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12 OCT 2002

SYMMETRIES OF NEUTRINO MIXING

1) TRI-BIMAXIMAL MIXING...

HPS hep-ph/0202074 PLB530 (2002) 167

$$\left(|U_{l\nu}|^2\right) = \begin{array}{c} \\ e \\ \mu \\ \tau \end{array} \begin{array}{ccc} \nu_1 & \nu_2 & \nu_3 \\ \multicolumn{3}{c}{\begin{pmatrix} 2/3 & 1/3 & 0 \\ 1/6 & 1/3 & 1/2 \\ 1/6 & 1/3 & 1/2 \end{pmatrix}} \end{array}$$

2) SYMMETRIES AND GENERAL...

HS hep-ph/0203209 PLB535 (2002) 163.

i) ZERO CP VIOL. $J = 0$

ii) μ-τ REFLECTION SYM

$$|U_{\mu i}| = |U_{\tau i}|$$

iii) ν_2-TRIMAXIMAL

$$|U_{e2}| = |U_{\mu 2}| = |U_{\tau 2}| = \frac{1}{\sqrt{3}}$$

Lepton mixing and CP violation-2

I cannot resist quoting a paper I wrote in 1978 (Phys. Lett 72B, 333–335):

"maximal neutrino mixing requires CP violation"

By maximal I mean that all matrix elements of the lepton mixing matrix V_L should have equal size.

Since $V_L V_L^\dagger = 1$ solution to this problem is essentially unique and necessarily complex:

$$\sqrt{3} \times V_L = \begin{vmatrix} 1 & x & x^2 \\ x & x^2 & 1 \\ x^2 & 1 & x \end{vmatrix}; \quad x = \exp(\frac{2\pi i}{3})$$

" We are probably far from this solution, but not very far. . . . "

TRIMAX $\rightarrow$ TRI-BIMAX

$$\omega = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$$

$$\bar{\omega} = -\frac{1}{2} - i\frac{\sqrt{3}}{2}$$

$$\nu_1 \quad + \quad \nu_3 \quad \rightarrow \quad \nu_1'$$

$$\begin{pmatrix} 1 \\ \omega \\ \bar{\omega} \end{pmatrix} + \begin{pmatrix} 1 \\ \bar{\omega} \\ \omega \end{pmatrix} \rightarrow \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}$$

$$\nu_1 \quad - \quad \nu_3 \quad \rightarrow \quad \nu_3'$$

$$\begin{pmatrix} 1 \\ \omega \\ \bar{\omega} \end{pmatrix} - \begin{pmatrix} 1 \\ \bar{\omega} \\ \omega \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

HS hep-ph/9406351 PLB 333 (1994) 471

CIRCULANT MASS MATRIX:

$$a = \frac{1}{3} m_1^2 + \frac{1}{3} m_2^2 + \frac{1}{3} m_3^2$$

$$b = \frac{1}{3} m_1^2 + \frac{\omega}{3} m_2^2 + \frac{\varpi}{3} m_3^2 \qquad \omega = \exp(+i2\pi/3)$$

$$\varpi = \exp(-i2\pi/3)$$

$$b^* = \frac{1}{3} m_1^2 + \frac{\varpi}{3} m_2^2 + \frac{\omega}{3} m_3^2$$

$$U^+ \quad \left(M_\nu^2 := \; M_\nu M_\nu^+ \right) \quad U$$

$$\begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{\varpi}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{\omega}{\sqrt{3}} \\ \frac{\omega}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{\varpi}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} a & b & b^* \\ b^* & a & b \\ b & b^* & a \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{\omega}{\sqrt{3}} & \frac{\varpi}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{\varpi}{\sqrt{3}} & \frac{\omega}{\sqrt{3}} \end{pmatrix}$$

$$\Updownarrow$$

$$\begin{pmatrix} m_1^2 & & \\ & m_2^2 & \\ & & m_3^2 \end{pmatrix}$$

EXCLUDED:

$$\begin{pmatrix} 1/2 & 1/2 & 0 \\ 1/4 & 1/4 & 1/2 \\ 1/4 & 1/4 & 1/2 \end{pmatrix}$$

BIMAXIMAL

V. Barger et al.
PLB 437 (1998) 107
hep-ph/9806387

$$\begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$$

TRIMAXIMAL

HPS/HS
PLB 349 (1995) 137
PLB 333 (1994) 471
hep-ph/9406351

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ 1/6 & 1/6 & 2/3 \\ 1/3 & 1/3 & 1/3 \end{pmatrix}$$

DEMOCRATIC

H. Fritzsch, Z. Xing
PLB 372 (1996) 265
hep-ph/9509389

MIXING MATRIX

$$|U|^2 \simeq \begin{array}{c|ccc|} & \nu_1 & \nu_2 & \nu_3 \\ \hline e & & 1/3 & \varepsilon^2 \\ \mu & & & 1/2 \\ \tau & & & \end{array}$$

