Proton Decay: The Missing Link of Grand Unification

Jogesh C. Pati

(Ness, 2002
ν Conference, YITP, 2002)

1) K. S. Babu, J. C. Pati & F. Wilczek - "Fermion Masses, Neutrino Oscillations & Proton Decay in the light of SuperKamiokande", hep-ph/981538, Nuc. Phys. B 566, 33 (2000)
2) J. C. Pati, "Confronting the Conventional Ideas of Grand Unification .....", ICTP, Trieste Lectures (2001), hep-ph/0106082

I. Evidence Favoring SUSY Grand Unification

1) Family Structure – Quantum Nos. } G(224)/SO(10) ↕ ν_R // B-L

All Members in One Multiplet

2) Quantization of Q_{em}

3) Meeting of 3 gauge couplings } SUSY SO(10)/SU(5)/ string G(224)

$M_X \sim 2\times10^{16}$ GeV

4) $\boxed{\Delta m^2(\nu_\mu-\nu_\tau)\sim(1/20\ \text{eV})^2}$ ⟷ G(224)/SO(10)

↕

$m(\nu^\tau)_{\text{Dirac}} = m_t(M_X) \approx 120\,\text{GeV}$ ⟷ $SU(4)^C$

$M(\nu_R^\tau) \approx M_{GUT}^2/M_{st} \approx 5\times10^{14}\,\text{GeV}$.

5) $\boxed{m_b^0 \approx m_\tau^0}$

6) $\boxed{\Theta(\nu_\mu-\nu_\tau)\approx\pi/4 \longleftrightarrow V_{cb}\approx 0.04}$ ⟷ G(224)/SO(10).

7) LEPTOGENESIS → $Y_B \approx 5\times10^{-11}$ ⟷ ν_R // B-L

Success of All 7 features Seems Nontrivial

↕

Together make Strong Case in favor of SUSY GRAND UNIFICATION // G(224) or SO(10) in 4D

① Evidence For Grand Unification: Family Multiplet structure

$$G(213) = SU(2)_L \times U(1)_{Y_W} \times SU(3)^c$$

$$\begin{pmatrix} u_r & u_y & u_b \\ d_r & d_y & d_b \end{pmatrix}_L^{1/3} ; \; (u_{r,y,b})_R^{4/3} ; \; (d_{r,y,b})_R^{-2/3} , \; \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L^{-1} ; \; (e^-_R)^{-2}$$

$$Q_{em} = I_{3L} + Y_W/2$$

5 disconnected multiplets in 1 Family // Y_W ? //

Q_{em} ? // $Q_{e^-} = -Q_p$ // Co-exist of (q, ℓ) // g_1, g_2, g_3 ?

$$\Downarrow$$

JCP & Salam 72-73

$$\boxed{G(224) = [SU(2)_L \times SU(2)_R \times SU(4)^c] \otimes (L \leftrightarrow R)}$$

$$F^e_{L,R} = \begin{bmatrix} u_r & u_y & u_b & \nu_e \\ d_r & d_y & d_b & e^- \end{bmatrix}_{L,R}$$

All 16 in one L-R Conj. mult / Y_W / Q_{em} quantized /

$Q_{e^-} = -Q_p$ / $\{q, \ell\}$ unif / (W, E, S) / $\boxed{\nu_R}$ / $\boxed{B-L}$ Generator

