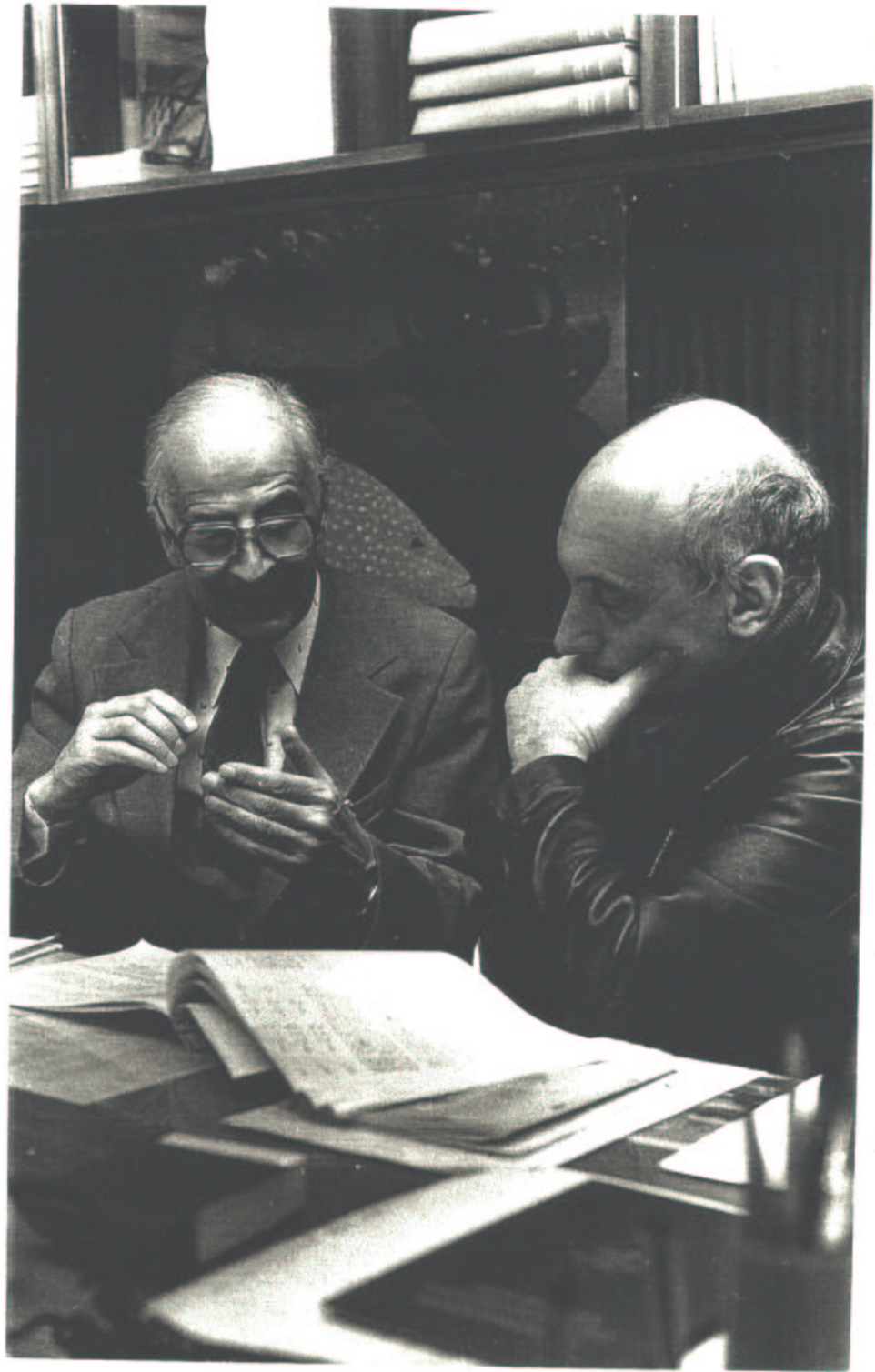


SOLAR NEUTRINO DATA, NEUTRINO MASS  
SPECTRUM, CP-VIOLATION AND  
 $(\beta\beta)_{0\nu}$ -DECAY

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BULGARIA

"NEUTRINOS AND..."  
YITP, SUNY  
STONY BROOK  
OCTOBER 10-13, 2002



## EVIDENCES FOR $\nu$ -OSCILLATIONS:

-  $\nu_{\text{ATM}}$  : SK

UP-DOWN ASYMMETRY

(ZENITH ANGLE DEPENDENCE)

MULTI-GEV  $\mu$ -LIKE SAMPLE

DOMINANT

$$\overline{\nu}_{\mu} \rightarrow \overline{\nu}_{\tau}$$

K2K; MINOS, CNGS.

-  $\nu_{\odot}$  :

HOMESTAKE, KAMIOKANDE,

SAGE, GALLEX/GNO,

SUPER-KAMIOKANDE,

SNO

DOMINANT

$$\nu_e \rightarrow \nu_{\mu, \tau}$$

KAMLAND; BOREXINO, ...

- LSND

$$\overline{\nu}_{\mu} \rightarrow \overline{\nu}_e$$

MINIBOONE

$$\nu_{\ell L} = \sum_{j=1} U_{\ell j} \nu_{j L}, \quad \ell = e, \mu, \tau$$

$\nu$ - FACTORIES : 3- $\nu$  MIXING, LMA MSW

$L \sim (3000 - 7000) \text{ km.}$

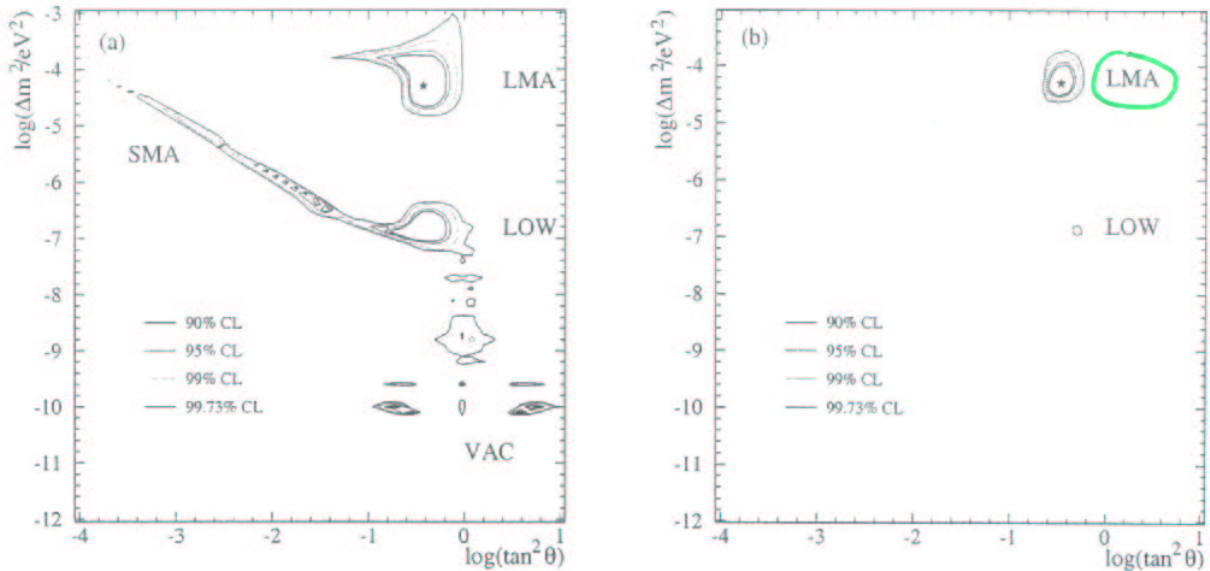


FIG. 4: Allowed regions of the MSW plane determined by a  $\chi^2$  fit to (a) SNO day and night energy spectra and (b) with additional experimental and solar model data. The star indicates the best fit. See text for details.