SNO/SK → SNO $\simeq 0.34 ^{+.05}_{-.04}$

$|e2|^2 \approx 0.32 ^{+0.10}_{-0.06}$

CHOOZ

$|e3|^2 \cong \varepsilon^2$

$\varepsilon^2 \lesssim 0.03$ 95%CL

SUPER-K

$|\mu 3|^2 \simeq \frac{1}{2} \pm \sqrt{\frac{2P-1}{2}}$

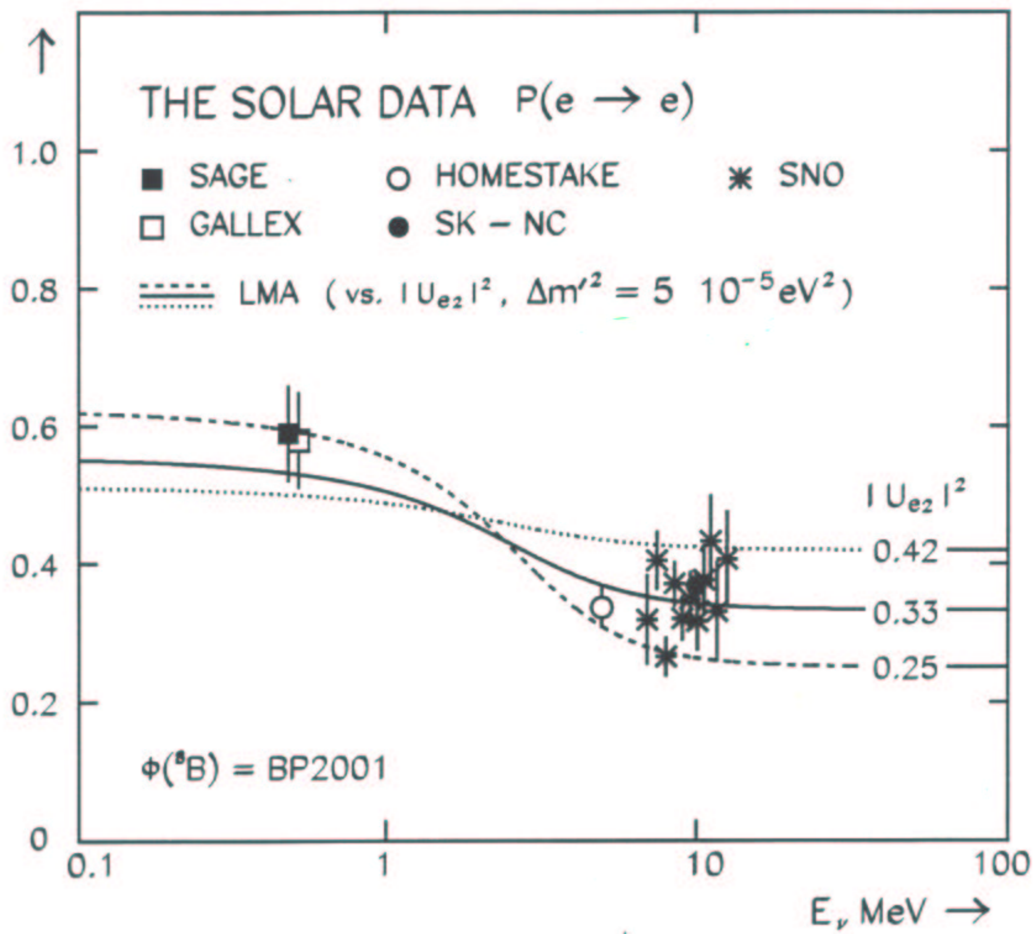
$\simeq 0.50 \pm 0.14$

SOLAR DATA

WRT BP2001 :-

SNO $\simeq$ 0.347 $\pm$ 0.029

