OVERVIEW OF SUSY GUT MODELS OF NEUTRINO MIXING

WHERE DOES ν MASS COME FROM?

GUTs (SEE-SAW MECHANISM)

or NEW PHYSICS AT LOW ENERGY?

(or EXTRA DIMENSIONS?)

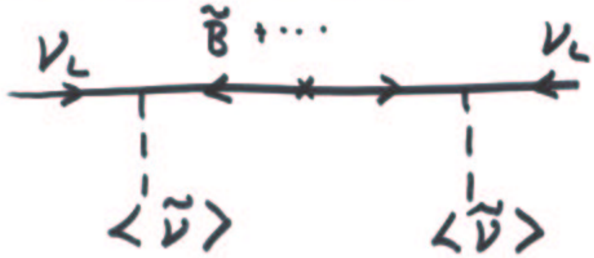
EXAMPLES OF POSSIBLE LOW ENERGY NEW PHYSICS THAT PRODUCE m_ν.

ZEE MODEL

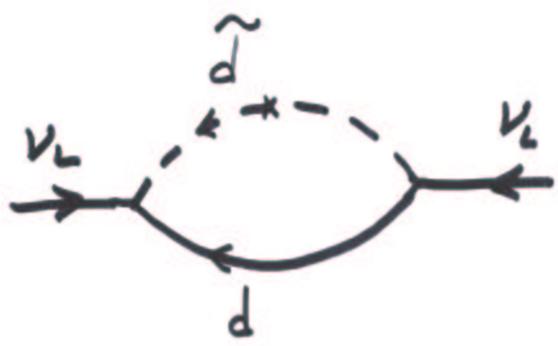
$$h^+ L_i L_j + h^+ \Phi_\alpha \Phi_\beta \Rightarrow$$

SUSY with $\cancel{R}$

$\Rightarrow$



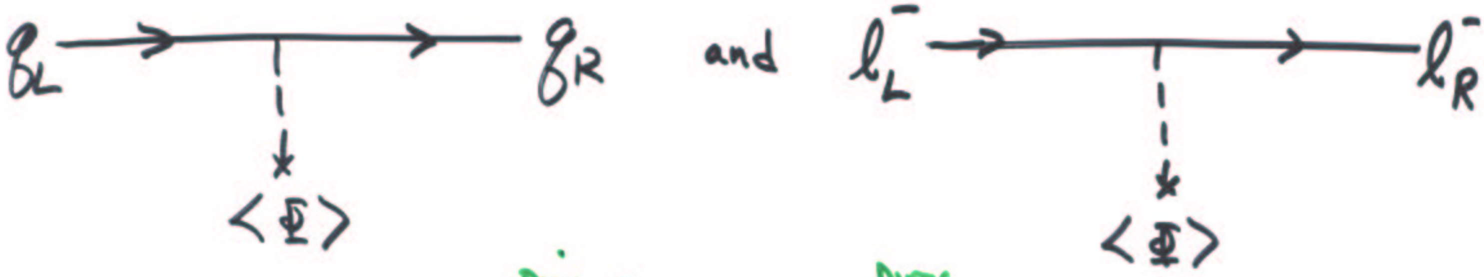
and/or



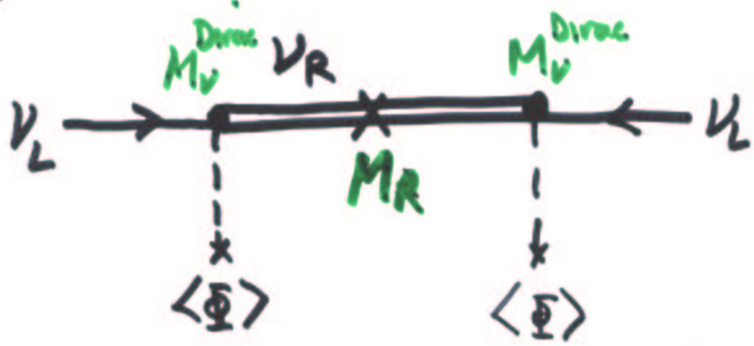
- $m_\nu^{loop} \sim \dfrac{\Pi(\lambda)}{16\pi^2} M_W \Rightarrow \Pi(\lambda) \lesssim 10^{-12}$ WHY?
- NEW PHYSICS NOT STRONGLY MOTIVATED ON OTHER GROUNDS.