- SNO web site: <http://sno.phy.queensu.ca>
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- [13] S. Nakamura *et al.* (2002), nucl-th/0201062.
- [14] J. N. Bahcall, H. M. Pinsonneault and S. Basu, Astrophys. J **555**, 990 (2001).
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- [16] J. N. Abdurashitov *et al.*, Phys. Rev. C **60**, 055801 (1999).
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$$\Delta m_{\odot}^2 [\text{eV}^2] > 0 \quad \sin^2 2\theta_{\odot}$$

B. F. V.:

$$5 \times 10^{-5}$$

$$0.75$$

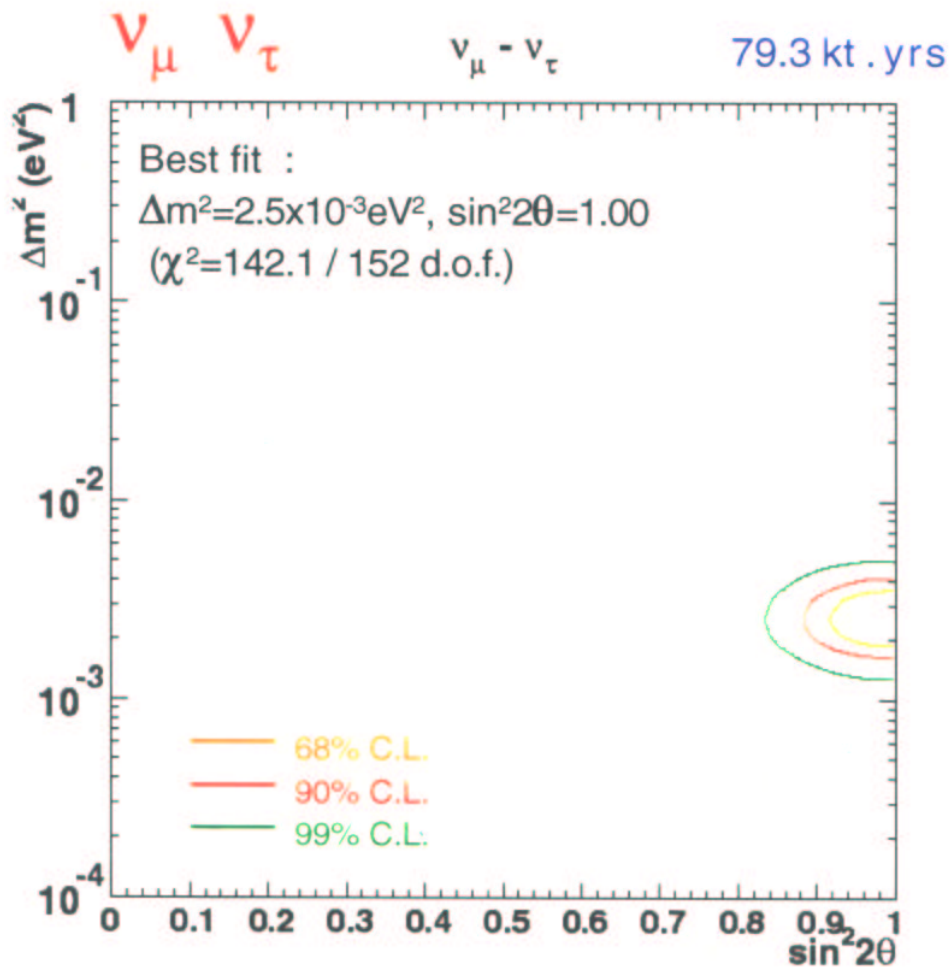
95% C.L.

$$(2.5 - 10) \times 10^{-5}$$

$$0.56 - 0.89$$

$$\cos 2\theta_{\odot} \geq 0.26 \quad \text{AT} \quad 99.73\% \text{ C.L.}$$

Allowed region  
(FC + PC + UP-thru + UP-stop)



SK combined result

$\Delta m^2 = (1.7 \sim 4) \times 10^{-3} \text{eV}^2$

$\sin^2 2\theta > 0.89$  (90% C.L.)

**sign( $\Delta m^2$ ) - UNDETERMINED**

**3 → MIXING :**  $m_1 < m_2 < m_3$  - **NH**

OR  $m_3 < m_1 < m_2$  - **IH**

CHOOZ :  $\bar{\nu}_e \rightarrow \bar{\nu}_e$

$\sim 1 \text{ km}$   
 $E_{\nu} \sim 2 \text{ MeV}$

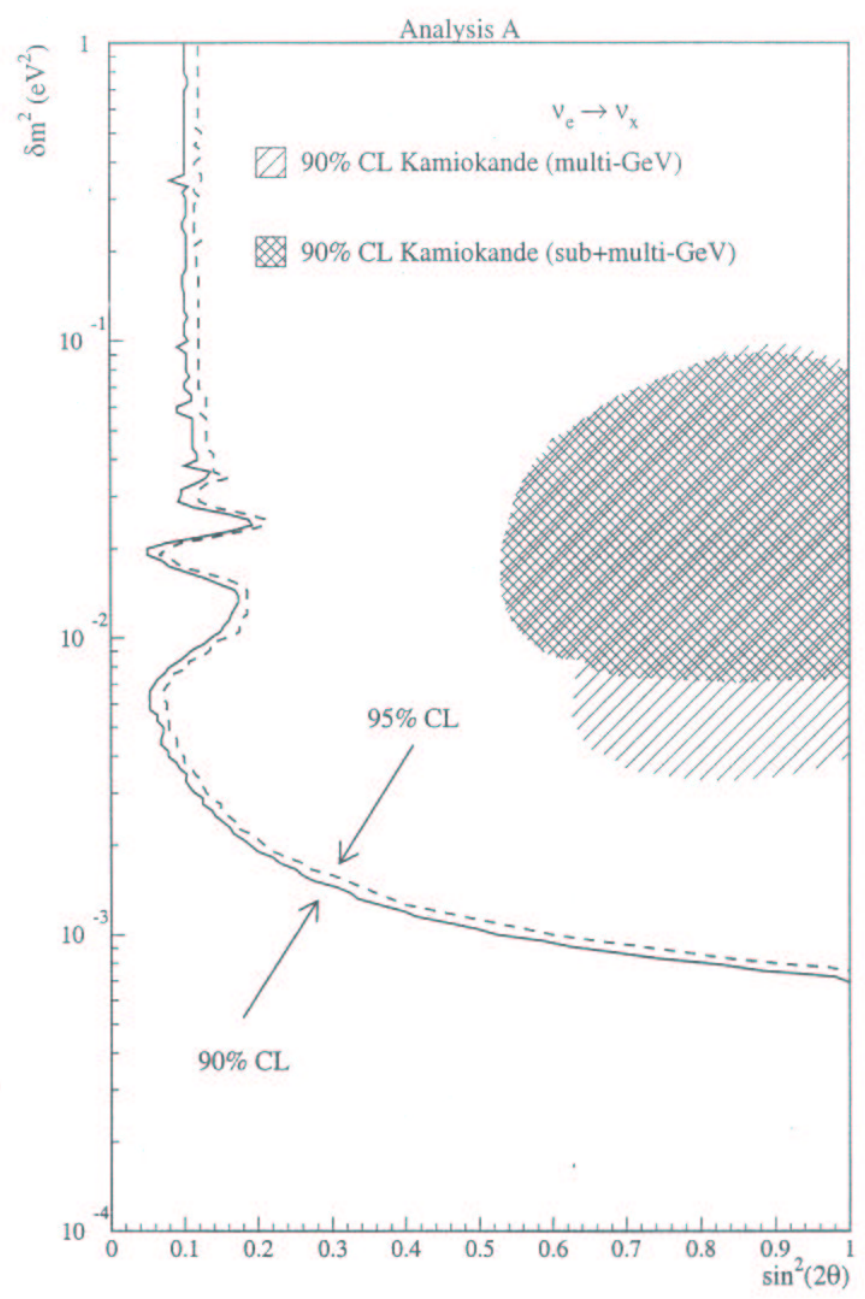


Figure 9: Exclusion plot for the oscillation parameters based on the absolute comparison of measured vs. expected positron yields.