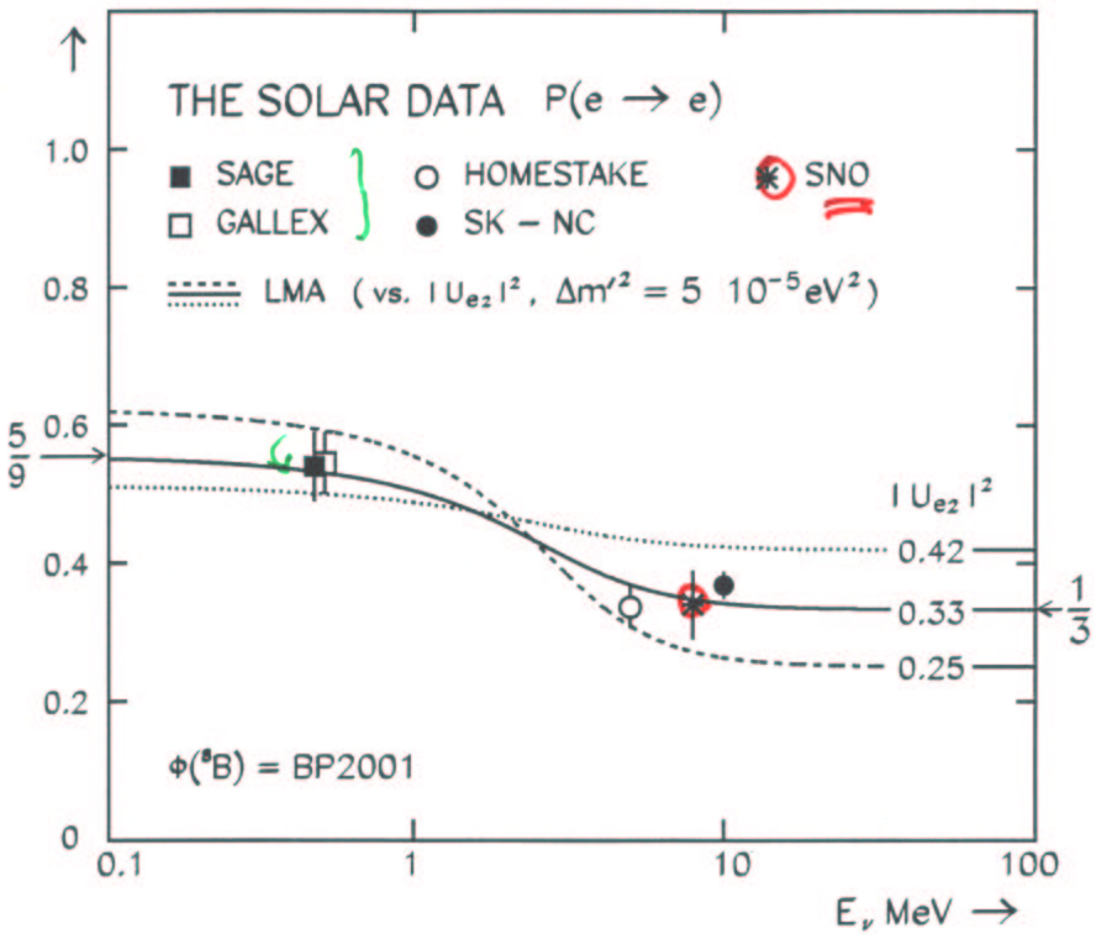
HOME $\simeq$ 0.337 $\pm$ 0.030



FLUX INDPT :-

$$\frac{SNO}{SK-SNO} \simeq 0.32 \begin{array}{l} +0.10 \\ -0.06 \end{array}$$

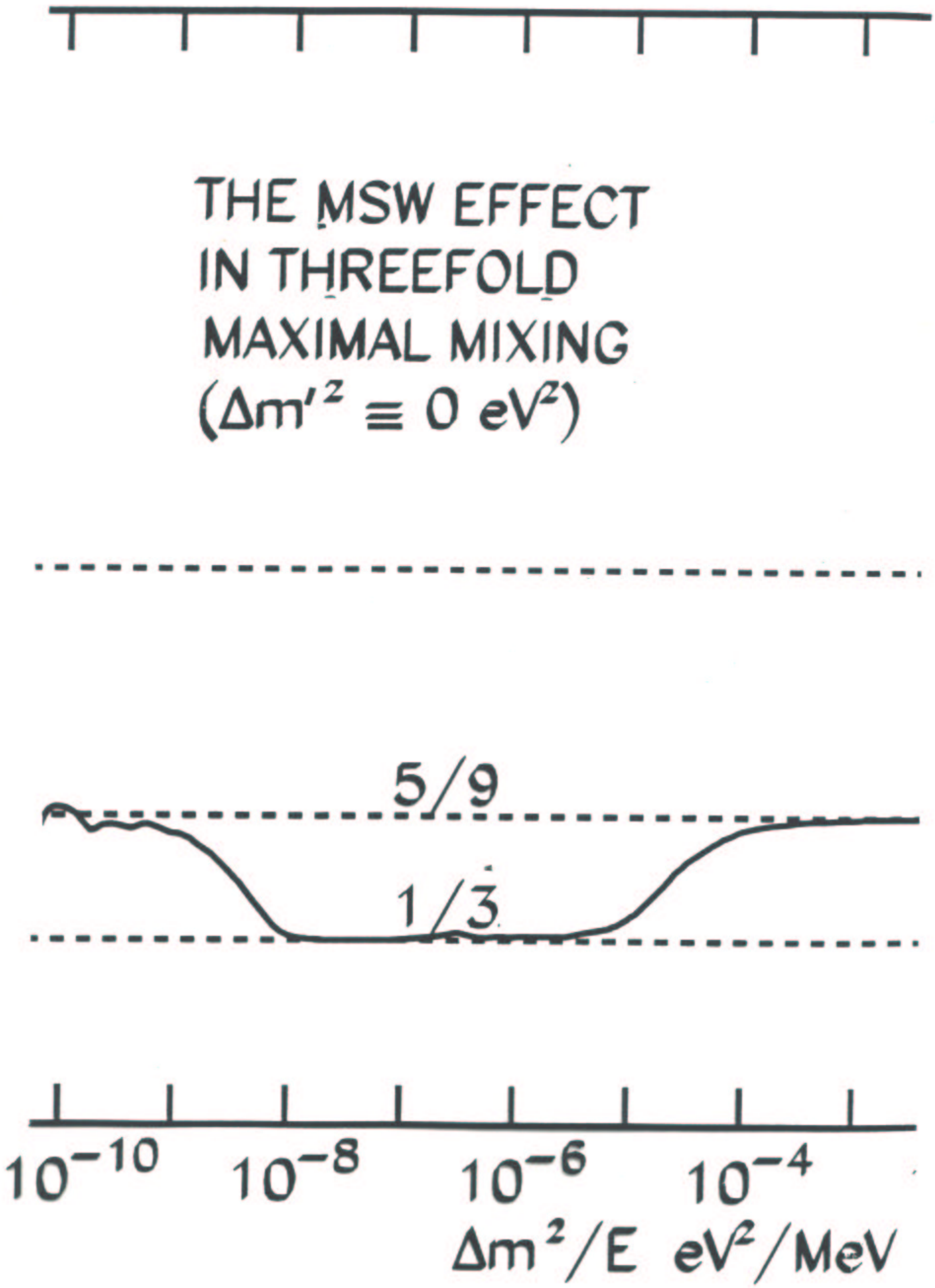
SOLAR DATA *APRIL 2002



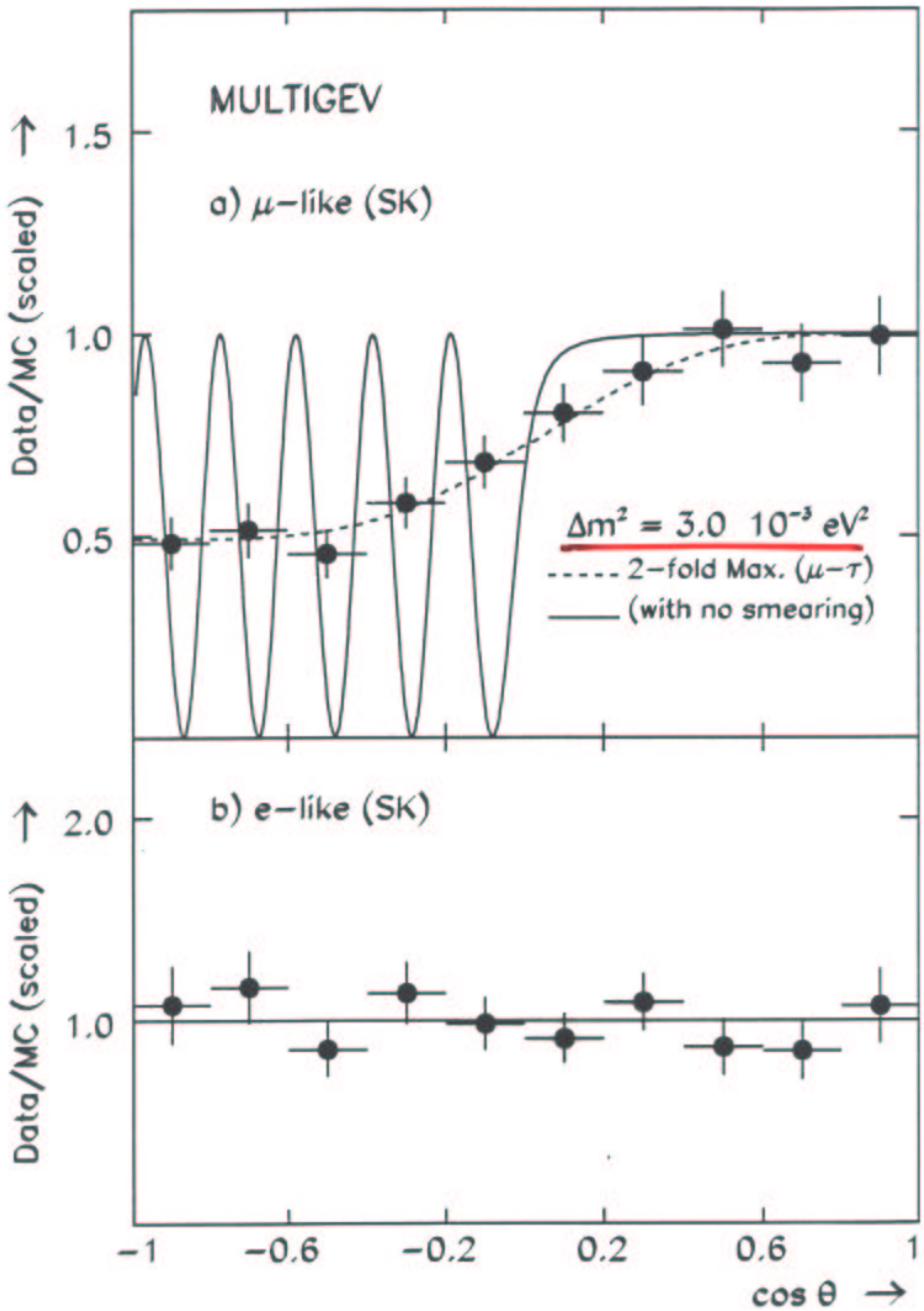
FLUX INDPT :-

$$\frac{\text{SNO-CC}}{\text{SNO-NC}} \simeq \underline{0.34^{+0.05}_{-0.04}}$$