$$\boxed{Q_{em} = I_{3L} + I_{3R} + \frac{B-L}{2}}$$

$$\Downarrow$$

$G(213) \rightarrow SU(5)$

$\bar{5} + 10$

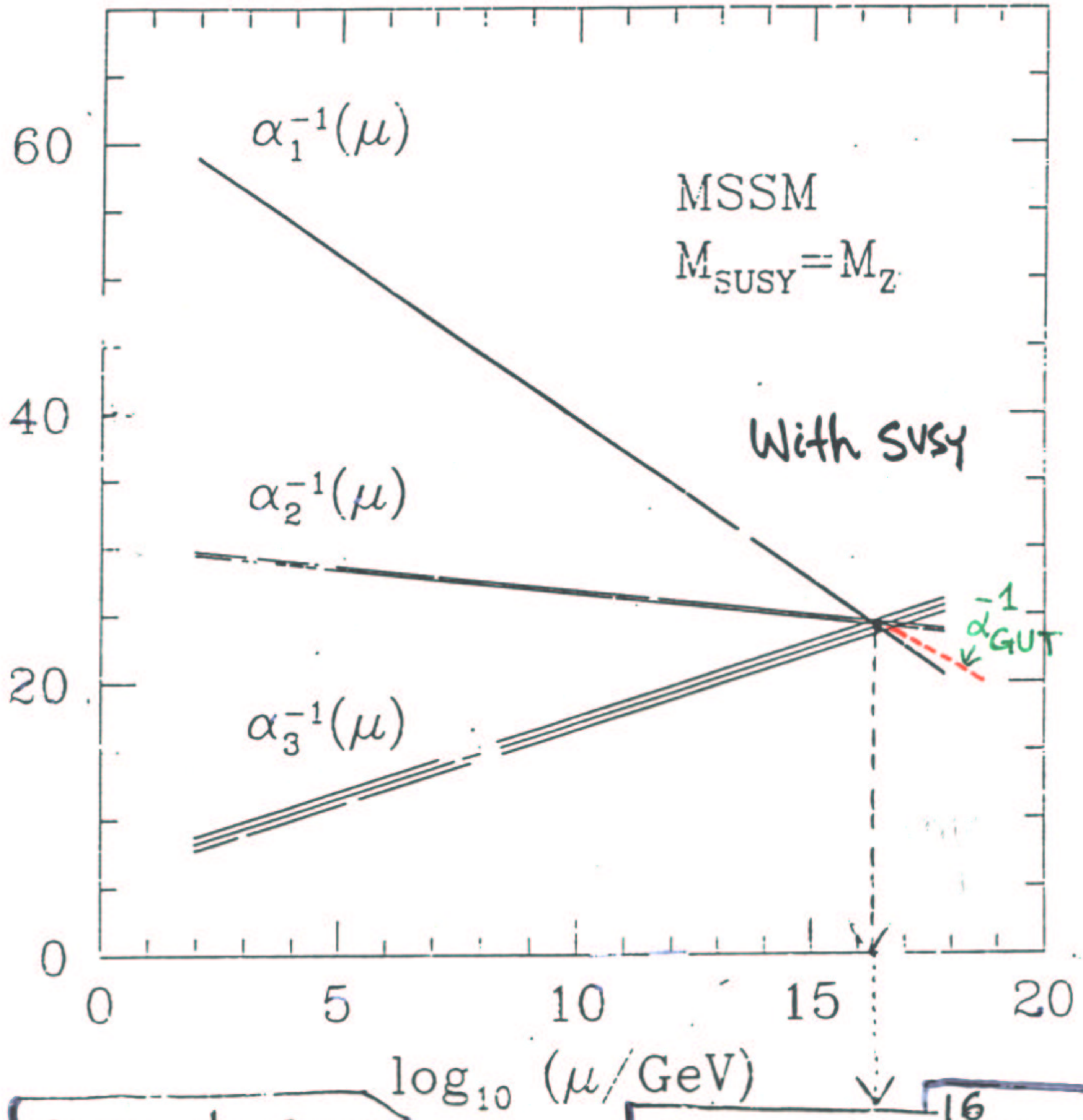
Georgi & Glashow (74)

NO ν_R, NO B-L

$SO(10)$: $\underset{\sim}{16}$ (1 gauge coupling / $g_1 = g_2 = g_3$ at M_U)

(Georgi // Fritzsch, Minkowski (74/75))

② Gauge Coupling Unification



Supports SUSY Unification

$M_{GUT} \approx 2 \times 10^{16}$ Gev

$\sin^2\Theta_W(m_Z)_{th} = 0.2315 \pm .003$

$\sin^2\Theta_W(m_Z)_{EXPT} = 0.23124 \pm .00017$

$\alpha_3(m_Z) \simeq 0.125 - 0.13$

$\alpha_3(m_Z)_{obs} = 0.118 \pm .003$

③ $\sqrt{\Delta m^2(\nu_\mu \nu_\tau)_{\text{SuperK}}} \approx \frac{1}{20}\,\text{eV}$

SeeSaw

$$\boxed{m(\nu_L^\tau) \approx \frac{m(\nu^\tau_{\text{Dirac}})^2}{M(\nu_R^\tau)}}$$

Yanagida// Gell-Mann et al// Mohapatra & Senjanovic

(a) $\boxed{m(\nu^\tau_{\text{Dirac}}) \approx m_t(M_X) \approx 120\,\text{GeV}} \Leftarrow$ SU(4) - Color

$S\cancel{U}(5)$, $[S\cancel{U}(3)]^3$

(b) Get $M(\nu_R^\tau)$ from SUSY Unif. Scale: $M_X \approx 2\times 10^{16}$ GeV

$$\underset{(\approx 1)}{f_{33}}\,\frac{16_3 16_3 \langle \overline{16}_H\rangle\langle \overline{16}_H\rangle}{M_{Pl} \rightarrow 2\times 10^{18}\text{GeV}} \Rightarrow M(\nu_R^\tau) \approx \frac{(2\times 10^{16}\,\text{GeV})^2}{2\times 10^{18}\,\text{GeV}}$$

$$\approx \boxed{2\times 10^{14}\,\text{GeV}}$$

$$\Downarrow$$

$$\boxed{m(\nu_L^\tau) \approx \frac{(120\,\text{GeV})^2}{2\times 10^{14}\,\text{GeV}} \approx \left(\frac{1}{17}\,\text{eV}\right)\left(\frac{1}{2}\text{ to }2\right)}$$

Also get $m(\nu_L^\mu) \sim \frac{m(\nu_L^\tau)}{10} \Rightarrow \sqrt{\Delta m^2(\nu_\mu \nu_\tau)_{\text{Th}}} \sim \left(\frac{1}{17}\text{eV}\right)\left(\frac{1}{2}-2\right)$

Thus SuperK result brings to light the existence of ν_R // reinforces the ideas of (a) SUSY Unif, (b) SU(4) - Color, & (c) Seesaw.