IN SUSY GUTs

- NEW PHYSICS IS VERY WELL MOTIVATED
- EXPLANATION OF ORIGIN OF m_ν SIMILAR TO EXPLANATION OF OTHER LIGHT FERMION MASSES. i.e. EXISTENCE OF RIGHT-HANDED ν.
- MAGNITUDE OF m_ν EXPLAINED.



SIMILARLY



$$\boxed{M_\nu = - M_\nu^{Dirac\,T} M_R^{-1} M_\nu^{Dirac}}$$

$$m_{\nu_\tau} \sim m_t \, M_R^{-1} \, m_t$$

$$M_R \sim m_t^2 / m_{\nu_\tau} \sim \frac{(175\ \mathrm{GeV})^2}{(6 \times 10^{-11}\ \mathrm{GeV})}$$

$$M_R \sim \frac{1}{2} \times \underline{10^{15}\ \mathrm{GeV}}$$

- GRATIFYINGLY CLOSE TO M_{GUT} FROM RGE.
- STRONG PRESUMPTION IN FAVOR OF SUSY GUT/SEE SAW EXPLANATION OF m_ν.
- GUTs ALSO ALLOW POSSIBILITY OF PREDICTIVE MODELS

HOWEVER, DISCOVERY OF LARGE θ_{atm} CAME AS A SURPRISE FROM GUT POINT OF VIEW.

PUZZLING FEATURES

- GUTs RELATE QUARKS AND LEPTONS

  eg. MINIMAL SU(5) $\Rightarrow M_D = M_L^T$

  MINIMAL SO(10) $\Rightarrow M_D = M_L \propto M_u = M_\nu^{Dirac}$

  = SYMMETRIC.

  SO WHY AREN'T LEPTONIC ANGLES SMALL LIKE QUARK (CKM) ANGLES?

- THERE HAD SEEMED TO BE A RELATION, FOR QUARKS, BETWEEN MASS RATIOS AND ANGLES. SO WHY, FOR LEPTONS, MASS RATIOS SMALL ($m_e \ll m_\mu \ll m_\tau$ AND $\Delta m^2_{sol} \ll \Delta m^2_{atm}$) WHEREAS ANGLES LARGE ($\sin^2 2\theta_{atm} = 1.0$, $\tan^2\theta_{sol} \simeq 0.4$ for LMA, BUT $U_{e3} < 0.16$)?

Old empirical relation $\tan\theta_c \cong \sqrt{m_d/m_s}$

explained by "texture":

$$\begin{bmatrix} 0 & \varepsilon \\ \varepsilon & 1 \end{bmatrix} m \longrightarrow \left.\begin{matrix} \tan 2\theta = 2\varepsilon \Rightarrow \theta \cong \varepsilon \\ |m_d/m_s| \cong \varepsilon^2 \end{matrix}\right\}$$

[Weinberg; Wilczek and Zee; Fritzsch (1979)]

This was generalized to 3 families by Fritzsch, and many models since then predict relations between angles and mass ratios.

eg Babu and Nandi (1999)

$$M_D \approx \begin{bmatrix} \epsilon^6 & \epsilon^6 & \epsilon^6 \\ \epsilon^6 & \epsilon^4 & \epsilon^4 \\ \epsilon^6 & \epsilon^4 & \epsilon^2 \end{bmatrix} m \; ; \quad M_u \approx \begin{bmatrix} \epsilon^6 & \epsilon^4 & \epsilon^4 \\ \epsilon^4 & \epsilon^2 & \epsilon^2 \\ \epsilon^4 & \epsilon^2 & 1 \end{bmatrix} m$$

Note characteristic "hierarchical" form
$\equiv$ elements get smaller up and to left of diagonal elements.
$\Longrightarrow$ small mass ratios and angles, related.

However, for neutrinos

$$\Delta m^2_{sol} / \Delta m^2_{atm} \lesssim 10^{-2}$$

But $\theta_{atm} \cong \pi/4$

$\theta_{sol} \approx \pi/6$

(On other hand $|U_{e3}| \leq 0.16$)

Two ways to explain large leptonic angle:
From M_ν or from M_L.