HPS PLB 374(1996) 111
hep-ph/9601346



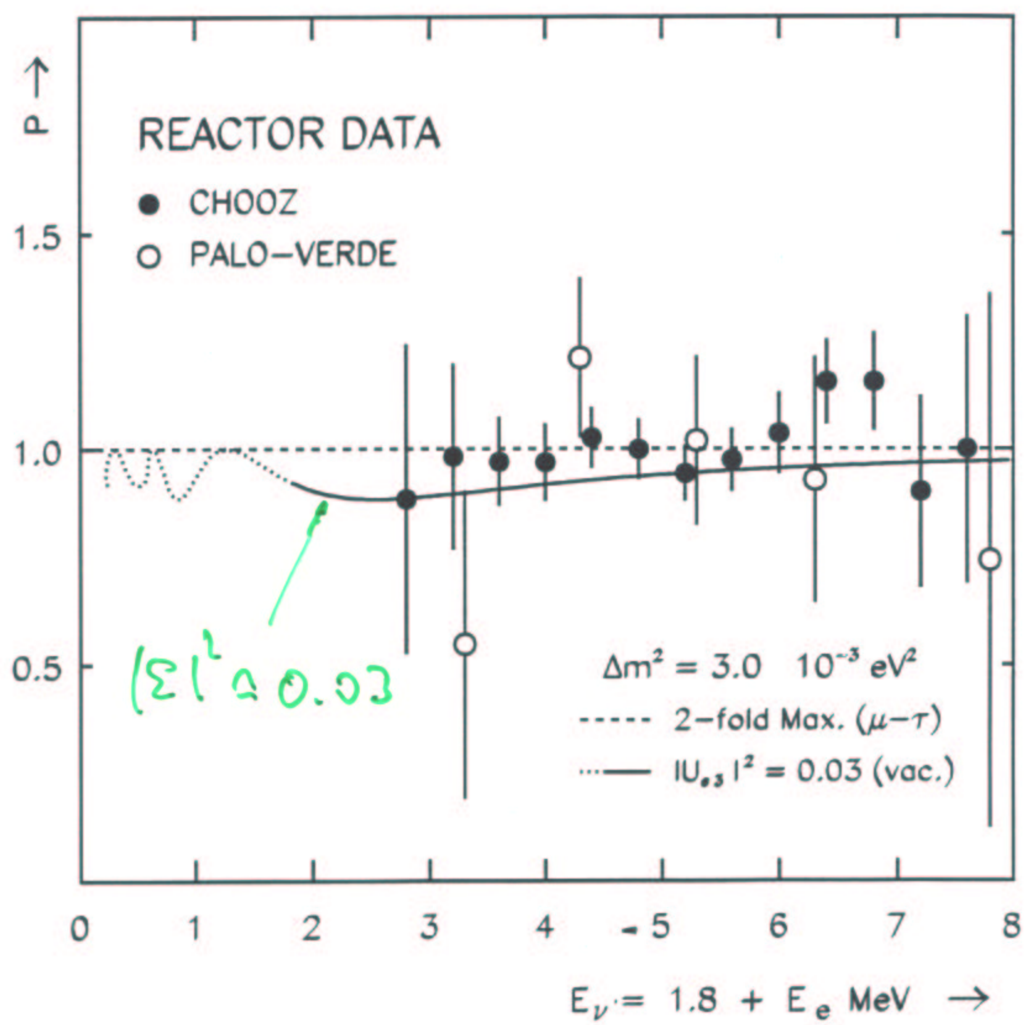
SUPER-K (ATMOS)



$$P(\mu \to \mu) \simeq 0.50 \pm 0.04 \ (?)$$

REACTOR DATA

CHOOZ
PALO-VERDE.



$$|U_{e3}|^2 \equiv |\varepsilon|^2$$

$$|\varepsilon|^2 \lesssim 0.03 \quad 95\%\ CL$$

K2K

G Fogli et al.
hep-ph/0110089