In short, just this single piece of information ($\Delta m^2(\nu_\mu \nu_\tau) \sim \left(\frac{1}{20}\ \text{eV}\right)^2$)

Disfavors

- ~~$SU(5)$~~, ~~$[SU(3)]^3$~~
- ~~Intermediate Scale ($\lesssim 10^{12}$ GeV) Breaking of B-L~~
- ~~Large Extra Dim.~~

Favors

$SU(2)_L \times SU(2)_R \times SU(4)^c$ / SO(10) Route To Higher Unification with SUSY //

Single step Breaking of G(224) / SO(10) at $M_X \sim 2\times 10^{16}$ GeV

④ Fermion Masses & Mixings in SO(10)

Minimal Higgs For SO(10) - Breaking

$\boxed{45_H, 16_H, \overline{16}_H, 10_H}$, $\cancel{126}_H, \cancel{120}_H$

Allowed by string solns

Too large GUT-scale Threshold Corrections // Also Seems disallowed by strings

$$\begin{tikzcd} SO(10) \arrow[d, "\langle 45_H\rangle \propto B-L \sim M_{GUT}"', "\boxed{\langle 16_H\rangle^{\tilde{\bar{\nu}}_R} \text{ Breaks } (B-L)} \sim M_{GUT}"] \\ SU(2)_L \times U(1)_Y \times SU(3)^C \arrow[d, "\langle 10_H\rangle \approx v_{EW} \approx 247\text{GeV}"] \\ U(1)_{em} \times SU(3)^C \end{tikzcd}$$

Babu, Pati & Wilczek (1998//2000)

(i) • $16_i\, 16_j\, 10_H \rightarrow$ Symmetric in $i \leftrightarrow j$, indep of B-L

(ii) • But need B-L dep. $\rightarrow m_s^0 \sim m_\mu^0/3, \ldots$

(iii) • Need Antisymm. Piece to explain smallness of $V_{bc} \simeq 0.04$ & Largeness of $\Theta^{osc}_{\nu_\mu \nu_\tau}$

(iv) Need M_U Not proportional to M_D: $V_{CKM} \neq \mathbb{1}$

But 10_H can't give (ii), (iii) & (iv).

↓

A Concrete Example: Minimal Higgs: $\{45_H, 16_H, \overline{16}_H, 10_H\} + \text{"}S\text{"}$

Only μ & τ Families (Flavor sym: $\mu \neq \tau$)

$$\mathcal{L}_{mass} = h_{33}\, 16_3\, 16_3\, \langle 10_H \rangle \rightarrow \text{3rd Family: } \underset{\cdots\cdots}{m_b^0 = m_\tau^0}$$

$$\propto \textcircled{1} \leftarrow\!\!-\!-\; h_{33}$$

$$\propto \textcircled{σ} \leftarrow\!-\!- + h_{23}\, 16_2\, 16_3\, \langle 10_H \rangle \langle S/M \rangle \rightarrow \text{2nd Family}$$

$$\propto \textcircled{ϵ} \leftarrow\!-\!- + a_{23}\, 16_2\, 16_3\, \underbrace{\overbrace{\langle 10_H \rangle}^{\text{ANTISYMMETRIC}} \langle 45_H \rangle}_{M} \rightarrow m_\mu^0 \neq m_s^0$$

ANTISYMMETRIC, $\propto B-L$

$$\propto \textcircled{$\hat{\eta}$} \leftarrow\!-\!- + g_{23}\, 16_2\, 16_3\, \underline{\langle 16_H \rangle}\, \langle 16_H \rangle_{EW}^{D} / M$$

$\hookrightarrow$ CKM $\neq 1$

$$10_H \rightarrow (2 \leftrightarrow 2) \ll (2 \leftrightarrow 3) \ll (3,3) \text{ Flavor}_{\text{sym}}$$

$$U = \begin{array}{c} \begin{matrix} c & \quad t \end{matrix} \\ \begin{pmatrix} 0 & \epsilon+\sigma \\ -\epsilon+\sigma & 1 \end{pmatrix} \end{array} m_U^0 \qquad D = \begin{array}{c} \begin{matrix} s & \quad b \end{matrix} \\ \begin{pmatrix} 0 & \epsilon+\eta \\ -\epsilon+\eta & 1 \end{pmatrix} \end{array} m_D^0$$

$$N_{\text{Dirac}} = \begin{array}{c} \begin{matrix} \nu_\mu & \quad \nu_\tau \end{matrix} \\ \begin{pmatrix} 0 & -3\epsilon+\sigma \\ +3\epsilon+\sigma & 1 \end{pmatrix} \end{array} m_U^0 \qquad L = \begin{array}{c} \begin{matrix} \mu & \quad \tau \end{matrix} \\ \begin{pmatrix} 0 & -3\epsilon+\eta \\ 3\epsilon+\eta & 1 \end{pmatrix} \end{array} m_D^0$$

Note $q-\ell$ correlation (SU(4)-color)

up-down " $(SU(2)_L \times SU(2)_R)$

$$\boxed{\eta \equiv \hat{\eta} + \sigma}$$

Including $m_U^0 \to$ 7 param ($\eta, \epsilon, \sigma, \eta', \epsilon', m_U^0, m_D^0$)

describing $9 \times 4 = 36$ entries → Will it work ?