Recall:

$$\underset{(\text{MNS})}{U_{\text{leptonic}}} = U_L^\dagger U_\nu = \begin{array}{cc} & \begin{array}{ccc} & \text{solar} & \end{array} \\ & \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix} \begin{array}{c} \\ \leftarrow \text{atm} \\ \\ \end{array} \end{array}$$

$$\cong \underbrace{\begin{bmatrix} 1 & & \\ & C_a & S_a \\ & -S_a & C_a \end{bmatrix}}_{R_{23}(\theta_{atm})} \underbrace{\begin{bmatrix} C_s & S_s & \\ -S_s & C_s & \\ & & 1 \end{bmatrix}}_{R_{12}(\theta_{sol})} = \begin{bmatrix} C_s & S_s & 0 \\ -S_sC_a & -C_sC_a & S_a \\ S_sS_a & -C_sS_a & C_a \end{bmatrix}$$

Three simple possibilities

A) $U_L \cong I, \qquad U_\nu \cong R_{23}(\theta_{atm})\, R_{12}(\theta_{sol})$

i.e both large mixings from ν sector
[first 23, then 12 rotation to diagonalize M_ν]

B) $U_\nu \cong I, \qquad U_L \cong R_{12}(-\theta_{sol})\, R_{23}(-\theta_{atm})$

i.e both large mixings from ℓ^- sector
[first 12, then 23 rotation to diagonalize M_L]

C) $U_L \cong R_{23}(-\theta_{atm}) \qquad U_\nu \cong R_{12}(\theta_{sol})$

θ_{atm} from ℓ^- sector, θ_{sol} from ν sector.

How could large θ_{atm} come from M_ν (case A) in GUT/see-saw models?

Obvious Problem: • M_U, M_D presumably hierarchical.

- GUTs would tend to give M_ν^{Dirac} hierarchical also (for SO(10), for example).
- Then see-saw formula $M_\nu = -M_\nu^{Dirac\,T} M_R^{-1} M_\nu^{Dirac}$ would tend to give M_ν hierarchical.

eg. Let $M_\nu^{Dirac} \propto \begin{bmatrix} \varepsilon' & & \\ & \varepsilon & \\ & & 1 \end{bmatrix}$, $\varepsilon' \ll \varepsilon \ll 1$

$$M_R^{-1} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$$

$$\Rightarrow \quad M_\nu = \begin{bmatrix} \varepsilon'^2 a_{11} & \varepsilon'\varepsilon\, a_{12} & \varepsilon' a_{13} \\ \varepsilon'\varepsilon\, a_{12} & \varepsilon^2 a_{22} & \varepsilon\, a_{23} \\ \varepsilon' a_{13} & \varepsilon\, a_{23} & a_{33} \end{bmatrix} = \text{hierarchical (typically)}$$

$$\theta_{23} \cong \frac{\varepsilon\, a_{23}}{a_{33} - \varepsilon^2 a_{22}}$$

$\ll 1$ unless, say, $a_{33}/a_{23} \sim \varepsilon$

(Conspiracy between Majorana, Dirac matrices)
$\qquad\qquad\qquad\qquad\qquad M_R \qquad M_\nu^{Dirac}$

However, large θ_{atm} can come from M_ν, even with M_ν^{Dirac} = hierarchical and with no "conspiracy".

EXAMPLE 1 (Jezabek and Sumino, 1998)

$$M_\nu^{Dirac} = \begin{bmatrix} \varepsilon'\varepsilon^2 & & \\ & \varepsilon & \varepsilon \\ & O(\varepsilon^2) & 1 \end{bmatrix} m, \quad M_R = \begin{bmatrix} & & A \\ & 1 & \\ A & & \end{bmatrix} m_R$$

$$\Rightarrow \quad M_\nu = \begin{bmatrix} 0 & O(\varepsilon'\varepsilon^4/A) & \varepsilon'\varepsilon^2/A \\ O(\varepsilon'\varepsilon^4/A) & \varepsilon^2 & \varepsilon^2 \\ \varepsilon'\varepsilon^2/A & \varepsilon^2 & \varepsilon^2 \end{bmatrix} \frac{m^2}{m_R}$$

$$R_{23}(\pi/4)^T M_\nu R_{23}(\pi/4) \cong \begin{bmatrix} 0 & -\frac{\varepsilon'\varepsilon^2}{\sqrt{2}A} & \frac{\varepsilon'\varepsilon^2}{\sqrt{2}A} \\ -\frac{\varepsilon'\varepsilon^2}{\sqrt{2}A} & 0 & 0 \\ \frac{\varepsilon'\varepsilon^2}{\sqrt{2}A} & 0 & 2\varepsilon^2 \end{bmatrix} \frac{m^2}{m_R}$$