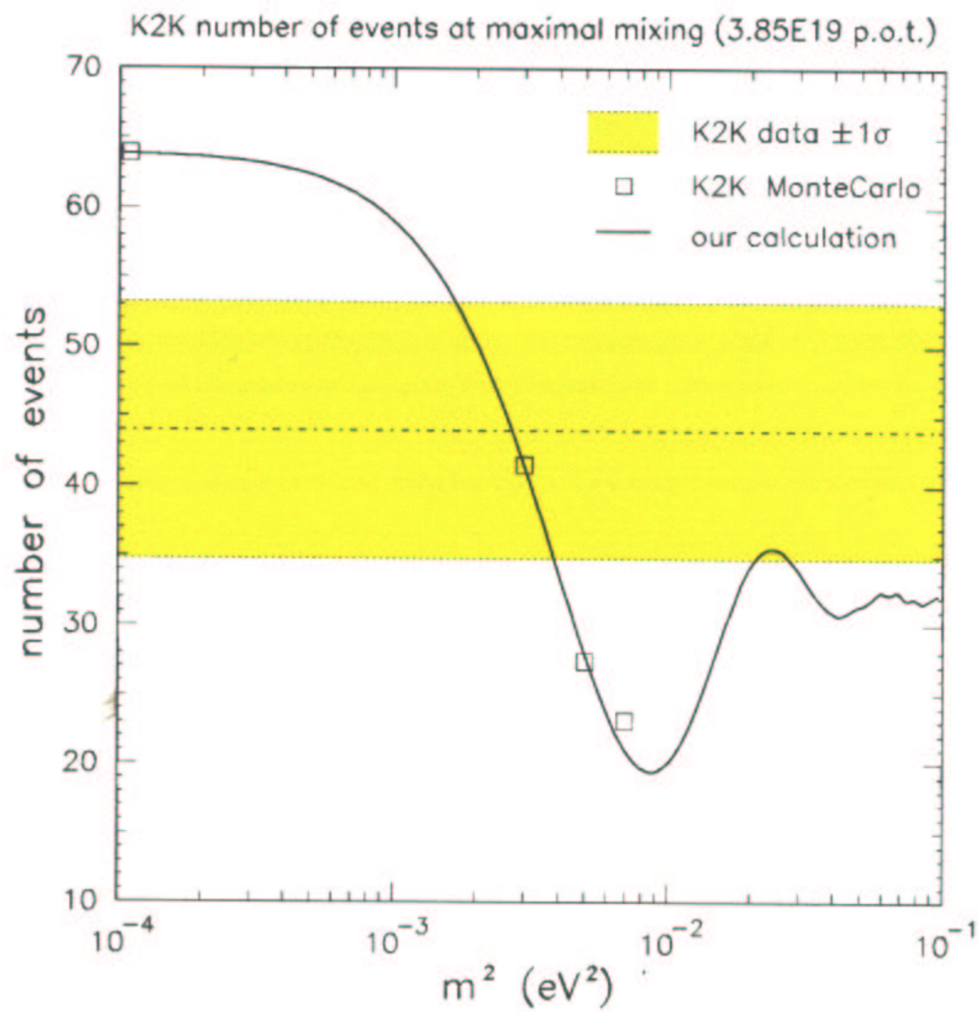


FIG. 1. Number of events in K2K for the current exposure (3.85×10^{19} p.o.t.), as a function of m^2 (at $\tan^2 \psi = 1$). Our calculation (solid curve) is benchmarked by the K2K MC simulation (square markers). The horizontal gray band represents the current K2K data within one standard deviation.