Input: Assume all param real for a moment

$m_t^{phys} = 174$ GeV ; $m_c(m_c) = 1.37$ GeV,
$m_s(1\,\text{GeV}) = 116$ MeV, m_μ, m_τ, $m_u(M_X) = 1.5$ MeV, m_e

$\Downarrow$

$\sigma \simeq 0.110$, $\eta \simeq 0.151$, $\epsilon \simeq -0.095$,
$\epsilon' = \sqrt{m_u/m_c}\,(m_c/m_t) \simeq 2 \times 10^{-4}$; $\eta' \simeq \sqrt{m_e/m_\mu}\left(\frac{m_c}{m_t}\right) \simeq 2 \times 10^{-4}$
$m_U^0 = m_t(M_X) \simeq 110$ GeV ; $m_D^0 \simeq 1.5$ GeV

Majorana Mass of ν_R's : $f_{ij}\, 16_i\, 16_j\, \overline{16}_H\, \overline{16}_H / M$

$$\Rightarrow f_{ij}\left(\nu_R^{i\,T} C^{-1} \nu_R^j\right) \langle 16_H \rangle^2 / M$$

$$M_R^\nu = \begin{pmatrix} x & 0 & z \\ 0 & 0 & y \\ z & y & 1 \end{pmatrix} \boxed{M_R}$$

determined by $m_{\nu_2}/m_{\nu_3} \approx 1/8$

Calculated $\approx 5 \times 10^{14}$ GeV.

6 New observables.

Summary on Fermion Masses & Mixings

Predictions	Observations
$m_b(m_b) \approx (4.7-4.9)\,\text{GeV}$	$\approx 4.2\,\text{GeV}$
$m(\nu_\tau) \approx \left(\frac{1}{20}\,\text{eV}\right)\left(\frac{1}{2}-2\right)$	$\approx (1/15 - 1/25)\,\text{eV}$ ⊛
$V_{cb} \approx 0.042$	≈ 0.04
$\sin^2 2\Theta^{osc}_{\nu_\mu \nu_\tau} \approx 0.92 \leftrightarrow \boxed{0.99}$	$\approx 0.92 \leftrightarrow 1$
$V_{us} \approx 0.22$	≈ 0.21
$V_{ub} \approx 0.0032$	$\approx 0.003 - 0.004$
$m_d(1\,\text{GeV}) \simeq 8\,\text{MeV}$	$\approx 8-10\,\text{MeV}$

* * *

$$m(\nu_\mu) \approx (2-10)\times 10^{-3}\,\text{eV} \longleftrightarrow \begin{cases} \text{SMA} \sim 3\times 10^{-3}\,\text{eV} \\ \boxed{\text{LMA} \approx 7\times 10^{-3}\,\text{eV}} \end{cases}$$ ⊛

$$m(\nu_e) \sim (1\text{ to few})\times 10^{-3}\,\text{eV}$$

$\updownarrow$ Consistent with the framework

* * *

$$M\left(\nu_R^\tau, \nu_R^\mu, \boxed{\nu_R^e}\right) \approx \left(5\times 10^{14}, 10^{12}, \boxed{10^9 - 10^{10}}\right)\text{GeV}$$

Just right for Lepto/Baryogenesis

SMA or LMA ?

⊛ Assuming hierarchical pattern: $m(\nu_L^e) \ll m(\nu_L^\mu) \ll m(\nu_L^\tau)$

Predictions

Writing only for 2×2 (for simplicity)

$$U = \left(\begin{array}{cc} c & t \\ 0 & \epsilon+\sigma \\ -\varepsilon+\sigma & 1 \end{array}\right) m_U^0 \quad ; \quad D = \left(\begin{array}{cc} s & b \\ 0 & \varepsilon+\eta \\ -\varepsilon+\eta & 1 \end{array}\right) m_D^0$$

$$N = \begin{pmatrix} 0 & -3\varepsilon+\sigma \\ 3\varepsilon+\sigma & 1 \end{pmatrix} m_U^0 \quad ; \quad L = \left(\begin{array}{cc} \mu & \tau \\ 0 & -3\varepsilon+\eta \\ 3\varepsilon+\eta & 1 \end{array}\right) m_D^0$$

$$\Downarrow$$

$$m_b^0 \approx m_\tau^0(1-8\varepsilon^2) \Rightarrow \boxed{m_b(m_b) \simeq 4.7\ \text{GeV}}$$

$$V_{cb} = \left| \underbrace{\sqrt{\frac{m_s}{m_b}}}_{(0.156)} \underbrace{\left(\frac{\eta+\varepsilon}{\eta-\varepsilon}\right)^{1/2}}_{(1/2 \cdot 2)} - \sqrt{\frac{m_c}{m_t}} \left(\frac{\sigma+\varepsilon}{\sigma-\varepsilon}\right)^{1/2} \right| = |\sigma-\eta|$$

$$\boxed{= 0.042}$$

Suppressed

ENHANCED ≃ 1.8

$$\theta^{osc}_{\nu_\mu \nu_\tau} = \left| \theta^{l}_{\mu\tau} - \theta^{\nu}_{\mu\tau} \right| = \left| \sqrt{\frac{m_\mu}{m_\tau}} \left(\frac{\eta-3\varepsilon}{\eta+3\varepsilon}\right)^{1/2} + \sqrt{\frac{m_{\nu_2}}{m_{\nu_3}}} \right|$$