## 3-ν MIXING:

$$\begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}$$

$U_{PMNS}$

PARAMETERS:  $U_{PMNS} = n \times n$

	$n$	2	3	4
<u>ANGLES</u>	$\frac{n(n-1)}{2}$	1	3	6

CP-VIOLATING  
PHASES:

$\nu_j$  - DIRAC     $\frac{(n-1)(n-2)}{2}$     0    1    3

$\nu_j$  - MAJORANA     $\frac{n(n-1)}{2}$     1    3    6

BILENKY, HOSEK, PETCOV '80;  
DOI ET AL., '81.

## STANDARD PARAMETRIZATION:

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13}e^{i\alpha_{21}/2} & U_{e3}e^{i(\alpha_{12}/2+\delta)} \\ -s_{12}c_{23}-c_{12}s_{23}U_{e3}^* & (c_{12}c_{23}-s_{12}s_{23}U_{e3}^*e^{i\alpha_{21}}) & s_{23}c_{13}e^{i(\alpha_{12}/2+\delta)} \\ s_{12}s_{23}-c_{12}c_{23}U_{e3}^* & (c_{12}s_{23}-s_{12}c_{23}U_{e3}^*e^{i\alpha_{21}}) & c_{23}c_{13}e^{i(\alpha_{12}/2+\delta)} \end{pmatrix}$$

$$c_{ij} \equiv \cos \theta_{ij}, \quad s_{ij} \equiv \sin \theta_{ij}, \quad 0 \leq \theta_{12}, \theta_{13}, \theta_{23} \leq \pi/2$$

$$U_{e3} = s_{13}e^{-i\delta}, \quad \delta \in [0, 2\pi] - \text{DIRAC CP-VIOLATING PHASE}$$

$$\alpha_{21}, \alpha_{31} - \text{MAJORANA CP-VIOLATING PHASES}$$

IF  $\nu_j$  ARE MAJORANA PARTICLES, S.M. BILENKY, J. HOSOKI,  
S.T.P. '80  
CP-SYMMETRY CAN BE VIOLATED EVEN IN THE  
CASE OF  $n=2$  FAMILIES OF LEPTONS:

$$N_{CP}^M = \frac{n(n+1)}{2} - n = \frac{n(n-1)}{2}$$

# "PARAMETERS" :

$$\theta_{12}, \theta_{13}, \theta_{23}$$

$\nu_j$

DIRAC

MAJORANA

$$\delta$$

$$\delta, d_{21}, d_{31}$$

$$m_1, m_2, m_3$$

$m_{1,2,3}$  : MEASURED IN  $\nu$ -OSCILLATION EXPERIMENTS

$$\Delta m_{\odot}^2 = \Delta m_{21}^2 > 0, |\Delta m_{ATM}^2| = |\Delta m_{31}^2|$$

(A)

$$m_1 < m_2 < m_3 \quad \text{OR} \quad m_3 < m_1 < m_2$$

(B)

$$m_1 < m_2 < m_3 :$$

$$m_1, m_2, m_3 \Rightarrow m_1, \Delta m_{21}^2 > 0, \Delta m_{32}^2 > 0$$

$$m_2 = \sqrt{m_1^2 + \Delta m_{21}^2}$$

$$m_3 = \sqrt{m_1^2 + \Delta m_{21}^2 + \Delta m_{32}^2}$$

$$\Delta m_{ATM}^2 = \Delta m_{31}^2 = \Delta m_{21}^2 + \Delta m_{32}^2$$

## TWO POSSIBILITIES :

$$\Delta m_{\odot}^2 = \Delta m_{21}^2 - NH$$

$$\Delta m_{\odot}^2 = \Delta m_{32}^2 - IH$$

"DISCRETE" PARAMETER

## THE MAIN PROBLEMS IN THE STUDIES OF $\nu$ -MIXING :

- DETERMINATION OF  $\Delta m_{\odot}^2, \theta_{\odot}, \Delta m_{atm}^2, \theta_{atm}$  WITH A HIGH PRECISION
- MEASURE, OR IMPROVE BY AT LEAST A FACTOR OF  $\sim 10$  THE EXISTING LIMITS ON,  $|U_{e3}|^2 = \sin^2 \theta_{13}$
- DETERMINE THE TYPE OF  $\nu$ -MASS SPECTRUM,  
 $m_1 \ll m_2 \ll m_3$ , NH  
 $m_1 \ll m_2 \cong m_3$ , IH  
 $m_1 \cong m_2 \cong m_3, m_j^2 \gg \Delta m_{atm}^2$ , QD
- DETERMINE, OR OBTAIN CONSTRAINTS ON, THE ABSOLUTE  $\nu$ -MASS SCALE (LIGHTEST  $\nu$  MASS)
- DETERMINE THE NATURE OF  $\nu_j$  (DIRAC VS MAJORANA)
- ESTABLISH WHETHER CP-SYMMETRY IS VIOLATED IN THE LEPTON SECTOR  
1. DUE TO THE DIRAC PHASE,  $\delta$ ,  
2. DUE TO THE MAJORANA PHASES  $\alpha_{21}, \alpha_{31}$ , IF  $\nu_j$  - MAJORANA
- FIND A THEORY WHICH EXPLAINS THE DATA (THE THEORY)

$\nu_e$  - DATA :  $\Delta m_{21}^2, \theta_{12}$

SNO: D-N, CC/NC, SPECTRUM CC

$A_{D-N} < 10\%$ ,  $R_{CC/NC} < 0.50$  CAN  
SIGNIFICANTLY REDUCE THE LMA  
REGION

$A_{D-N} > 10\%$  - ELLIMINATES LOW-QVO

$A_{D-N} > 1\%$  - ELLIMINATES QVO

KAMLAND: LMA  $L \approx 180$  km

IF  $\Delta m_{21}^2 \approx 2 \times 10^{-4} \text{ eV}^2$ , DIST KAMLAND  $L \sim 20$  km

BOREXINO: D-N LOW, ...