"Pseudo-Dirac"
(after rotating away 13, 31 elements) $\Longrightarrow$ $\theta_{sol} \cong \pi/4$

EXAMPLE 2 (Altarelli, Feruglio + Masina, 1999)

$$M_\nu = \begin{bmatrix} \varepsilon' & \varepsilon' & \varepsilon' \\ \varepsilon' & \varepsilon & \varepsilon \\ \varepsilon' & \varepsilon & 1 \end{bmatrix} \begin{bmatrix} - & & \\ & 1 & \\ & & - \end{bmatrix} \begin{bmatrix} \varepsilon' & \varepsilon' & \varepsilon' \\ \varepsilon' & \varepsilon & \varepsilon \\ \varepsilon' & \varepsilon & 1 \end{bmatrix} \frac{m^2}{m_R}$$

Order
$\varepsilon' \ll \varepsilon \ll 1$

$M_R^{-1} \cong$ projection
one very light ν_R (second family)

$$\Rightarrow \quad M_\nu = \begin{bmatrix} \varepsilon'^2 & \varepsilon'\varepsilon & \varepsilon'\varepsilon \\ \varepsilon'\varepsilon & \varepsilon^2 & \varepsilon^2 \\ \varepsilon'\varepsilon & \varepsilon^2 & \varepsilon^2 \end{bmatrix} \frac{m^2}{m_R}$$

Lesson: Require special forms.
Also need further conditions satisfied to get LMA solar solution from M_ν.

However, there is a simpler way to get large θ_{atm}: from M_L (case B, or case C)

Suppose M_ν = hierarchical (as is typical in GUTs)

but $M_L = \begin{bmatrix} - & - & - \\ - & - & \rho \\ - & - & 1 \end{bmatrix} m$, $\rho \sim 1$

$$\Rightarrow \quad U_L^\dagger \cong \begin{bmatrix} 1 & & \\ & c_a & +s_a \\ & -s_a & c_a \end{bmatrix} \qquad \tan\theta_{atm} \cong -\rho$$

Then small m_2/m_3 comes from M_ν whereas large $\theta_{23} = \theta_{atm}$ comes from M_L

(Resolves a puzzle)

But then, why isn't there also a large quark angle(s), since M_D related to M_L?

There is, but it is harmless

minimal SU(5) $\Rightarrow$ $M_D = \begin{bmatrix} - & - & - \\ - & - & - \\ - & \rho & 1 \end{bmatrix} m$

produces large mixing of right handed b, s.

Idea of "lopsided" models.

* large θ_{atm} from M_L (not M_ν), * M_L highly asymmetric
* M_L related to M_D^T by SU(5)

θ_{sol} can come either from M_L (B) or M_ν (C)

Example of a highly predictive SO(10)
model based on "lopsided" idea (Albright, Babu, Barr)

$$M_U = \begin{bmatrix} \eta & 0 & 0 \\ 0 & 0 & -\varepsilon/3 \\ 0 & \varepsilon/3 & 1 \end{bmatrix} m_U, \qquad M_D = \begin{bmatrix} 0 & \delta & \delta' \\ \delta & 0 & -\varepsilon/3 \\ \delta' & \rho + \varepsilon/3 & 1 \end{bmatrix} m_D$$

$$M_\nu^{Dirac} = \begin{bmatrix} \eta & 0 & 0 \\ 0 & 0 & \varepsilon \\ 0 & -\varepsilon & 1 \end{bmatrix} m_U, \qquad M_L = \begin{bmatrix} 0 & \delta & \delta' \\ \delta & 0 & \rho + \varepsilon \\ \delta' & -\varepsilon & 1 \end{bmatrix} m_D$$

With six terms, fits well all quark and charged lepton masses, CKM parameters, and ν mixing (atm).