$$= 0.437 + \sqrt{m_{\nu_2}/m_{\nu_3}}$$

$$\Rightarrow \boxed{\sin^2 2\theta^{osc}_{\nu_\mu\nu_\tau} = 0.92 \leftrightarrow 0.99} \qquad \overset{\text{Expt}}{.92 \leftrightarrow 1}$$

$$m_{\nu_2}/m_{\nu_3} = 1/15 \longleftrightarrow 1/8 \ \text{LMA.}$$

Summary on Fermion Masses & Mixings in the G(224)/SO(10) Framework

Given the bizarre pattern of masses & mixings of quarks, charged leptons and neutrinos, it seems remarkable that the simple pattern of fermion mass matrices*, motivated in large part by the group th of G(224)/SO(10) and the assumption of minimality of the Higgs system makes 7 predictions in agreement with observation.

→ Study Proton Decay, $\cancel{CP}$ within this framework

→ Extend to ESSM

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* Need to understand the origin of flavor symmetries.
→ Hierarchical entries.

SMA or LMA ?

$f_{ij}\ 16_i\, 16_j\, \overline{16}_H\, \overline{16}_H / M$

Just with M_R^{ν} (Superheavy Maj. Masses of ν_R's)

SMA rather generic

$m(\nu_L^e) \sim 2\times10^{-5} - 2\times10^{-6}\,\mathrm{eV}$ // $m(\nu_L^\mu) \sim 3\times10^{-3}\,\mathrm{eV}$

$$\Theta^{osc}_{\nu_e\nu_\mu} = \Theta^{\ell}_{e\mu} - \Theta^{\nu}_{e\mu} \approx 0.05$$

Situation alters once allow for direct Maj. masses of ν_L's — Most likely to arise through Higher Dim. op. involving GUT & EW VEV's - Through tiny $\sim 10^{-3}$ eV entries → IMPORTANT For (ν_e-ν_μ)

$$W \supset g_{12}\ \underbrace{16_1\ 16_2}\ \underset{\downarrow}{16_H}\ \underset{\downarrow}{16_H}\ \underbrace{10_H\ 10_H}/M^3_{GUT}$$

$$g_{12}\,(\nu_L^e\,\nu_L^\mu)\left(\langle\tilde{\nu}_{RH}\rangle/M_{GUT}\right)^2 (v_u^2)$$

$(\sim 175\ \mathrm{GeV})^2$

$$\boxed{\sim g_{12}(\nu_L^e\,\nu_L^\mu)\,(1.5-6)\times10^{-3}\,\mathrm{eV}} \quad (\langle 16_H\rangle \approx (1-2)\,M_{GUT})$$

$$\begin{array}{cc} \nu_L^e & \nu_L^\mu \\ \end{array}$$

$$\begin{bmatrix} \approx 0 & (3-4) \\ (3-4) & 7 \end{bmatrix} \times 10^{-3}\,\mathrm{eV} \Longrightarrow \begin{bmatrix} \Theta^{\nu}_{\nu_e\nu_\mu} \approx 1/2 \\ \sin^2 2\Theta^{osc}_{\nu_e\nu_\mu} \sim 0.7 \end{bmatrix}$$

Quite Plausible

Thus LMA not strictly a prediction, but perfectly plausible within the framework.

$$m(\nu_L^e \nu_L^\tau)_{\text{Non-seesaw}} \sim (2-6) \times 10^{-3}\,\text{eV}$$

$$\Theta_{13} \sim \frac{(2-6) \times 10^{-3}\,\text{eV}}{5 \times 10^{-2}\,\text{eV}}$$

$$\sim .03 - .1$$

⑤ Leptogenesis $\xleftrightarrow{\text{Sphalerons}}$ Baryogenesis

Kuzmin, Rubakov, Shaposhnikov // Fukugita, Yanagida

Use the Dirac & Majorana Mass-Matrices of the G(224)/SO(10)-framework

$$\nu_R \longrightarrow \ell + \phi_H \,,\quad \bar{\ell} + \bar{\phi}_H \Big\} \Delta(B-L) \neq 0$$

$$\downarrow$$

LEPTON - Asymmetry

$$\Big\downarrow \text{EW Sphalerons}$$

$$(Y_B)_{Th} / (\sin 2\phi_{eff}) \approx (4-37)\times 10^{-11}$$

$$\updownarrow$$

$$(Y_B)_{obs} \approx (3-9)\times 10^{-11} \quad (BBN)$$

Agrees with observation for natural values of $\phi_{eff} \approx 1/3 - 1/8$ in accord with Gravitino Constraint