$\nu_{ATM}$  - DATA (SK):  $|\Delta m_{31}^2|, \theta_{23}, \theta_{13}?$

K2K, MINOS, CNRS

SUPERBEAMS

VLBL ( $\nu$ -FACTORY):  $\delta, \theta_{13}, (\theta_{23})$

$\nu$  - MASS SPECTRUM:  $^3\text{H}$   $\beta$ -DECAY,  $(\beta\beta)_{0\nu}$ -DECAY

MAJORANA  $\nu_j$ :  $(\beta\beta)_{0\nu}$ -DECAY  
 $\alpha_{21}, \alpha_{31}$

## KAMLAND:

- OBSERVES  $e^+$ -SPECTRUM DISTORTIONS

$$\Delta m_{\odot}^2 \cong (2 \cdot 10^{-5} - 10^{-4}) \text{ eV}^2$$

- DOES NOT OBSERVE  $e^+$ -SPECTRUM DISTORTION

BUT MEASURES JUST A REDUCTION OF THE TOTAL EVENT RATE BY

$$\sim \left(1 - \frac{1}{2} \sin^2 2\theta_{\odot} (1 - |U_{e3}|^2)^2\right)$$

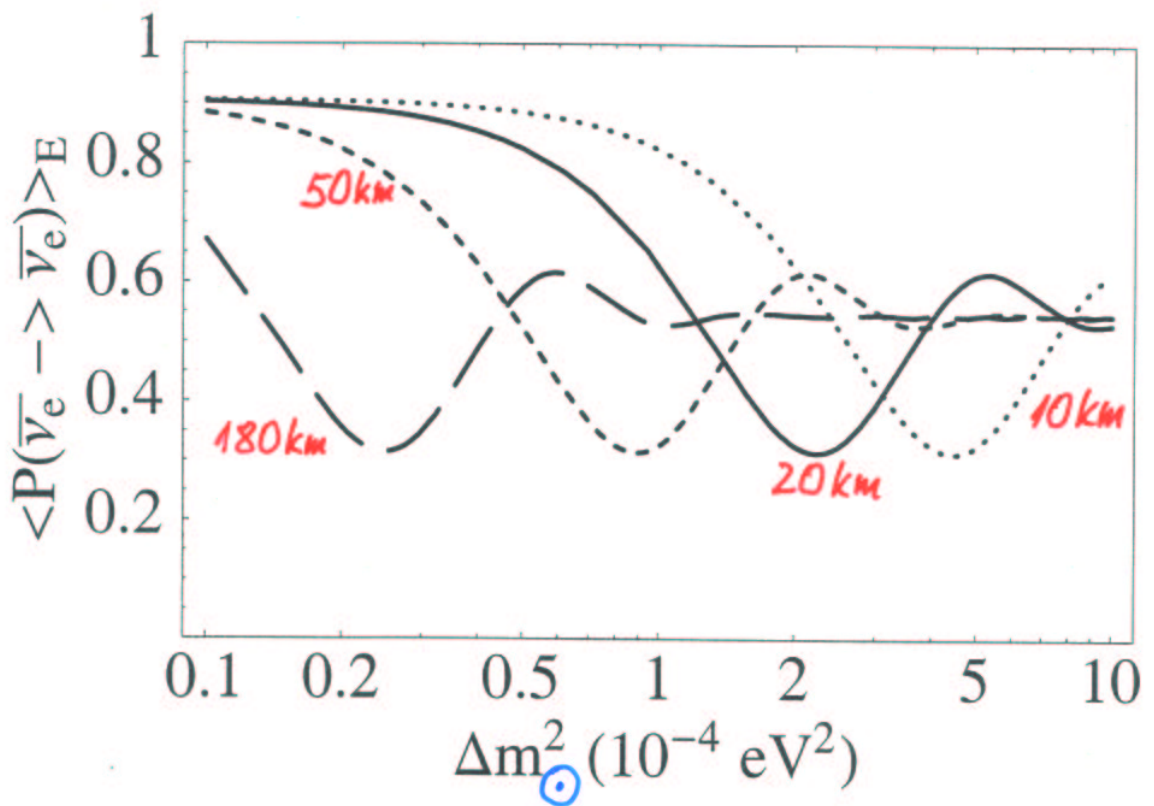
$$\Delta m_{\odot}^2 \gtrsim 2 \cdot 10^{-4} \text{ eV}^2$$

- DOES NOT OBSERVE ANY DEVIATIONS FROM THE  $ST$  PREDICTIONS

PROBLEM WITH LMA?

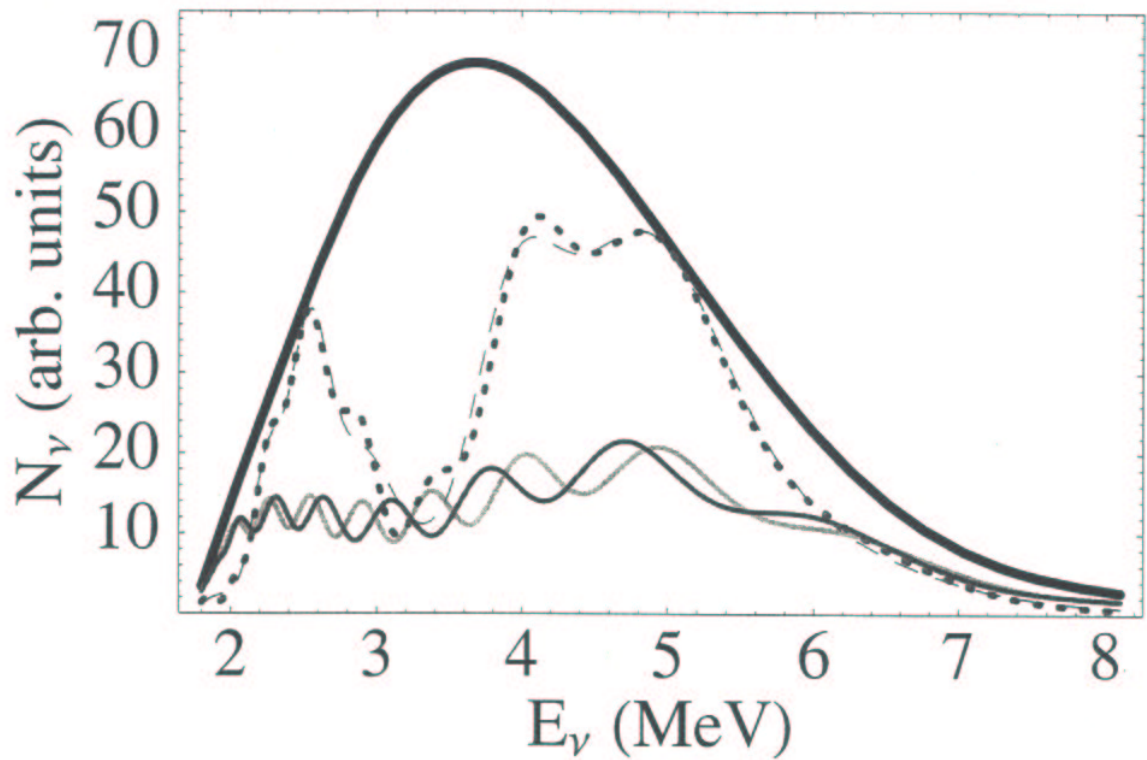
LOW,  $QVQ$ ,  $VQ$ ,  $RSFP$ ,  $FCNC$ , ... ?

FIGURE N. 1



$\Delta m_{31}^2 = 2.5 \cdot 10^{-3} \text{ eV}^2$ ,  $\sin^2 2\theta_{\odot} = 0.8$ ,  $\sin^2 \theta_{CH} = 0.05$

FIGURE N. 2



$$\Delta m_{\odot}^2 = 2 \cdot 10^{-4} \text{ eV}^2; 6 \cdot 10^{-4} \text{ eV}^2$$

$$L = 20 \text{ km}$$

$$10^{-4} \text{ eV}^2 \leq \Delta m_{\odot}^2 \leq 8 \cdot 10^{-4} \text{ eV}^2$$