In this model, large θ_{sol} can also arise through M_ν. (Case C). (Albright, Barr 1999)

eg. $M_R = \begin{bmatrix} & & A \\ & 1 & \\ A & & \end{bmatrix} m_R$ or $\begin{bmatrix} & A & \\ A & & \\ & & 1 \end{bmatrix} m_R$

$$\Rightarrow \quad M_\nu = M_\nu^{Dirac\,T} M_R^{-1} M_\nu^{Dirac} = \begin{bmatrix} 0 & \varepsilon\eta/A & \eta/A \\ \varepsilon\eta/A & 0 & 0 \\ \eta/A & 0 & \varepsilon^2 \end{bmatrix} \frac{m_U^2}{m_R}$$

After rotating away 13, 31 elements $(M_\nu)_{11} = -(\eta/\varepsilon A)^2$

$\Rightarrow$ 12 block "pseudo Dirac" $\Longrightarrow$ $\theta_{sol} \cong \pi/4$

Note, no difficulty getting large θ_{sol} from M_ν since m_1/m_2 not known to be small, could be ~ 1.

Another simple way to get large θ_{sol} in lopsided model: from M_L also (case B) "doubly lopsided". (Babu, Barr 1996, 2002)

Consider M_ν = hierarchical

$$M_L = \begin{bmatrix} - & - & \rho' \\ - & - & \rho \\ - & - & 1 \end{bmatrix} m, \qquad \rho, \rho' \sim 1$$

Rotate first from left by $R_{12}(\theta_{sol})$, where $\boxed{\tan\theta_{sol} = \rho'/\rho}$

$$M_L \longrightarrow \begin{bmatrix} - & - & - \\ - & - & \sqrt{\rho^2+\rho'^2} \\ - & - & 1 \end{bmatrix} m$$

Then rotate from left by $R_{23}(\theta_{atm})$, where $\boxed{\tan\theta_{atm} = \sqrt{\rho^2+\rho'^2}}$

$$M_L \longrightarrow \begin{bmatrix} - & - & - \\ - & - & - \\ - & - & \sqrt{1+\rho^2+\rho'^2} \end{bmatrix} m$$

So $U_{leptonic} = U_L^\dagger U_\nu \cong \begin{bmatrix} 1 & & \\ & c_a & s_a \\ & -s_a & c_a \end{bmatrix} \begin{bmatrix} c_s & s_s & \\ -s_s & c_s & \\ & & 1 \end{bmatrix} = \begin{bmatrix} c_s & s_s & 0 \\ -s_s c_a & c_s c_a & s_a \\ s_s s_a & -s_s c_a & c_a \end{bmatrix}$

$$\tan\theta_{atm} \cong \sqrt{\rho^2+\rho'^2} \sim 1$$

$$\tan\theta_{sol} \cong \rho'/\rho \sim 1$$

Question: Why is θ_{atm} so close to maximal, i.e. $\pi/4$?

This requires some non-abelian flavor symmetry

<u>CASE A</u> If θ_{atm} primarily from M_ν

then need $M_\nu \cong \begin{bmatrix} - & - & - \\ - & A & B \\ - & B & A \end{bmatrix}$

$\Rightarrow$ $\theta_{atm} \cong \pi/4$.

$|m_2/m_3| = \left|\frac{A-B}{A+B}\right| \ll 1 \Rightarrow A \cong B$

<u>CASES B,C</u>

Let us take lopsided (or doubly lopsided) form, and by non-abelian flavor symmetry (eg $\mu_L^- \leftrightarrow \tau_L^-$) make $\rho = 1$

$$M_L = \begin{bmatrix} - & - & \rho' \\ - & - & 1 \\ - & - & 1 \end{bmatrix} m$$

(while angles from U_ν small)

$\Rightarrow$ $\tan^2\theta_{sol} \cong (\rho'/\rho)^2 = \rho'^2$

$\tan^2\theta_{atm} \cong \rho^2 + \rho'^2 = 1 + \rho'^2$

$\Rightarrow \tan^2\theta_{atm} = 1 + \tan^2\theta_{sol} \Rightarrow \sin^2 2\theta_{atm} = \dfrac{1+\tan^2\theta_{sol}}{(1+\frac{1}{2}\tan^2\theta_{sol})^2}$

eg. $\tan^2\theta_{sol} = 0.4 \rightarrow \sin^2 2\theta_{atm} = 0.97$

Lopsided mass matrices and b-τ unification.