TRI-BIMAXIMAL MIXING

$$|U|^2 \simeq \begin{array}{c|ccc} & \nu_1 & \nu_2 & \nu_3 \\ \hline e & 2/3 & 1/3 & 0 \\ \mu & 1/6 & 1/3 & 1/2 \\ \tau & 1/6 & 1/3 & 1/2 \end{array}$$

NB. $|\Sigma|^2 \equiv 0 \to \underline{\underline{J \equiv 0}}$

HS hep-ph/0203209 PLB 535 (2002) 163.

THE `CIRCULANT' BASIS:

$$a = \frac{1}{3} m_e^2 + \frac{1}{3} m_\mu^2 + \frac{1}{3} m_\tau^2$$

$$b = \frac{1}{3} m_e^2 + \frac{\omega}{3} m_\mu^2 + \frac{\varpi}{3} m_\tau^2 \qquad \begin{aligned} \omega &= \exp(+i2\pi/3) \\ \varpi &= \exp(-i2\pi/3) \end{aligned}$$

$$b^* = \frac{1}{3} m_e^2 + \frac{\varpi}{3} m_\mu^2 + \frac{\omega}{3} m_\tau^2$$

$$U^+ \left(M_l^2 := M_l \; M_l^+ \right) U$$

$$\begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{\varpi}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{\omega}{\sqrt{3}} \\ \frac{\omega}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{\varpi}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} a & b & b^* \\ b^* & a & b \\ b & b^* & a \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{\omega}{\sqrt{3}} & \frac{\varpi}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{\varpi}{\sqrt{3}} & \frac{\omega}{\sqrt{3}} \end{pmatrix}$$

$$\Updownarrow$$

$$\begin{pmatrix} m_e^2 & & \\ & m_\mu^2 & \\ & & m_\tau^2 \end{pmatrix}$$

See also:

HS hep-ph/9406351 PLB 333 (1994) 471.

3 x 3 CIRCULANT INVARIANT
UNDER CYCLIC (C3) PERMS.

$$P^{\dagger} \qquad \left(M_l^2 \right) \qquad P$$

$$\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} a & b & b^* \\ b^* & a & b \\ b & b^* & a \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\Downarrow$$

$$\begin{pmatrix} a & b & b^* \\ b^* & a & b \\ b & b^* & a \end{pmatrix}$$

THE NEUTRINO MASS MATRIX:

$$x = \frac{1}{2} m_1^2 + \frac{1}{2} m_3^2$$

$$y = \frac{1}{2} m_1^2 - \frac{1}{2} m_3^2$$

$$z = m_2^2$$

(IN THE
(CHARGED-LEPTON)
CIRCULANT
BASIS).

$$U^+ \left(M_\nu^2 := M_\nu \; M_\nu^+ \right) U$$

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{-1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} x & 0 & y \\ 0 & z & 0 \\ y & 0 & x \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{-1}{\sqrt{2}} \end{pmatrix}$$

$$\Updownarrow$$

$$\begin{pmatrix} m_1^2 & & \\ & m_2^2 & \\ & & m_3^2 \end{pmatrix}$$

2 x 2 CIRCULANT INVARIANT UNDER CYCLIC (C2) PERMS.

$$P^{\dagger} \qquad \left(M_l^2\right) \qquad P$$

$$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} x & 0 & y \\ 0 & z & 0 \\ y & 0 & x \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\Downarrow$$

$$\begin{pmatrix} x & 0 & y \\ 0 & z & 0 \\ y & 0 & x \end{pmatrix}$$

TRI-BIMAXIMAL MIXING:

$$
\begin{array}{c} \\ e \\ \mu \\ \tau \end{array}
\begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{\varpi}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{\omega}{\sqrt{3}} \\ \frac{\omega}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{\varpi}{\sqrt{3}} \end{pmatrix}
\begin{array}{c} \nu_1 \quad \nu_2 \quad \nu_3 \\ \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{-1}{\sqrt{2}} \end{pmatrix} \end{array}
=
\begin{array}{cc} & \nu_1 \quad \nu_2 \quad \nu_3 \\ \begin{array}{c} e \\ \mu \\ \tau \end{array} & \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-i}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{i}{\sqrt{2}} \end{pmatrix} \end{array}
$$

TRI -ϕ MAXIIMAL MIXING

$$M_\nu{}^2 = \begin{pmatrix} x & 0 & y^* \\ 0 & z & 0 \\ y & 0 & x \end{pmatrix}$$

$$x = \frac{1}{2}m_1^2 + \frac{1}{2}m_3^2$$

$$|y| = \frac{1}{2}m_3^2 - \frac{1}{2}m_1^2$$

$$y = -|y|\exp(i2\phi)$$

$$z = m_2^2$$

$$U(\phi) =$$

$$\begin{array}{cc}
 & \begin{array}{ccc} \qquad\nu_1\qquad & \qquad\nu_2\qquad & \qquad\nu_3\qquad \end{array} \\
\begin{array}{c} e \\[2ex] \mu \\[2ex] \tau \end{array} &
\begin{pmatrix}
\sqrt{\frac{2}{3}}\cos\phi & \frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}}\sin\phi \\
-\frac{\cos\phi}{\sqrt{6}} - \frac{\sin\phi}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{\cos\phi}{\sqrt{2}} - \frac{\sin\phi}{\sqrt{6}} \\
-\frac{\cos\phi}{\sqrt{6}} + \frac{\sin\phi}{\sqrt{2}} & \frac{1}{\sqrt{3}} & -\frac{\cos\phi}{\sqrt{2}} - \frac{\sin\phi}{\sqrt{6}}
\end{pmatrix}
\end{array}$$