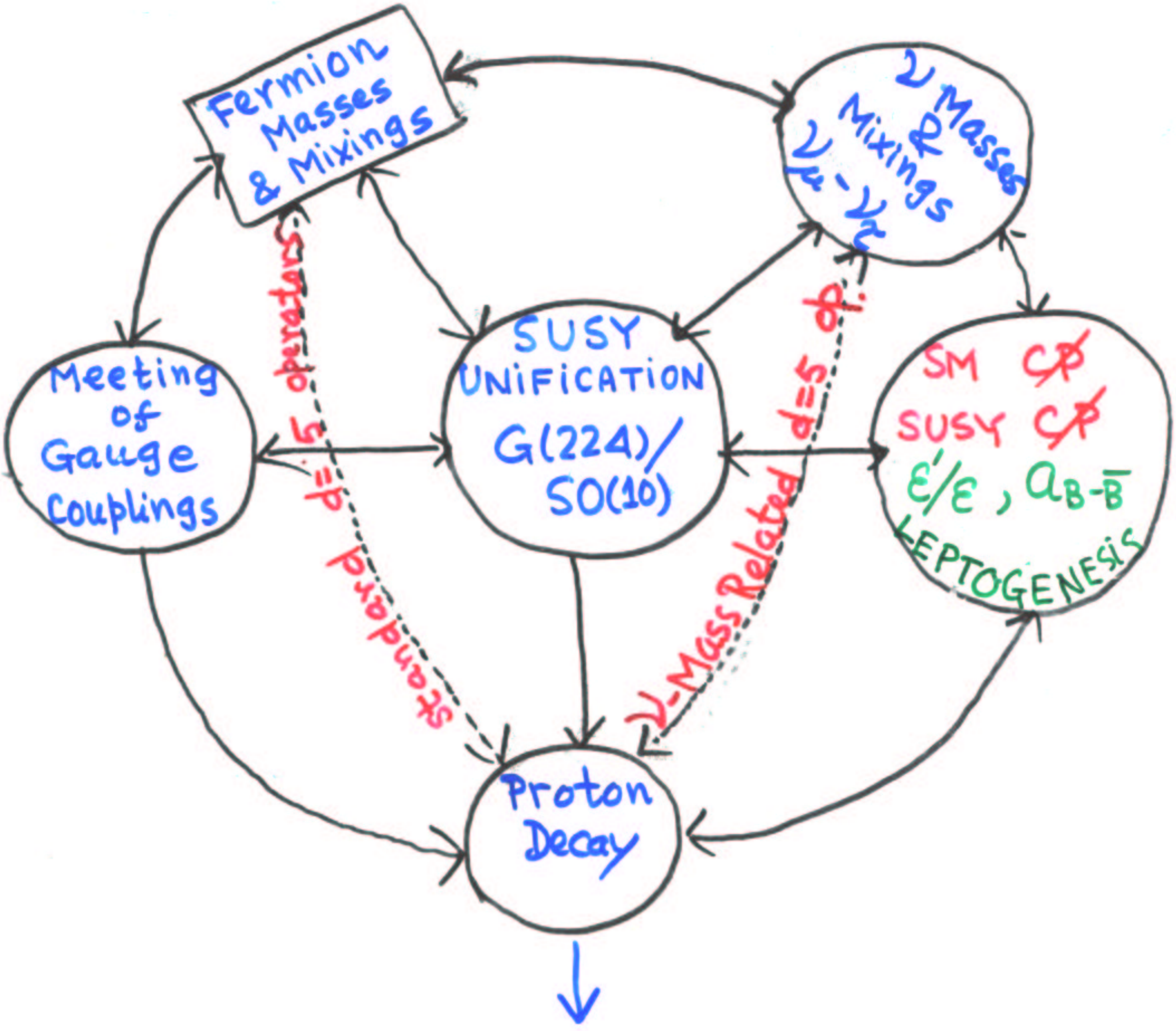
Pati/hep-ph/0209160

Supports the Exist. of ν_R // Spont. Viol of B-L at High Temp // G(224)/SO(10) Mass Matrices,

Exist of ν_R // Non-Zero ν-Masses &

ν-Oscillation $\longrightarrow$ $\boxed{\text{A NECESSARY SUBTELITY}}$

② UNIFICATION LINKS



The Missing Link

III Proton Decay : The Hall Mark of Grand Unification

One can argue that a gauge unification of $\{q \text{ and } \ell\}$

$\Rightarrow$ (B, L) Non-Conservation

$\Downarrow$ Simplest

$\boxed{\text{Proton Decay}}$ 1973

$$
\begin{array}{llll}
\text{proton} & \longrightarrow & e^+ \; \pi^0 & , \bar{\nu} K^+ , \bar{\nu} \pi^+ \\
B = +1 & & 0 \quad 0 & \\
L = 0 & & -1 \quad 0 &
\end{array}
\left.\right\} \begin{array}{l} \Delta B \neq 0 \\ \Delta L \neq 0 \end{array}
$$

Expt. (SuperK) $\Gamma^{-1}(p \to e^+ \pi^0) \gtrsim 6 \times 10^{33}$ yrs

$\Gamma^{-1}(p \to \bar{\nu} K^+) \gtrsim 2 \times 10^{33}$ yrs

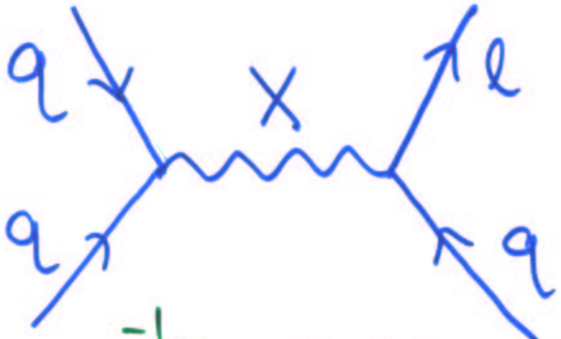
Minimal Non-SUSY SU(5) } $\Gamma^{-1}(p \to e^+ \pi^0)_{\text{Theory}} < 10^{31}$ yrs