Minimal SU(5): $M_D = M_L^T \Rightarrow$

	actual
$m_b^0 = m_\tau^0$	✓
$m_s^0 = m_\mu^0$	$\simeq \times \frac{1}{3}$ (approx)
$m_d^0 = m_e^0$	$\simeq \times 3$ (approx)

$\det M_D^0 \cong \det M_L^0$

Old Georgi-Jarlskog idea

$$M_D^0 = \begin{bmatrix} & b & \\ b & -\frac{1}{3}a & \\ & & 1 \end{bmatrix} m, \quad M_L^0 = \begin{bmatrix} & b & \\ b & a & \\ & & 1 \end{bmatrix} m$$

So $\det M_D^0 = \det M_L^0$.

$10_2 \, \bar{5}_2 \langle \overline{45}_H \rangle$

others: $10_i \, \bar{5}_j \langle \bar{5}_H \rangle$

How good is $m_\tau^0 = m_b^0$?

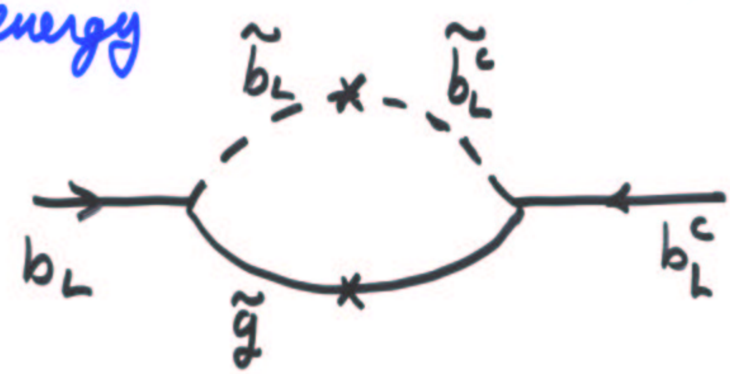
$\tan\beta$	m_τ^0 / m_b^0
1.85	1.03
2.0	1.09
2.5	1.15
3.0	1.17
5.0	1.20
10.0	1.21
20.0	1.20
40.0	1.16
50.0	1.11
55.0	1.06
58.0	1.00

(• $\alpha_s(M_Z) = 0.118$
• $m_b(m_b) = 4.25$ GeV
• $m_\tau(m_\tau) = 1.777$ GeV
• All SUSY masses taken to be degenerate at m_t.
• From m_t to M_G use 2-loop MSSM RGE.
• From 1 GeV to M_Z use 3-loop QCD, 1-loop QED RGE.)

Discrepancy can be explained by
$\tan\beta \approx 1$, or $\tan\beta \approx m_t/m_b$

or by one-loop corrections to m_b at low-energy

gluino loop



$$W \supset h_b (b_L b_L^c) H_d + \mu H_u H_d$$
$$\rightarrow \mu h_b \tilde{b}_L \tilde{b}_L^c \langle H_u \rangle^* = \mu m_b \tan\beta\, \tilde{b}_L \tilde{b}_L^c$$

$$\delta m_b / m_b \Big|_{gluino} = \frac{2\alpha_3}{3\pi} (\mu M_3)\, I(m^2_{\tilde{b}_1}, m^2_{\tilde{b}_2}, M_3^2) \tan\beta$$

(also chargino loop)

Can give 15% correction (for large $\tan\beta$) needed

Need $\mathrm{sgn}(\mu M_3) < 0$ generally to get $\delta m_b < 0$ and explain discrepancy.

However, in mSUGRA typically that makes chargino loop contribution to $b \to s\gamma$ add constructively to SM + charged Higgs loops and $b \to s\gamma$ too big unless sparticles heavy.
[See S. Komine and M. Yamaguchi, PRD 65, 075013 (2001).]

Perhaps m_τ^0 / m_b^0 really is > 1.

[To know, we would need to know $\tan\beta$, and μ, M_3, $m^2_{\tilde{b}_i}$.]