CAN BE EXPLORED

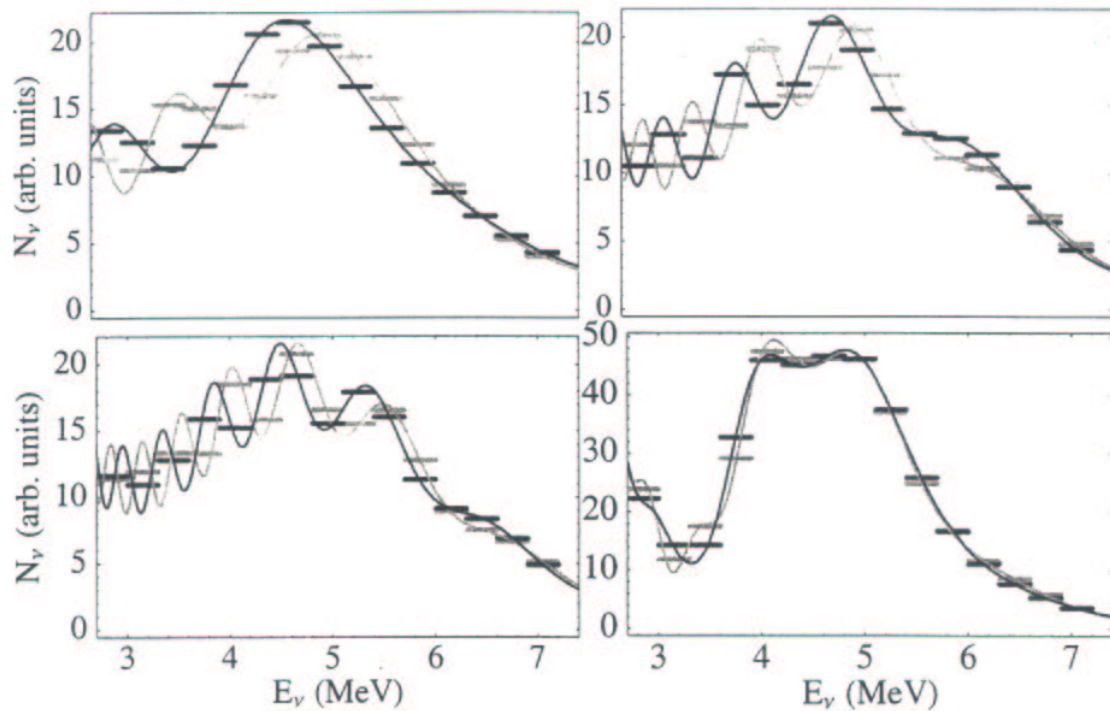


Figure 3: Comparison between the predicted event rate spectrum at  $L = 20$  km, measured in energy bins having a width of  $\Delta E_\nu = 0.3$  MeV in the cases of normal (light grey) and inverted (dark grey) neutrino mass hierarchy. The two upper and the lower left figures are for  $\Delta m_{21}^2 = 2 \times 10^{-4}$  eV<sup>2</sup>,  $\sin^2 2\theta_{12} = 0.8$ ,  $\sin^2 \theta = 0.05$ , and  $\Delta m_{31}^2 = 1.3; 2.5; 3.5 \times 10^{-3}$  eV<sup>2</sup> respectively. The lower right figure was obtained for  $\Delta m_{21}^2 = 6 \times 10^{-4}$  eV<sup>2</sup> and  $\Delta m_{31}^2 = 2.5 \times 10^{-3}$  eV<sup>2</sup>.

Normal vs. Inverted hierarchy:

$L = 20$  km, KAMLAND LIKE DETECTOR

$$2 \cdot 10^{-4} \text{ eV}^2 \lesssim \Delta m_{21}^2 \lesssim 5 \cdot 10^{-4} \text{ eV}^2$$

EXTREMELY CHALLENGING!

## 2 The $\bar{\nu}_e$ Survival Probability

We assume 3-neutrino mixing. The neutrinos with definite mass in vacuum  $\nu_j$ ,  $j = 1, 2, 3$ , are numbered (without loss of generality) in such a way that their masses obey  $m_1 < m_2 < m_3$ . We do not assume any of the relations  $m_1 \ll m_2 \ll m_3$ , or  $m_1 \lesssim m_2 \ll m_3$ , or  $m_1 \ll m_2 \cong m_3$ .

In the case of normal hierarchy between the neutrino masses we have:

$$\Delta m_{\odot}^2 = \Delta m_{21}^2, \quad (6)$$

and

$$|U_{e1}| = \cos \theta_{\odot} \sqrt{1 - |U_{e3}|^2}, \quad |U_{e2}| = \sin \theta_{\odot} \sqrt{1 - |U_{e3}|^2}, \quad (7)$$

where

$$\theta_{\odot} = \theta_{12}, \quad |U_{e3}|^2 = \sin^2 \theta \equiv \sin^2 \theta_{13}, \quad (8)$$

$\theta_{12}$  and  $\theta_{13}$  being two of the three mixing angles in the standard parameterization of the PMNS matrix.  $|U_{e3}|^2$  is constrained by the CHOOZ and Palo Verde results. The  $\bar{\nu}_e$  survival probability has the form:

$$\begin{aligned} P_{NH}(\bar{\nu}_e \rightarrow \bar{\nu}_e) &= 1 - 2 \sin^2 \theta \cos^2 \theta \left( 1 - \cos \frac{\Delta m_{31}^2 L}{2 E_{\nu}} \right) \\ &- \frac{1}{2} \cos^4 \theta \sin^2 2\theta_{\odot} \left( 1 - \cos \frac{\Delta m_{\odot}^2 L}{2 E_{\nu}} \right) \\ &+ 2 \sin^2 \theta \cos^2 \theta \sin^2 \theta_{\odot} \left( \cos \left( \frac{\Delta m_{31}^2 L}{2 E_{\nu}} - \frac{\Delta m_{\odot}^2 L}{2 E_{\nu}} \right) - \cos \frac{\Delta m_{31}^2 L}{2 E_{\nu}} \right). \end{aligned} \quad (9)$$

For the neutrino mass spectrum with inverted hierarchy one has:

$$\Delta m_{\odot}^2 = \Delta m_{32}^2, \quad (10)$$

and

$$|U_{e2}| = \cos \theta_{\odot} \sqrt{1 - |U_{e1}|^2}, \quad |U_{e3}| = \sin \theta_{\odot} \sqrt{1 - |U_{e1}|^2}. \quad (11)$$

The mixing matrix element constrained by the CHOOZ and Palo Verde data is now  $|U_{e1}|^2$ :

$$|U_{e1}|^2 = \sin^2 \theta. \quad (12)$$

The expression for the  $\bar{\nu}_e$  survival probability can be written in the form:

$$\begin{aligned} P_{IH}(\bar{\nu}_e \rightarrow \bar{\nu}_e) &= 1 - 2 \sin^2 \theta \cos^2 \theta \left( 1 - \cos \frac{\Delta m_{31}^2 L}{2 E_\nu} \right) \\ &- 12 \cos^4 \theta \sin^2 2\theta_\odot \left( 1 - \cos \frac{\Delta m_\odot^2 L}{2 E_\nu} \right) \\ &+ 2 \sin^2 \theta \cos^2 \theta \cos^2 \theta_\odot \left( \cos \left( \frac{\Delta m_{31}^2 L}{2 E_\nu} - \frac{\Delta m_\odot^2 L}{2 E_\nu} \right) - \cos \frac{\Delta m_{31}^2 L}{2 E_\nu} \right). \end{aligned} \quad (13)$$

# IF $\nu_j$ - MAJORANA PARTICLES,

BILENKY ET AL. '80  
DOI ET AL. '81

$U_{PMNS}$  - CONTAINS  
3D MIXING

$\delta$  - DIRAC  
 $\alpha_{21}, \alpha_{31}$  - MAJORANA  
PHYSICAL CP-VIOLATING  
PHASES

$\nu$  - OSCILLATIONS  $\bar{\nu}_l \rightleftharpoons \bar{\nu}_{l'}$ ,  $l, l' = e, \mu, \tau$

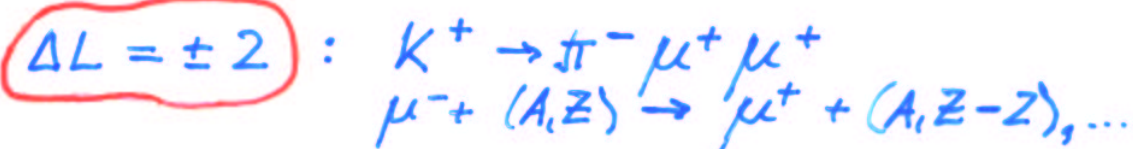
BILENKY, HOSEK, STP '80

- ARE NOT SENSITIVE TO THE NATURE OF  $\nu_j$ ;
- PROVIDE INFORMATION ON  $\Delta m_{jk}^2 = m_j^2 - m_k^2$ ,  $j > k = 1, 2, 3$   
BUT NOT ON THE ABSOLUTE VALUES OF  
NEUTRINO MASSES  $m_j$ .