Excluded by expt

Also excluded by Gauge Coupling Unif or Prediction of $\sin^2\theta_W$

d = 6 Gauge Mediated : SUSY SU(5)/SO(10)

①



$\Rightarrow$ Amp $(p \to e^+\pi^0)$ $\propto g^2/M_X^2$

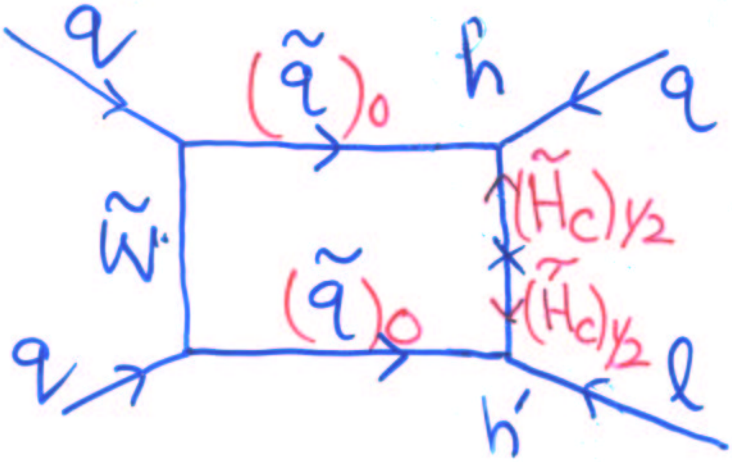
$\Gamma^{-1}(p \to e^+\pi^0) \approx 10^{35.3\pm1.3}$ yrs (Theory)

Can be as short as $\approx \boxed{10^{34}\text{ yrs}}$

② In Supersymmetry

Color triplet Higgsino Mediated

Sakai, Yanagida//
Weinberg.



$qqq \to l$

$$\boxed{\begin{aligned}\tilde{H}_c \subset 10_H &= (2,2,1)_H \\ &+ (1,1,6)\end{aligned}}$$

"Standard" d=5 operator

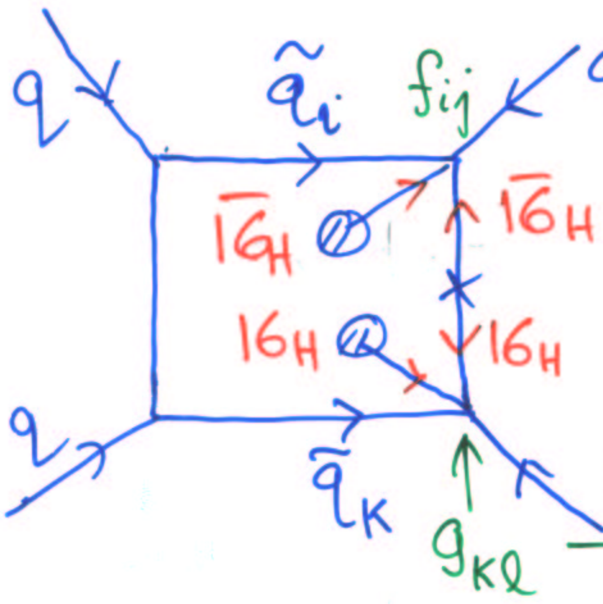
$$\text{Amp} \propto \frac{hh'}{M_{H_c}}\left(m_{\tilde{W}}/m_{\tilde{q}}^2\right)\alpha_2$$

$p \to \bar{\nu} K^+$ dominant

$\to \mu^+ K^0$ (For "standard" d=5, suppressed)

$$\boxed{BR(p \to \mu^+ K^0)^{\text{std}}_{d=5} \sim 10^{-3}}$$

③ New $d=5$ operators ↔ Majorana Masses of ν_R's



$\Updownarrow$

$f_{ij}\ 16_i\, 16_j\, \overline{16}_H\, \overline{16}_H / M$

$\downarrow$

Generically $16_i\, \overline{16}_H$ in 45 & 1 of SO(10)

ENTERS INTO DIRAC MASS MATRICES ↔ $V_{CKM} \neq \mathbb{1}$

Babu, Pati & Wilczek (1997)

$\Downarrow$

$$\Gamma^{-1}(p \to \bar{\nu} K^+)_{\text{New } d=5} \approx 10^{33} - 10^{34} \text{ yrs.}$$

$\Downarrow$

$$BR(p \to \mu^+ K^0)_{\text{New } d=5} \approx (10-30)\%$$

SUSY SO(10) or G(224)

Note that these contribs would generically be present, even if "standard" $d=5$ operators med. by color triplets $\subset 10_H$ are absent, as in SUSY G(224)

