theta_{13}$ | CP Violation |
|-------|-----------|-------------|-----------------------|--------------|
| AB | Normal | 0 - 0.035 | 0 - 0.005 | Small |
| BPW | Normal | ? | ? | ? |
| RV-S | Normal | ~ 0.07 | ~ 0.02 | ? |

Summary

- A number of $SO(10)$ SUSY GUT models have been proposed in the literature. Some have been or are on the verge of being eliminated, while some still survive and are able to explain all the known quark and lepton mass and mixing data.
- Long baseline experiments which can determine whether the neutrino mass hierarchy is normal or inverted appear to have a direct bearing on the survival of $SO(10)$ vs Zee-type models.
- The observed value of $\sin^2 2\theta_{13}$ will further narrow down the list of viable models. Some predict that θ_{13} lies just below the CHOOZ bound and will be observable with **off-axis beams** and/or **Superbeams**. Others favor such low values of θ_{13} that a **Neutrino Factory** may be required to determine its value.
- The issue of proton decay via dim-5 operators is potentially a serious one, if proton decay is not detected shortly. On the other hand, by formulating an $SO(10)$ model in 5 dimensions, one can eliminate the dim-5 operator contributions entirely. Albright, Barr