HOW CAN ONE OBTAIN INFORMATION ON

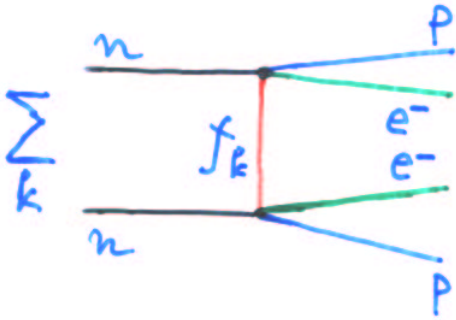
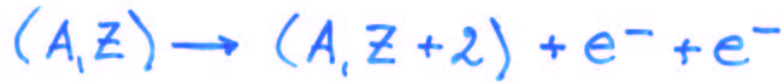
- THE NATURE OF  $\nu_j$  ?
- $m_1, m_2, m_3$ , i.e., ON THE  $\nu$ -MASS SPECTRUM ?
- ON THE CP-VIOLATION IN THE LEPTON SECTOR, INDUCED BY THE MAJORANA CP-VIOLATING PHASES ?

THE MAJORANA NATURE OF  $\nu_j$  CAN MANIFEST ITSELF  
IN THE EXISTENCE OF PROCESSES



THE MOST SENSITIVE PROCESS -  $(\beta\beta)_{\nu\nu}$  - decay  
OF CERTAIN EVEN-EVEN NUCLEI :

$(\beta\beta)_{0\nu}$  - decay:



$$\vec{j}_{eL}(\alpha) = \sum_k U_{ek} \underbrace{f_{kL}(\alpha)}_{m_k}$$

$$\mathcal{H}_W^{\beta} = \frac{G_F}{\sqrt{2}} 2 (\bar{e}_L(\alpha) \gamma_\alpha \vec{j}_{eL}(\alpha)) \underbrace{f_{\alpha}^R(\alpha)}_{m_k} + h.c.$$

$$S^{(2)} = - \frac{(-i)^2}{2} 4 \left( \frac{G_F}{\sqrt{2}} \right)^2 \int dx_1 dx_2 \times T_{\alpha\beta}^R(x_1, x_2) \times$$

$$\times N \left[ \bar{e}_L(x_1) \gamma_\alpha \overbrace{\vec{j}_{eL}(x_1) \vec{j}_{eL}^T(x_2)} \gamma_\beta^T \bar{e}_L^T(x_2) \right]$$

|  $A(\beta\beta)_{0\nu} \sim \langle m_\nu \rangle$ ,

$$\langle m_\nu \rangle = \sum_k |U_{ek}|^2 \frac{1}{i} \underbrace{\eta^{CP}(f_k)}_{m_k} m_k$$

$m_k \lesssim \text{few MeV}$   
CP - inv.

Data:  
 $^{76}\text{Ge}$

$$|\langle m_\nu \rangle| < (1 \div 2) \text{ eV}$$

The Majorana nature of the massive neutrinos can manifest itself in the existence of  $L \neq 0$ ,  $\Delta L = 2$ , processes. The process most sensitive to the existence of massive Majorana neutrinos (coupled to the electron) is the neutrinoless double  $\beta$  ( $(\beta\beta)_{0\nu}$ ) decay of certain even-even nuclei

$$(A, Z) \rightarrow (A, Z + 2) + e^- + e^- \quad (1)$$

If the  $(\beta\beta)_{0\nu}$ -decay is generated *only by the left-handed (LH) charged current weak interaction through the exchange of virtual massive Majorana neutrinos*,  $A((\beta\beta)_{0\nu})$  is proportional to the so-called “effective Majorana mass”

$$A((\beta\beta)_{0\nu}) \sim \langle m \rangle \equiv \sum_{j=1} U_{ej}^2 m_j, \quad m_j \lesssim \text{few MeV}, \quad (2)$$

where  $m_j$  is the mass of the Majorana neutrino  $\nu_j$  and  $U_{ej}$  is the element of neutrino (lepton) mixing matrix  $U$ .

A large number of experiments are searching for  $(\beta\beta)_{0\nu}$ -decay of different nuclei at present. No indications that this process takes place were found.

$^{76}\text{Ge}$  Heidelberg-Moscow experiment:

$$|\langle m \rangle| < 0.35 \text{ eV}, \quad 90\% \text{ C.L.} \quad (3)$$

Taking into account a factor of 3 uncertainty associated with the calculation of the relevant nuclear matrix element

$$|\langle m \rangle| < (0.35 \div 1.05) \text{ eV}, \quad 90\% \text{ C.L.} \quad (4)$$

The IGEX collaboration has obtained [30]:

$$|\langle m \rangle| < (0.33 \div 1.35) \text{ eV}, \quad 90\% \text{ C.L.} \quad (5)$$

Considerably higher sensitivity to the value of  $|\langle m \rangle|$  is planned to be reached in several  $(\beta\beta)_{0\nu}$ -decay experiments of a new generation:

- the **NEMO3** experiment scheduled to start in 2002, will search for  $(\beta\beta)_{0\nu}$ -decay of  $^{100}\text{Mo}$  and  $^{82}\text{Se}$ ; will reach a sensitivity to  $|\langle m \rangle| \cong 0.1 \text{ eV}$ .

- **CUORE**: a similar sensitivity is planned to be reached with the cryogenic detector CUORE; will search for the  $(\beta\beta)_{0\nu}$ -decay of  $^{130}\text{Te}$ .

- **GENIUS**: sensitivity to  $|\langle m \rangle| \cong 10^{-2} \text{ eV}$ , is planned to be achieved utilizing one ton of enriched  $^{76}\text{Ge}$ .

- **EXO**: proposal to study the  $(\beta\beta)_{0\nu}$ -decay of  $^{136}\text{Xe}$  in a background-free experiment with detection of the two  $e^-$  and the  $^{+}\text{Ba}$  atom in the final state; the estimated sensitivity of this experiment is  $|\langle m \rangle| \cong (1 - 5) \times 10^{-2} \text{ eV}$ .