This could be result of same large offdiagonal elements in M_L that give large θ_{atm}. i.e. effect of lopsidedness.
(Barr + Dorsner 2002)

Consider
$$M_L = \begin{bmatrix} - & - & \rho' \\ - & - & \rho \\ - & - & 1 \end{bmatrix} m$$

$$M_D = \begin{bmatrix} - & - & - \\ - & - & - \\ c\rho' & c\rho & 1 \end{bmatrix} m$$

"Clebsch"

$$\frac{m_\tau^0}{m_b^0} \cong \frac{\sqrt{1+\rho^2+\rho'^2}}{\sqrt{1+c^2(\rho^2+\rho'^2)}}$$

If angles in U_ν negligible:
$$\tan\theta_{atm} = \sqrt{\rho^2+\rho'^2} \qquad \text{(page 10)}$$

$$\frac{m_\tau^0}{m_b^0} \cong \frac{\sqrt{1+\tan^2\theta_{atm}}}{\sqrt{1+c^2\tan^2\theta_{atm}}} \cong \sqrt{\frac{2}{1+c^2}}$$

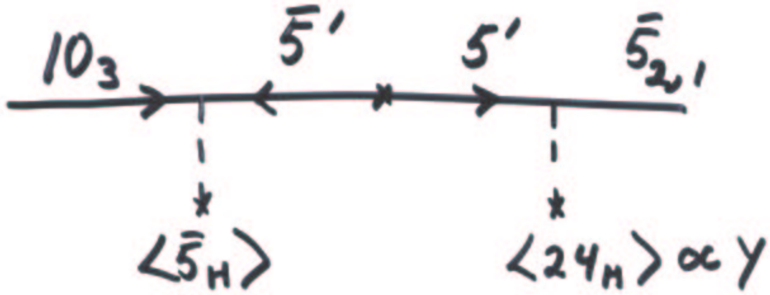
$$m_\tau^0 / m_b^0 \cong \sqrt{\frac{2}{1+c^2}}$$

(a) If ρ, ρ' come from $10_3\, \bar{5}_{2,1}\, \langle \overline{45}_H \rangle$ (a la Georgi Jarlskog)

$\Rightarrow$ $\boxed{c = -1/3}$ $\quad m_\tau^0 / m_b^0 \cong \sqrt{\frac{9}{5}} = \underline{1.34}$

(b) If ρ, ρ' come from $10_3\, \bar{5}_{2,1}\, \langle \bar{5}_H \rangle \frac{\langle 24_H \rangle}{M}$

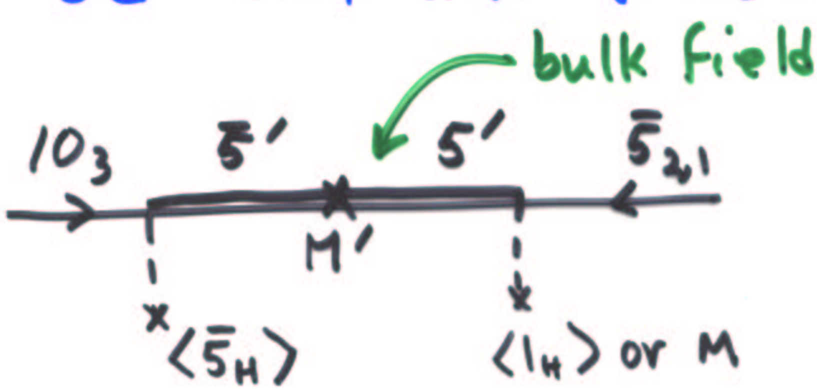
by diagram:



with $M_{5'} \gg \langle 24_H \rangle$

$\Rightarrow$ $\boxed{c = -2/3}$ $\quad m_\tau^0 / m_b^0 \cong \sqrt{18/13} = \underline{1.17}$

(c) ρ, ρ' come from similar diagram in 5d SUSY GUT ($S^1/Z_2 \times Z_2'$ orbifold)



with certain assumptions ($M, M' \ll 1/R$) one finds $\boxed{c \ll 1.}$ $\quad m_\tau^0 / m_b^0 \cong \sqrt{2} = \underline{1.41}$

"LOPSIDED MODELS"

- RELATIVELY SIMPLE WAY TO GET LARGE ν ANGLES
- CAN MAKE HIGHLY PREDICTIVE (AND SUCCESSFUL) MODELS.
- MAY HAVE SOMETHING TO DO WITH $m_t^0/m_b^0 \gtrsim 1$
  (INTERESTING PREDICTIONS FROM SPECIAL CASES)
- MAY ALLOW RELATIVELY SIMPLE WAY TO EXPLAIN $\theta_{atm} \cong$ MAXIMAL.