$J = 0$

TRI -χ MAXIIMAL MIXING

$$M_\nu{}^2 = \begin{pmatrix} x & 0 & y \\ 0 & z & 0 \\ y & 0 & w \end{pmatrix}$$

$$x = \frac{m_1^2 + m_3^2}{2} + \frac{m_1^2 - m_3^2}{2}\sin 2\chi$$

$$w = \frac{m_1^2 + m_3^2}{2} - \frac{m_1^2 - m_3^2}{2}\sin 2\chi$$

$$y = \frac{m_1^2 - m_3^2}{2}\cos 2\chi$$

$$z = m_2^2$$

$$U(\chi) =$$

$$\begin{array}{cc} & \begin{array}{ccc} \nu_1 & \nu_2 & \nu_3 \end{array} \\ \begin{array}{c} e \\ \mu \\ \tau \end{array} & \begin{pmatrix} \sqrt{\frac{2}{3}}\cos\chi & \frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}}\sin\chi \\ -\frac{\cos\chi}{\sqrt{6}} - i\frac{\sin\chi}{\sqrt{2}} & \frac{1}{\sqrt{3}} & i\frac{\cos\chi}{\sqrt{2}} - \frac{\sin\chi}{\sqrt{6}} \\ -\frac{\cos\chi}{\sqrt{6}} + i\frac{\sin\chi}{\sqrt{2}} & \frac{1}{\sqrt{3}} & -i\frac{\cos\chi}{\sqrt{2}} - \frac{\sin\chi}{\sqrt{6}} \end{pmatrix} \end{array}$$

$$J = \frac{1}{6\sqrt{3}} \sin 2\chi$$

$\searrow$ INTERPOLATES

TRI $\chi\phi$ MAXIIMAL MIXING

$$M_\nu{}^2 = \begin{pmatrix} x & 0 & y^* \\ 0 & z & 0 \\ y & 0 & w \end{pmatrix}$$

$$x = \frac{m_1^2 + m_3^2}{2} + \frac{m_1^2 - m_3^2}{2}\sin 2\chi$$

$$w = \frac{m_1^2 + m_3^2}{2} - \frac{m_1^2 - m_3^2}{2}\sin 2\chi$$

$$y = \frac{m_1^2 - m_3^2}{2}\cos 2\chi \quad \exp(i2\phi)$$

$$z = m_2^2$$

$$U(\chi,\phi) =$$

$$\begin{array}{cc} & \begin{array}{ccc} \nu_1 & \nu_2 & \nu_3 \end{array} \\ \begin{array}{c} e \\ \mu \\ \tau \end{array} & \begin{pmatrix} \sqrt{\frac{2}{3}}c_\chi c_\phi + i\sqrt{\frac{2}{3}}s_\chi s_\phi & \frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}}s_\chi c_\phi + i\sqrt{\frac{2}{3}}c_\chi s_\phi \\ -\frac{c_\chi c_\phi - i s_\chi s_\phi}{\sqrt{6}} - \frac{c_\chi s_\phi + i s_\chi c_\phi}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{i c_\chi c_\phi - s_\chi s_\phi}{\sqrt{2}} - \frac{s_\chi c_\varepsilon + i c_\chi s_\phi}{\sqrt{6}} \\ -\frac{c_\chi c_\phi - i s_\chi s_\phi}{\sqrt{6}} + \frac{c_\chi s_\phi + i s_\chi c_\phi}{\sqrt{2}} & \frac{1}{\sqrt{3}} & -\frac{i c_\chi c_\phi - s_\chi s_\phi}{\sqrt{2}} - \frac{s_\chi c_\phi + i c_\chi s_\phi}{\sqrt{6}} \end{pmatrix} \end{array}$$

$c_\chi = \cos\chi;$ $\quad c_\phi = \cos\phi$

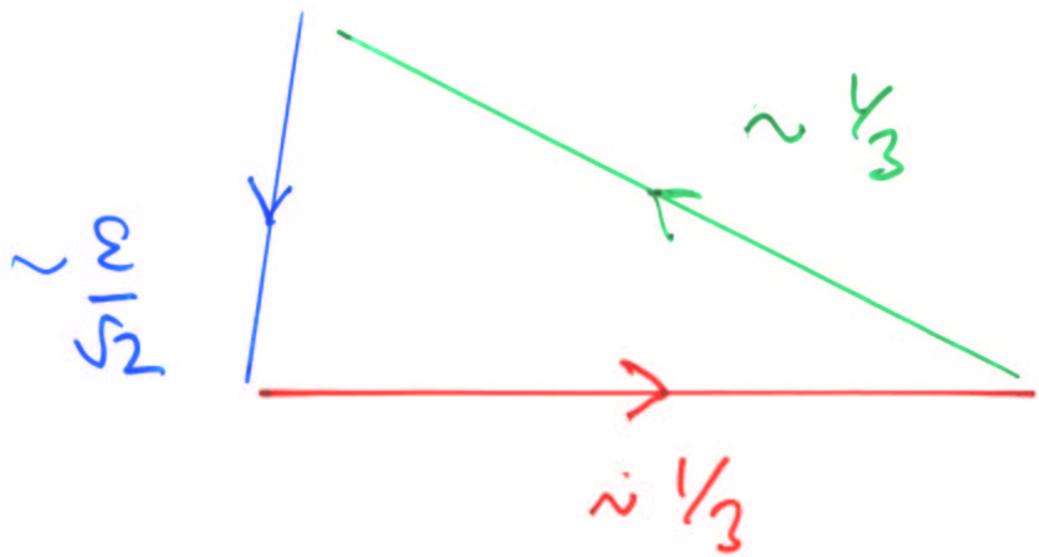
$s_\chi = \sin\chi;$ $\quad s_\phi = \sin\phi$