As is well known, the explanation of the atmospheric and solar neutrino data in terms of neutrino oscillations requires the existence of 3-neutrino mixing in the weak charged lepton current:

$$\nu_{lL} = \sum_{j=1}^3 U_{lj} \nu_{jL}, \quad (5)$$

where  $\nu_{lL}$ ,  $l = e, \mu, \tau$ , are the three left-handed flavour neutrino fields,  $\nu_{jL}$  is the left-handed field of the neutrino  $\nu_j$  having a mass  $m_j$  and  $U$  is a  $3 \times 3$  unitary mixing matrix - the Pontecorvo-Maki-Nakagawa-Sakata neutrino (lepton) mixing matrix. If  $\nu_j$  are Majorana neutrinos,

$$C(\bar{\nu}_j)^T = \nu_j, \quad j = 1, 2, 3,$$

$C$  is the charge conjugation matrix, and for  $m_j \lesssim \text{few MeV}$ ,

$$|\langle m \rangle| = |m_1 U_{e1}^2 + m_2 U_{e2}^2 + m_3 U_{e3}^2| \quad (6)$$

$$= |m_1 |U_{e1}|^2 + m_2 |U_{e2}|^2 e^{i\alpha_{21}} + m_3 |U_{e3}|^2 e^{i\alpha_{31}}| \quad (7)$$

where

$$U_{ej} = |U_{ej}| e^{\frac{i\alpha_j}{2}}$$

$$\alpha_{21} \equiv (\alpha_2 - \alpha_1), \quad \alpha_{31} \equiv (\alpha_3 - \alpha_1)$$

are two CP-violating phases. If CP-invariance holds, one has

$$\alpha_{21} = k\pi, \quad \alpha_{31} = k'\pi, \quad k, k' = 0, 1, 2, \dots$$

In this case

$$\eta_{21} \equiv e^{i\alpha_{21}} = \pm 1, \quad \eta_{31} \equiv e^{i\alpha_{31}} = \pm 1, \quad (8)$$

represent the relative CP-parities of the neutrinos  $\nu_1$  and  $\nu_2$ , and  $\nu_1$  and  $\nu_3$ , respectively.

We can numerate (without loss of generality) the neutrino masses in such a way that  $m_1 < m_2 < m_3$ . If we denote by  $\theta_\odot$  and  $\theta$  respectively the mixing angles constrained by the solar neutrino data and the data from the CHOOZ experiment, then depending on the type of the neutrino mass spectrum one has either

$$|U_{e1}| = \cos \theta_\odot \sqrt{1 - |U_{e3}|^2}, \quad |U_{e2}| = \sin \theta_\odot \sqrt{1 - |U_{e3}|^2}, \quad |U_{e3}|^2 = \sin^2 \theta, \quad (9)$$

or

$$|U_{e2}| = \cos \theta_\odot \sqrt{1 - |U_{e1}|^2}, \quad |U_{e3}| = \sin \theta_\odot \sqrt{1 - |U_{e1}|^2}, \quad |U_{e1}|^2 = \sin^2 \theta. \quad (10)$$

Relations (9) are valid for the hierarchical neutrino mass spectrum, while those in eq. (10) are realized for the neutrino mass spectrum with inverted hierarchy.

L. WOLFENSTEIN '81  
B. KAYSER '84  
BILENKY, NEDELICHEVA,  
S.T.P. '84

The neutrino oscillation experiments provide information on  $\Delta m_{jk}^2 = m_j^2 - m_k^2$  ( $j > k$ ). In the case of 3-neutrino mixing as an independent set of three neutrino mass parameters one can choose

$$m_1, \quad \sqrt{\Delta m_{21}^2}, \quad \sqrt{\Delta m_{32}^2}.$$

Then :

$$m_2 = \sqrt{m_1^2 + \Delta m_{21}^2}, \quad (11)$$

$$m_3 = \sqrt{m_1^2 + \Delta m_{21}^2 + \Delta m_{32}^2}. \quad (12)$$

The  $\Delta m^2$  inferred from the atmospheric neutrino data,

$$\Delta m_{\text{atm}}^2 = \Delta m_{31}^2 = \Delta m_{21}^2 + \Delta m_{32}^2, \quad (13)$$

while for the one deduced from the solar neutrino data,  $\Delta m_{\odot}^2$ , we have two possibilities:

$$\Delta m_{\odot}^2 \equiv \Delta m_{32}^2 \quad \text{or} \quad \Delta m_{\odot}^2 \equiv \Delta m_{21}^2. \quad (14)$$

Depending on the relative magnitudes of  $m_1$ ,  $\sqrt{\Delta m_{21}^2}$  and  $\sqrt{\Delta m_{32}^2}$ , one recovers the different possible types of neutrino mass spectrum:

1. if  $m_1 \ll \sqrt{\Delta m_{21}^2} \ll \sqrt{\Delta m_{32}^2}$ , one has  $m_1 \ll m_2 \ll m_3$ , i.e., hierarchical (H) neutrino mass spectrum;
2.  $m_1 \ll \sqrt{\Delta m_{32}^2} \ll \sqrt{\Delta m_{21}^2}$  implies  $m_1 \ll m_2 \simeq m_3$ , i.e., neutrino mass spectrum with inverted hierarchy (IH);
3. for  $\sqrt{\Delta m_{21}^2}, \sqrt{\Delta m_{32}^2} \ll m_1$ , we have  $m_1 \simeq m_2 \simeq m_3$ , i.e., quasi-degenerate (QD) neutrino mass spectrum;
4. if  $\sqrt{\Delta m_{21}^2} \ll \sqrt{\Delta m_{32}^2} \sim \mathcal{O}(m_1)$ , one finds  $m_1 \simeq m_2 < m_3$ , i.e., spectrum with "partial mass hierarchy"; interpolates between H and QD.
5. for  $\sqrt{\Delta m_{32}^2} \ll \sqrt{\Delta m_{21}^2} \sim \mathcal{O}(m_1)$ , we have  $m_1 < m_2 \simeq m_3$ , i.e., spectrum with "partial inverted mass hierarchy"; interpolates between IH and QD.

Given the values of  $\Delta m_{\odot}^2$ ,  $\theta_{\odot}$ ,  $\Delta m_{\text{atm}}^2$  and of  $\theta$ ,

$$|\langle m \rangle| = |\langle m \rangle| (m_1, \alpha_{21}, \alpha_{31}; S), \quad S = H, IH$$

The knowledge of  $m_1$  would allow to determine the neutrino mass spectrum.

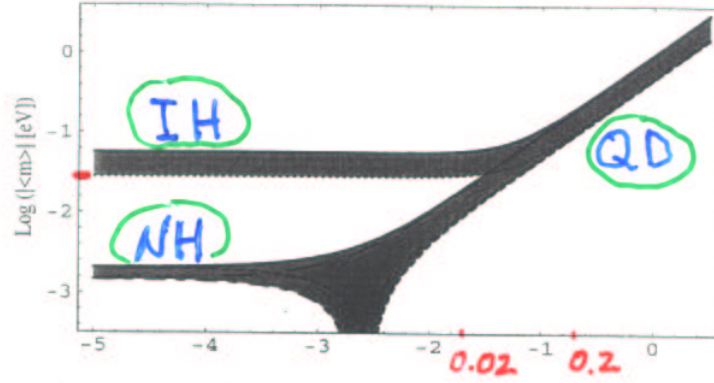
GIVEN THE VALUES OF  $\Delta m_{atm}^2$  AND  $\Delta m_{\odot}^2$ ,  
INFERRED FROM THE DATA, ONE HAS:

-  $m_1 \ll 0.02 \text{ eV}$  - HIERARCHICAL OR  
INVERTED HIERARCHY

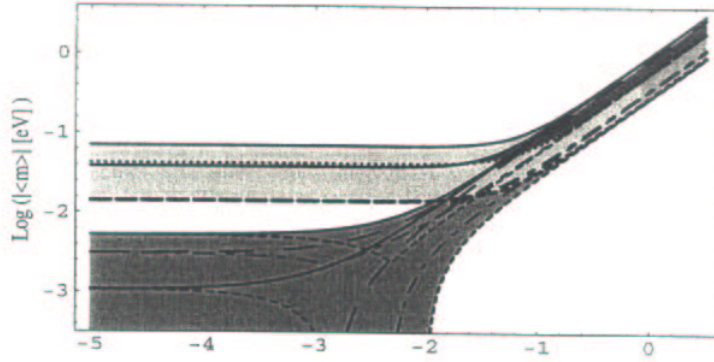
-  $0.02 \lesssim m_1 \lesssim 0.2 \text{ eV}$  - PH OR PIH

-  $m_1 > 0.2 \text{ eV}$  - QD

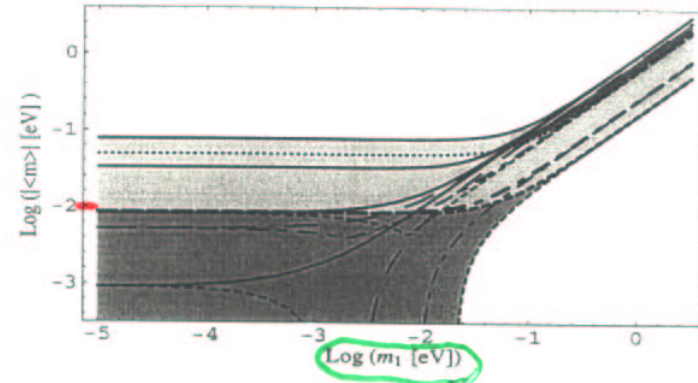
∇ - MASS SPECTRUM



B.F.V.