The $\mu^+ K^0$ mode A Signature of this Mechanism

$d = 5$ Rate : Update : JCP – ph/0204240

$$\Gamma^{-1}(p\to\bar{\nu}K^{+}) \approx (0.15\times10^{31}\,\text{yrs})\times(0.32/A_L)^2\times(0.93/A_s)^2$$

$$\times\left(\frac{0.014\,\text{GeV}^3}{\beta_H}\right)^2\times\left(\frac{1/6}{m_{\tilde{W}}/m_{\tilde{q}}}\right)^2\times\left(\frac{m_{\tilde{q}}}{1.2\,\text{TeV}}\right)^2\times\left[\frac{2\times10^{-24}\,\text{GeV}^{-1}}{\hat{A}(\bar{\nu})}\right]^2$$

Calculate in Correlation with

a) fermion Masses & Mixings

b) Unif. Scale Threshold Corr. (Right sign & magnitude) $\sim 5\%$ in $\alpha_3(m_Z)$

c) New $d=5$ op related to ν Masses

① Matrix Element $\beta_H = (0.014\,\text{GeV}^3)(1/2 - 2)$

② $\tan\beta \geq 3$

③ $m_{\tilde{q}} \approx (1.2\,\text{TeV})\left(\frac{1}{\sqrt{2}} - 2\right)$ (Focus Point SUSY)

④ $(m_{\tilde{W}}/m_{\tilde{q}}) \approx (1/6)\left(\frac{1}{2} - 2\right)$

⑤ $A_L \approx 0.32$ (2 loop)

⑥ $A_s \approx 0.93$

$$\Downarrow$$

$$\boxed{\Gamma_{\text{now}} \approx 16\,\Gamma_{\text{before}}}$$

Summary on proton Decay : d = 5

SUSY SU(5) / MSSM } $\boxed{\Gamma^{-1}(p\to\bar{\nu}K^{+}) \lesssim 2\times10^{31}\ \text{yrs}}$ Excluded by SuperK // IMB

SUSY SO(10) / MSSM } $\boxed{\Gamma^{-1}(p\to\bar{\nu}K^{+}) \lesssim 1.9\times10^{33}\ \text{yrs}}$ Tightly Constrained (Std. d=5)

SUSY SO(10) / ESSM } $\boxed{\Gamma^{-1}(p\to\bar{\nu}K^{+}) \lesssim 2\times10^{34}\ \text{yrs}}$ Std. d = 5

SUSY G(224)/SO(10) / MSSM or ESSM } $\boxed{\Gamma^{-1}(p\to\bar{\nu}K^{+}) \approx 10^{33}-10^{34}\ \text{yrs}}$ New d = 5 $\updownarrow$ (ν Masses)

Last Two fully Compatible with present limits, Must be seen with improvement by factor of 5-10, or else SUSY SO(10)/ or String G(224), which is otherwise so successful, would have a major set back.

proton decay : d = 6 : SUSY SU(5)/SO(10) //

and Flipped SU(5) $\boxed{\Gamma^{-1}(p\to e^{+}\pi^{0}) \approx 10^{35.3\pm1.3}\ \text{yrs}}$

I) PROBING INTO NEW PHYSICS

Process	Tests	Dist. Scale
PROTON DECAY $p \to \underbrace{\bar{\nu} K^+, \mu^+ K^0}, \underbrace{e^+ \pi^0}$ $\lambda\, QQQL/M$; g^2/M_X^2	q-ℓ unif. q-$\bar{q}$, q-$\bar{\ell}$, ℓ-$\bar{\ell}$ unif SUSY // Und. Fermion Masses // UNITY OF Forces	$\lesssim 10^{-30}$ cm
NEUTRINO Mass $m(\nu_\tau) \approx m(\nu_\tau)^2_{DIRAC}/M_R$ $\approx 1/20$ eV	$M_R \sim 10^{15}$ GeV $\Delta L = 2$; See Saw q-ℓ unif ↔ SU(4)-col SUSY unif. scale	$\sim 10^{-29}$ cm
n-$\bar{n}$ oscillation	$\Delta B = 2$	10^{-19}-10^{-20} cm
$\mu \to e\gamma$, $\tau \to \mu\gamma$, ..	Flavor Viol, SUSY	10^{-17}-10^{-18} cm
$\delta(g-2)_\mu$	SUSY // Heavy Fermions	10^{-16}-10^{-18} cm
K^0-$\bar{K}^0$, B^0-$\bar{B}^0$	Flavor, $\cancel{CP}$	10^{-17}-10^{-18} cm
$(edm)_{n,e}$	$\cancel{CP}$ & $\cancel{P}$	10^{-17}-10^{-18} cm

Proton Decay is the Hallmark of Grand Unification.

It Provides a unique window to view physics at truly short distances $\lesssim 10^{-30}$ cm.

$p \to \bar{\nu} K^{+}$ if seen $\Longrightarrow$ { $q-\ell$, $q-\bar{q}$, $q-\bar{\ell}$ unif // SUSY unif // $M_c \sim M_X \sim 10^{16}-10^{17}$ GeV

↓

$p \to \mu^{+} K^{0}$ if seen $\Longrightarrow$ { ⊕ Relevance of ν-Mass Related operator

THUS PROTON DECAY IF SEEN WOULD PROVIDE A WEALTH OF KNOWLEDGE THAT CAN NOT BE GAINED BY ANY OTHER MEANS

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Discovery Potential High

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THIS is WHY NEED NEXT-GENERATION DETECTOR FOR PROTON DECAY & NEUTRINO-OSCILLATION → UNO // HYperKamiokande.