$J = \frac{1}{6\sqrt{3}} \sin 2\chi$

$e \to \mu$ UNITARITY TRIANGLE

$$|\varepsilon|^2 < 0.03$$

$$|\varepsilon| < 0.17$$



$$|J| \lesssim \frac{|\varepsilon|}{3\sqrt{2}}$$

$$\lesssim 4 \times 10^{-2}$$

TO EXPONENTIATE A MATRIX:

DIAGONALISE:

$$\left(U^{\dagger}\right)\left(M^{2}\right)\left(U\right)=\begin{pmatrix} m_1^2 & & \\ & m_2^2 & \\ & & m_3^2 \end{pmatrix}$$

EXPONENIATE:

$$\begin{pmatrix} m_1^2 & & \\ & m_2^2 & \\ & & m_3^2 \end{pmatrix} \rightarrow \begin{pmatrix} e^{-im_1^2L/2E} & & \\ & e^{-im_2^2L/2E} & \\ & & e^{-im_3^2L/2E} \end{pmatrix}$$

UN-DIAGONLISE

$$\left(U\right)\begin{pmatrix} e^{-im_1^2L/2E} & & \\ & e^{-im_2^2L/2E} & \\ & & e^{-im_3^2L/2E} \end{pmatrix}\left(U^{\dagger}\right)$$

$$=\left(e^{-iHt}\right)$$

$\mu-\tau$ REFLECTION SYMMETRY

HS : RAL-TR-2002-023

$(\equiv \ \mu \leftrightarrow \tau \ + \ CP \ )$

FLAVOUR BASIS:

$$M^2 = \begin{array}{c} \\ e \\ \mu \\ \tau \end{array} \!\!\! \begin{array}{c} \begin{array}{ccc} e & \mu & \tau \end{array} \\ \begin{pmatrix} z & w & w^* \\ w^* & x & y \\ w & y^* & x \end{pmatrix} \end{array}$$

$$\Downarrow$$

$$x = m_1^2 + (m_2^2 - m_1^2)\,|V_2|^2 + (m_3^2 - m_1^2)\,|V_3|^2$$

$$y = (m_2^2 - m_1^2)V_2^2 + (m_3^2 - m_1^2)V_3^2$$

$$z = m_1^2 + (m_2^2 - m_1^2)U_2^2 + (m_3^2 - m_1^2)U_3^2$$

$$w = (m_2^2 - m_1^2)U_2V_2^* + (m_3^2 - m_1^2)U_3V_3^*$$

$$U(U_1, U_2) = \begin{array}{c} \\ e \\ \mu \\ \tau \end{array} \!\!\! \begin{array}{c} \begin{array}{ccc} \nu_1 & \nu_2 & \nu_3 \end{array} \\ \begin{pmatrix} U_1 & U_2 & U_3 \\ V_1 & V_2 & V_3 \\ V_1^* & V_2^* & V_3^* \end{pmatrix} \end{array}$$

$\mu - \tau$

REFLECTION $(\equiv\ \mu \leftrightarrow \tau\ +\ CP\ )$

SYMMETRY

$$M_\nu^2 = \begin{pmatrix} \text{REAL} \end{pmatrix}$$

(CIRCULANT BASIS)

DIAGAGONALISED BY ORTHOG: O

$$O^+ M_\nu^2 O \rightarrow \begin{pmatrix} m_1^2 & & \\ & m_2^2 & \\ & & m_3^2 \end{pmatrix}$$

$$\Downarrow$$

$$U(U_1, U_2) = \begin{array}{c c} & \begin{array}{ccc} \nu_1 & \nu_2 & \nu_3 \end{array} \\ \begin{array}{c} e \\ \mu \\ \tau \end{array} & \begin{pmatrix} U_1 & U_2 & U_3 \\ V_1 & V_2 & V_3 \\ V_1^* & V_2^* & V_3^* \end{pmatrix} \end{array}$$

$(U_i\ REAL)$

CONCLUSIONS

1) TRI-MAX **EXCLUDED**

2) TRI-BIMAX **OK**
(WS hep-ph/9909481)

3) SYMMETRIES IN
CIRCULANT BASIS:
(FOR FLAVOUR BASIS: hep-ph/0204049.
Z. Xing PLB 533 (2002) 55)

TEXTURE ZEROS $\rightarrow$ ν_2 TRIMAX
TRI $\chi\phi$ MAX MIX.

REAL $\rightarrow$ μ-τ **REFLECTION** SYMMETRY.

4) TRI-BIMAX IS A VERY CLASSY MIXING MATRIX !!