90% C.L.



99.73% C.L.

Figure 3: The dependence of  $|\langle m \rangle|$  on  $m_1$  in the case of the LMA solution, for  $\Delta m_{\odot}^2 = \Delta m_{21}^2$  and  $\Delta m_{\odot}^2 = \Delta m_{32}^2$ , and for the best fit values (upper panel) and the 90% C.L. allowed values (middle panel) of the neutrino oscillation parameters found in refs. [1, 41]. The lower panel is obtained by using the 99.73% C.L. allowed values of  $\Delta m_{\odot}^2$  and  $\cos 2\theta_{\odot}$  from [1] and the 99% C.L. allowed values of  $\Delta m_{\text{atm}}^2$  and  $\sin^2 \theta$  from [41] (the latter article does not include results at 99.73% C.L.). In the case of CP-conservation, the allowed values of  $|\langle m \rangle|$  are constrained to lie: for i)  $\Delta m_{\odot}^2 = \Delta m_{21}^2$  and the middle and lower panels (upper panel) - in the medium-grey and light-grey regions a) between the two lower thick solid lines (on the lower thick solid line) if  $\eta_{21} = \eta_{31} = 1$ , b) between the two long-dashed lines and the axes (on the long-dashed line) if  $\eta_{21} = -\eta_{31} = 1$ , c) between the two thick dash-dotted lines and the axes (on the dash-dotted lines) if  $\eta_{21} = -\eta_{31} = -1$ , d) between the three thick short-dashed lines and the axes (on the short-dashed lines) if  $\eta_{21} = \eta_{31} = -1$ ; and for ii)  $\Delta m_{\odot}^2 = \Delta m_{32}^2$  and the middle and lower panels (upper panel) - in the light-grey regions a) between the two upper thick solid lines (on the upper thick solid line) if  $\eta_{21} = \eta_{31} = \pm 1$ , b) between the dotted and the doubly-thick short-dashed lines (on the dotted line) if  $\eta_{21} = -\eta_{31} = -1$ , c) between the dotted and the doubly-thick dash-dotted lines (on the dotted line) if  $\eta_{21} = -\eta_{31} = +1$ . In the case of CP-violation, the allowed regions for  $|\langle m \rangle|$  cover all the grey regions. Values of  $|\langle m \rangle|$  in the dark grey regions signal CP-violation.

V. BARGER ET AL.:

'NO-GO FOR CP-VIOLATION IN  $(\beta\beta)_{0\nu}$ -DECAY"

TOO PESSIMISTIC... (PASCOLI, S.T.P.)

CONSIDER, FOR EXAMPLE,

IH SPECTRUM ( $m_i < 0.02$  eV),  $|U_{ei}|^2$ -NEGLECTIBLE

$$\sqrt{\Delta m_{atm}^2} \cos 2\theta_{\odot} \leq |\langle m \rangle| \leq \sqrt{\Delta m_{atm}^2}$$

'JUST CP-VIOLATING' REGION:

$$(|\langle m \rangle|)_{\text{MAX}} < \sqrt{(\Delta m_{atm}^2)_{\text{MIN}}}$$

$$(|\langle m \rangle|)_{\text{MIN}} > \sqrt{(\Delta m_{atm}^2)_{\text{MAX}}} (\cos 2\theta_{\odot})_{\text{MAX}}$$

$$|\langle m \rangle| = \xi \left[ (|\langle m \rangle|_{\text{EXP}})_{\text{MIN}} \pm \Delta \right]$$

OBTAINED FOR THE MAXIMAL VALUE OF THE CORRESPONDING  $M_{\text{NUCLEAR}}$

$$\xi \geq 1.$$

NECESSARY CONDITION FOR ESTABLISHING CP-VIOLATION:

$$\xi < \frac{\sqrt{(\Delta m_{atm}^2)_{\text{MIN}}}}{\sqrt{(\Delta m_{atm}^2)_{\text{MAX}}} (\cos 2\theta_{\odot})_{\text{MAX}} + 2\Delta} \cong \frac{1}{(\cos 2\theta_{\odot})_{\text{MAX}}}$$

CP - VIOLATION CAN BE ESTABLISHED PROVIDED :

- $|\langle m \rangle|$  IS MEASURED WITH AN ERROR  $\lesssim (15-20)\%$ ;
- $\left. \begin{array}{l} \Delta m_{atm}^2 \text{ (IH)} \\ m_{\nu_e}^2 \text{ (QD)} \end{array} \right\}$  IS MEASURED WITH AN ERROR  $\lesssim 10\%$
- $\tan^2 \theta_{12} \gtrsim 0.55$
- $\left. \begin{array}{l} \alpha_{32} \text{ (IH)} \\ \alpha_{21} \text{ (QD)} \end{array} \right\}$  LIE IN THE INTERVALS  $\sim (\pi/2 - 3\pi/4)$  OR  $(5\pi/4 - 3\pi/2)$
- $\xi < 2$

(PASCOLI, PETCOV, RODEJOHANNI  
HEP-PH/0209059)

## CONCLUSIONS

WE ARE AT THE BEGINNING OF THE ERA OF HIGH PRECISION MEASUREMENTS OF THE  $\nu$ -MIXING PARAMETERS

THE STRATEGIES TO BE FOLLOWED DEPEND CRUCIALLY ON THE RESULTS THE KAMLAND EXPERIMENT WILL OBTAIN

IF LMA SOLUTION CONFIRMED,

-  $\Delta m_{21}^2, \theta_{21}$

- OR IF  $\Delta m_{21}^2 \approx 2 \cdot 10^{-4} \text{ eV}^2$ ,

POST KAMLAND AT  $L \sim 20 \text{ km}$ :  $\Delta m_{31}^2, \theta_{13}, \text{NH VERSUS IH}$  (VERY CHALLENGING)

- OPENS THE POSSIBILITY TO SEARCH FOR CP-VIOLATION IN  $\nu$ -OSCILLATIONS ( $\delta$ )

-  $\Delta m_{\text{atm}} = \Delta m_{31}^2$  - MINOS

-  $\theta_{13}$ : SK (HK, UNO)

SUPERBEAMS %  $|\Delta m_{\text{atm}}^2|, \theta_{23}, \theta_{13}, \delta$

$\nu$ -FACTORIES %  $\Delta m_{\text{atm}}^2, \theta_{23}, \theta_{13}, \delta$

$(\beta\beta)_{0\nu}$  - DECAY EXPERIMENTS:

- MAJORANA NATURE OF  $\nu_j$
- TYPE OF SPECTRUM (NH, IH, QD)
- ABSOLUTE  $\nu$ -MASS SCALE

!  ${}^3\text{H}$   $\beta$ -DECAY, COSMOLOGY...